

## Photodisintegration of Helium\*†

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In this paper, we extend the sum-rule calculations of Levinger and Bethe to include two-body Heisenberg and tensor forces. We apply these sum rules to calculate for the alpha particle the bremsstrahlung-weighted cross section ( $\sigma_b$ ) and the integrated cross section ( $\sigma_{\text{int}}$ ) using Irving's wave function for tensor force. We compare our results with values found from experiments integrated to 150 Mev. The experimental values are  $\sigma_b = 2.7$  mb and  $\sigma_{\text{int}} = 124$  Mev-mb, while the theoretical values are found to be  $\sigma_b = 1.23$  mb and  $\sigma_{\text{int}} = 60[1 + 0.70(x + \frac{1}{2}y) + 0.18(x' + \frac{1}{2}y')]$  Mev-mb. Here  $x$  and  $x'$  are the fractions of central and tensor potentials, respectively, that have Majorana exchange character;  $y$  and  $y'$  represent fractions of the central and tensor potentials that have Heisenberg exchange character. The low theoretical values for  $\sigma_b$  may be related to the small root-mean-square radius of the alpha particle given by Irving's wave function. The photodisintegration experiments seem to support Hofstadter's value for the size of the alpha particle, when one takes account of the proton size in his electron-scattering experiments.

### I. INTRODUCTION

THE cross section and angular distribution of the protons produced in the photodisintegration of the deuteron,<sup>1,2</sup> when compared with the experiments, serve to test the basic theory of photonuclear reactions. An over-all check of these calculations can be made by sum-rule calculations on the photodisintegration of the deuteron.<sup>3</sup> Calculations on the photodisintegration of the He<sup>4</sup> nucleus have been made by Flowers and Mandl<sup>4</sup> and by Gunn and Irving.<sup>5</sup> Very little is known of the wave function for the ground state of the helium nucleus and much less is known of the wave function for the excited states. Further, use of a plane wave to describe the motion of the emitted particle in the final state, though justified for the deuteron at moderate energies, seems to be quite dubious for He<sup>4</sup>. In sum-rule calculations, we sum over all excited states and use closure for the matrix elements, so that the results depend only on the wave function assumed for the ground state.

Flowers and Mandl<sup>4</sup> have calculated the  $(\gamma, p)$  cross section using *S*-state Gaussian wave functions to describe the ground state of the  $\alpha$  particle while Gunn and Irving<sup>5</sup> have made similar calculations using both Gaussian and exponential-type wave functions. A comparison of these calculations with experiment indicates that the experimentally measured<sup>6</sup>  $(\gamma, p)$  cross section is in better agreement with the calculations made by Gunn and Irving, using the exponential-type wave

function. In the region above 26 Mev a discrepancy is indicated between the energy dependence given by Gunn and Irving and that given by the experimental curve of Fuller. This discrepancy may be due to the crudeness of the experiment; but may be possibly attributed to the omission of "*D*" terms in the wave function or to the assumption of a free final state. Irving<sup>7</sup> has pointed out the need for the inclusion of a tensor force in the nuclear interaction in order to obtain the correct binding for the helium nucleus. It therefore seems necessary to include a tensor force in the photodisintegration calculations also.

For the following calculations, we have chosen Irving's<sup>7</sup> later wave function, in which Irving has taken tensor two-body forces into consideration and has assumed the ground state wave function to be a mixture of the <sup>1</sup>S<sub>0</sub> and the principal <sup>5</sup>D<sub>0</sub> state. (The possible terms in the He<sup>4</sup> wave functions are *S*, *P*, and *D*, and have been listed by Gerjuoy and Schwinger.<sup>8</sup>)

In Sec. II we calculate the rms radius of the particle using Irving's wave function and compare it with the size of the  $\alpha$  particle measured by Blankenbecler and Hofstadter<sup>9</sup> by electron scattering experiments. In Sec. III we calculate the dipole bremsstrahlung-weighted cross section ( $\sigma_b = \int (\sigma/W) dW$ ), and in Sec. IV the cross section integrated over the photon energy ( $\sigma_{\text{int}} = \int \sigma dW$ ). In Sec. V we compare the results for  $\sigma_b$  and  $\sigma_{\text{int}}$  with experiments, and we discuss the discrepancy between theory and experiments in Sec. VI.

### II. SIZE OF THE ALPHA PARTICLE

The complete wave function for the ground state of an alpha particle representing a mixture of the <sup>1</sup>S<sub>0</sub> and the principal <sup>5</sup>D<sub>0</sub> states may be written in the form<sup>7</sup>:

$$\psi = \frac{1}{(1+C^2)^{\frac{1}{2}}} (\psi_S + C\psi_D), \quad (1)$$

<sup>7</sup> J. Irving, Proc. Phys. Soc. (London) A66, 17 (1953).

<sup>8</sup> E. Gerjuoy and J. Schwinger, Phys. Rev. 61, 138 (1942).

<sup>9</sup> R. Hofstadter, Revs. Modern Phys. 28, 214 (1956).

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<sup>1</sup> L. I. Schiff, Phys. Rev. 78, 733 (1950).

<sup>2</sup> J. F. Marshall and E. Guth, Phys. Rev. 78, 738 (1950).

<sup>3</sup> J. S. Levinger, Phys. Rev. 97, 970 (1955).

<sup>4</sup> B. H. Flowers and F. Mandl, Proc. Roy. Soc. (London) A206, 131 (1951).

<sup>5</sup> J. C. Gunn and J. Irving, Phil. Mag. 42, 1353 (1951).

<sup>6</sup> E. G. Fuller, Phys. Rev. 96, 1306 (1954).

where

$$\psi_S = N_S \exp[-2\alpha(u^2 + v^2 + w^2)^{\frac{1}{2}}], \quad (2)$$

$$\psi_D = N_D \exp[-2\beta(u^2 + v^2 + w^2)^{\frac{1}{2}}] [6(\boldsymbol{\sigma}_1 \cdot \mathbf{v})(\boldsymbol{\sigma}_3 \cdot \mathbf{w}) + 6(\boldsymbol{\sigma}_1 \cdot \mathbf{w})(\boldsymbol{\sigma}_3 \cdot \mathbf{v}) - 4(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3)(\mathbf{v} \cdot \mathbf{w})], \quad (3)$$

$$\mathbf{v} = (\mathbf{r}_2 - \mathbf{r}_1)/2^{\frac{1}{2}}; \quad \mathbf{w} = (\mathbf{r}_4 - \mathbf{r}_3)/2^{\frac{1}{2}},$$

$$\mathbf{u} = \frac{1}{2}(\mathbf{r}_4 + \mathbf{r}_3 - \mathbf{r}_2 - \mathbf{r}_1), \quad (4)$$

$$\mathbf{R} = \frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4),$$

$$N_S^2 = 2^6 \alpha^9 / (3\pi^4); \quad N_D^2 = 2^7 \beta^{13} / (5^2 3^4 \pi^4), \quad (5)$$

and

$$r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2 = 4(u^2 + v^2 + w^2). \quad (6)$$

Here  $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3,$  and  $\mathbf{r}_4$  denote the position vectors of the particles; 1 and 2 denote the neutron and 3, 4 the proton coordinates.  $\mathbf{R}$  stands for the center of gravity of the  $\text{He}^4$  nucleus.  $C^2$  determines the amount of  $D$  state in the mixtures assuming that the "S" and "D" parts of the wave function have been separately normalized to unity.  $\alpha, \beta,$  and  $C$  are variation parameters and have been chosen to minimize the energy. The results fit the binding energy of  $\text{He}^4$  reasonably well. It is found<sup>7</sup> that

$$\alpha = 1.19 \times 10^{13} \text{ cm}^{-1}; \quad \beta = 1.792 \times 10^{13} \text{ cm}^{-1}, \quad (7)$$

and

$$C = -0.162.$$

In order to calculate the rms size of the charge distribution ( $b$ ) of the alpha particle, we shall have to evaluate

$$b = \left\{ \frac{1}{2} [(\mathbf{r}_3 - \mathbf{R})^2 + (\mathbf{r}_4 - \mathbf{R})^2]_{00} \right\}^{\frac{1}{2}}. \quad (8)$$

From Eq. (4), it is evident that

$$\mathbf{r}_3 - \mathbf{R} = \frac{1}{2}\mathbf{u} - \mathbf{w}/\sqrt{2} \quad \text{and} \quad \mathbf{r}_4 - \mathbf{R} = \frac{1}{2}\mathbf{u} + \mathbf{w}/\sqrt{2}. \quad (9)$$

Therefore

$$b^2 = \left[ \frac{1}{4}u^2 + \frac{1}{2}w^2 \right]_{00} = \frac{1}{(1+C^2)} \left\{ \int \psi_S^* \left( \frac{1}{4}u^2 + \frac{1}{2}w^2 \right) \psi_S d\mathbf{u}d\mathbf{v}d\mathbf{w} + C^2 \int \psi_D^* \left( \frac{1}{4}u^2 + \frac{1}{2}w^2 \right) \psi_D d\mathbf{u}d\mathbf{v}d\mathbf{w} \right\}. \quad (10)$$

Carrying out the integrations by Irving's method,<sup>7</sup> we obtain

$$\begin{aligned} \left[ \frac{1}{4}u^2 + \frac{1}{2}w^2 \right]_{00} &= \frac{1}{(1+C^2)} \left\{ \frac{45}{32\alpha^2} + \frac{91C^2}{32\beta^2} \right\} \\ &= \frac{1}{(1.026)} \{0.986 + 0.023\} \\ &= 0.98 \times 10^{-26} \text{ cm}^2. \end{aligned} \quad (11)$$

Therefore

$$b = \left( \left[ \frac{1}{4}u^2 + \frac{1}{2}w^2 \right]_{00} \right)^{\frac{1}{2}} = 0.99 \times 10^{-13} \text{ cm}, \quad (12)$$

which is about  $\frac{2}{3}$  of the experimental<sup>9</sup> value  $b = 1.61 \times 10^{-13}$  cm found from electron-scattering. Dalitz and Ravenhall<sup>9</sup> have recently computed an rms radius from the wave function of Clark<sup>10</sup> who also used a variation method to fit the binding energy of the alpha particle. The resulting radius is found to be  $\frac{2}{3}$  of the required size. Since Clark has included two  $D$  states in his wave function while Irving has used only one, we conclude that the additional  $D$  states have little effect on the rms radius of the alpha particle. Also note in Eq. (11) that the principal "D" state contributes only 2.5% to the mean square radius. There are several possibilities for this serious disagreement between theory and experiment.

(1) This application of Irving and Clark's wave functions to electron-scattering does not consider the finite size of the proton<sup>9</sup> ( $0.77 \times 10^{-13}$  cm). An inclusion of this finite size of the proton with treatment of the neutron as a point particle increases the rms radius of the  $\alpha$  particle to about  $1.30 \times 10^{-13}$  cm, which is still  $0.30 \times 10^{-13}$  below the experimental value. McIntyre's use<sup>11</sup> of this procedure gives agreement with his measurements of electron-deuteron scattering; but it violates the assumption of charge symmetry in nuclear physics.

(2) Recently Yamada *et al.*<sup>12</sup> have calculated the effect of a hard core in the nucleon-nucleon potential on the binding energy of  $\text{H}^3$  and  $\text{He}^3$  and they find that the hard core reduces the binding energy of  $\text{H}^3$ , and with properly chosen parameters decreases the Coulomb energy of  $\text{He}^3$  by 25% to give agreement with the experimental value. The rms radius of the 3-body system is increased by roughly 25%. We therefore believe that in addition to all  $D$  states, an inclusion of a hard core might possibly lead to the right binding energy and rms radius of the  $\alpha$  particle. (Other changes in the shape of the two-body potential, or the inclusion of many-body forces could have similar effects to those of a two-body repulsive core.)

(3) A third alternative suggested by McIntyre<sup>11</sup> is to assume that the charge density found from scattering experiments at high energies (400 Mev) cannot be directly related to  $\psi$ , the solution of the nuclear Schrödinger equation.<sup>13</sup>

### III. BREMSSTRAHLUNG-WEIGHTED CROSS SECTION

In this section we shall evaluate the electric dipole bremsstrahlung-weighted cross section  $\sigma_b$  using Irving's wave function.

<sup>10</sup> A. C. Clark, Proc. Phys. Soc. (London) **A67**, 323 (1953).

<sup>11</sup> J. A. McIntyre, Phys. Rev. **103**, 1464 (1956).

<sup>12</sup> Kikuta, Morita, and Yamada, Progr. Theoret. Phys. (Japan) **15**, 222 (1956).

<sup>13</sup> Yennie, Lévy, and Ravenhall, Revs. Modern Phys. **29**, 144 (1957).

Levinger and Bethe<sup>14</sup> evaluate the electric dipole bremsstrahlung-weighted cross section as

$$\sigma_b = \int (\sigma/W) dW = \frac{4\pi^2}{3} \left( \frac{e^2}{\hbar c} \right) \left\{ \left[ \sum_{i=3,4} (\mathbf{r}_i - \mathbf{R}) \right]^2 \right\}_{00}. \quad (13)$$

But from Eq. (9),

$$(\mathbf{r}_3 - \mathbf{R}) + (\mathbf{r}_4 - \mathbf{R}) = \mathbf{u}. \quad (14)$$

Therefore

$$\begin{aligned} \sigma_b &= \frac{4\pi^2}{3} \left( \frac{e^2}{\hbar c} \right) \frac{1}{(1+C^2)} \left[ \int \psi_S^* u^2 \psi_S d\mathbf{u} d\mathbf{v} d\mathbf{w} \right. \\ &\quad \left. + C^2 \int \psi_D^* u^2 \psi_D d\mathbf{u} d\mathbf{v} d\mathbf{w} \right] \\ &= \frac{4\pi^2}{3} \left( \frac{e^2}{\hbar c} \right) \frac{1}{(1+C^2)} \left( \frac{15}{8\alpha^2} + \frac{21C^2}{8\beta^2} \right) \\ &= 1.23 \text{ mb}, \end{aligned} \quad (15)$$

where the values of  $\alpha$ ,  $\beta$ , and  $C$  have been substituted from Eq. (7). The value of  $\sigma_b$  with Irving's central wave function<sup>15,16</sup> is only 0.8 mb.

#### IV. CROSS SECTION INTEGRATED OVER PHOTON ENERGY

Levinger and Bethe<sup>14</sup> have calculated an expression for the photodisintegration cross section of a nucleus integrated over the photon energy for a central potential with Majorana exchange. They find that

$$\begin{aligned} \sigma_{\text{int}} &= \int \sigma dW \\ &= \frac{2\pi^2 e^2 \hbar}{Mc} \left\{ \frac{NZ}{A} - \frac{Mx}{4\hbar^2} \left[ \sum_i \sum_j V(r_{ij}) P_{ij}^M, (\sum_i z_i - \sum_j z_j), (\sum_i z_i - \sum_j z_j) \right]_{00} \right\}. \end{aligned} \quad (16)$$

Here  $x$  is the fraction of the neutron-proton force that has a Majorana exchange character;  $i$  denotes proton and  $j$  neutron, the double sum being over all pairs of neutrons and protons;  $r_{ij}$  is the distance between proton " $i$ " and neutron " $j$ ";  $V(r_{ij})$  is the neutron-proton potential; and  $P_{ij}^M$  is the Majorana exchange operator.

Using the property that

$$\begin{aligned} [ [\sum_i \sum_j P_{ij}^M, \sum_i z_i], \sum_i z_i ] &= [ \sum_i \sum_j (z_j - z_i) P_{ij}^M, \sum_i z_i ] \\ &= \sum_i \sum_j (z_j - z_i)^2 P_{ij}^M, \end{aligned}$$

Levinger and Bethe obtain

$$\sigma_{\text{int}} = \frac{2\pi^2 e^2 \hbar}{Mc} \left\{ \frac{NZ}{A} - \frac{Mx}{3\hbar^2} \int \psi_0^* \sum_i \sum_j V(r_{ij}) r_{ij}^2 P_{ij}^M \psi_0 d\tau \right\}, \quad (17)$$

where  $\psi_0$  is the complete nuclear wave function.

In their paper, Levinger and Bethe have not considered tensor forces, or Bartlett and Heisenberg exchange operators. Since the tensor operator commutes with the space coordinate, Eq. (17) still holds, with  $V$  now including the tensor operator  $S_{ij}$ . The Bartlett operator exchanges the spin directions of the two particles, leaving their positions unaffected. Bartlett forces therefore will not contribute anything to  $\sigma_{\text{int}}$  because the Bartlett operator commutes with the space coordinates.

The Heisenberg operator  $P^H$  interchanges both position and spin coordinates, and for central forces of the form  $V(r_{ij})[1 + xP_{ij}^M + yP_{ij}^H]$ , we find that

$$\sigma_{\text{int}} = \frac{2\pi^2 e^2 \hbar}{Mc} \left\{ \frac{NZ}{A} - \frac{Mx}{3\hbar^2} \int \psi_0^* \sum_i \sum_j V(r_{ij}) r_{ij}^2 P_{ij}^M \psi_0 d\tau - \frac{My}{3\hbar^2} \int \psi_0^* \sum_i \sum_j V(r_{ij}) r_{ij}^2 P_{ij}^H \psi_0 d\tau \right\}. \quad (18)$$

Here  $x$  and  $y$  denote the fractions of Majorana and Heisenberg type forces. For most even-even nuclei, including the alpha particle, any pair of neutrons and protons will have a probability of  $\frac{3}{4}$  for being in a spin triplet state and  $\frac{1}{4}$  for being in the spin singlet state. Since the Bartlett operator gives  $+1$  when it acts on a spin triplet state and  $-1$  acting on a spin singlet state, and since  $P_{ij}^H = P_{ij}^M P_{ij}^B$ ,

$$[\psi_0^* P^H \psi_0] = \frac{1}{2} [\psi_0^* P^M \psi_0]. \quad (19)$$

<sup>14</sup> J. S. Levinger and H. A. Bethe, Phys. Rev. 78, 115 (1950).

<sup>15</sup> J. Irving, Phil. Mag. 42, 332 (1951).

<sup>16</sup> M. L. Rustgi, dissertation submitted to Louisiana State University, January, 1957 (unpublished).

where the factor  $\frac{1}{2}$  comes out of averaging over the nucleon spins. (See reference 16 for further discussion.) Thus, for an even-even nucleus, Eq. (18) reduces to

$$\sigma_{\text{int}} = \frac{2\pi^2 e^2 \hbar}{Mc} \left\{ \frac{NZ}{A} - \frac{M(x + \frac{1}{2}y)}{3\hbar^2} \int \psi_0^* \sum_i \sum_j V(r_{ij}) r_{ij}^2 P_{ij}^M \psi_0 d\tau \right\}. \quad (20)$$

For the present calculation, we shall use a general Yukawa potential including tensor interaction written as  $V(r_{ij}) = -V_0 \left[ \left(1 - x - \frac{1}{2}y - \frac{1}{2}g + \frac{1}{2}g(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\right) + xP_{ij}^M + yP_{ij}^H \right] J(r_{ij})$

$$+ \gamma \left\{ \left(1 - x' - \frac{1}{2}y' - \frac{1}{2}g + \frac{1}{2}g(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\right) + x'P_{ij}^M + y'P_{ij}^H \right\} K(r_{ij}) S_{ij}, \quad (21)$$

where the tensor operator

$$S_{ij} = r_{ij}^{-2} [3(\boldsymbol{\sigma}_i \cdot \mathbf{r}_{ij})(\boldsymbol{\sigma}_j \cdot \mathbf{r}_{ij})] - (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), \quad (22)$$

and

$$J(r_{ij}) = \frac{e^{-r_{ij}/r_c}}{(r_{ij}/r_c)}; \quad K(r_{ij}) = \frac{e^{-r_{ij}/r_t}}{(r_{ij}/r_t)}. \quad (23)$$

$V_0$ ,  $g$ ,  $\gamma$ ,  $r_c$ , and  $r_t$  are parameters and have been chosen to fit the binding energy of the triton by Pease and Feshbach.<sup>17</sup> They find

$$r_c = 1.184 \times 10^{-13} \text{ cm}, \quad r_t = 1.67 \times 10^{-13} \text{ cm}, \quad V_0 = 46.1 \text{ Mev}, \quad \gamma = 0.54, \quad \text{and} \quad g = -0.004.$$

For an alpha particle,

$$\sigma_{\text{int}} = \frac{2\pi^2 e^2 \hbar}{Mc} \left\{ 1 - \frac{MV_0}{(1+C^2)3\hbar^2} \int (\psi_S^* + C\psi_D^*) \left[ \sum_i \sum_j r_{ij}^2 \left[ (x + \frac{1}{2}y) J(r_{ij}) + (x' + \frac{1}{2}y') \gamma K(r_{ij}) S_{ij} \right] \right] (\psi_S + C\psi_D) d\tau \right\}.$$

From the orthogonality of the  $S$  and  $D$  states, it follows that

$$\sigma_{\text{int}} = \frac{2\pi^2 e^2 \hbar}{Mc} \left\{ 1 - \frac{MV_0}{(1+C^2)3\hbar^2} \left[ (x + \frac{1}{2}y) \int \psi_S^* \sum_i \sum_j J(r_{ij}) r_{ij}^2 \psi_S d\tau + 2C(x' + \frac{1}{2}y') \gamma \int \psi_S^* \sum_i \sum_j K(r_{ij}) S_{ij} r_{ij}^2 \psi_D d\tau \right. \right. \\ \left. \left. + C^2(x + \frac{1}{2}y) \int \psi_D^* \sum_i \sum_j J(r_{ij}) r_{ij}^2 \psi_D d\tau + C^2(x' + \frac{1}{2}y') \gamma \int \psi_D^* \sum_i \sum_j K(r_{ij}) S_{ij} r_{ij}^2 \psi_D d\tau \right] \right\}. \quad (24)$$

All the spin matrix elements needed are given by Irving<sup>7</sup> and are worked out in detail in reference 16.

Carrying out the spatial integrations by Irving's method, we obtain

$$\int \sigma dW = \frac{2\pi^2 e^2 \hbar}{Mc} \left\{ 1 + \frac{MV_0}{(1+C^2)3\hbar^2} \left[ \frac{(x + \frac{1}{2}y) N_S^2 (1-g) 2^3 \pi^4 (9!)}{(\sqrt{2}/r_c)^{11}} E(a_1) \right. \right. \\ \left. \left. + \frac{(x' + \frac{1}{2}y') N_S N_D 2^5 \pi^4 \gamma C (12!)}{(\sqrt{2}/r_t)^{13}} D(a_2) + \frac{(x + \frac{1}{2}y) N_D^2 C^2 2^6 \pi^4 (20)(13!)}{(\sqrt{2}/r_c)^{15}} F(a_3) - \frac{(x' + \frac{1}{2}y') N_D^2 C^2 2^3 \pi^4 \gamma (13!)}{(\sqrt{2}/r_t)^{15}} F(a_4) \right] \right\}, \quad (25)$$

where

$$a_1 = 2\sqrt{2}\alpha r_c, \quad a_2 = \sqrt{2}(\alpha + \beta) r_t, \quad a_3 = 2\sqrt{2}\beta r_c, \quad a_4 = 2\sqrt{2}\beta r_t. \quad (26)$$

$E$ ,  $D$ , and  $F$  are the integrals listed in the Appendix. (Though two of these integrals are listed by Irving,<sup>7</sup> we shall list them again for completeness.)

Evaluating (25), we obtain

$$\sigma_{\text{int}} = 60 \left[ 1 + 0.6710(x + \frac{1}{2}y) + 0.1815(x' + \frac{1}{2}y') + 0.0240(x + \frac{1}{2}y) - 0.0001(x' + \frac{1}{2}y') \right] \\ = 60 \left[ 1 + 0.695(x + \frac{1}{2}y) + 0.181(x' + \frac{1}{2}y') \right] \text{ Mev-mb}, \quad (27)$$

while for Irving's central force wave function<sup>16</sup>

$$\sigma_{\text{int}} = 60 \left[ 1 + 0.97(x + \frac{1}{2}y) \right] \text{ Mev-mb}. \quad (28)$$

Levinger<sup>18</sup> has made a sum-rule calculation of the electric-dipole transitions in the nuclear photoeffect using a simple harmonic oscillator independent-particle model with nuclear radius parameter  $r_0 = 1.2 \times 10^{-13}$  cm. For

<sup>17</sup> R. L. Pease and H. Feshbach, Phys. Rev. 88, 945 (1952).

<sup>18</sup> J. S. Levinger, Phys. Rev. 97, 122 (1955).

the particular case of  $\text{He}^4$ , he finds that  $\sigma_b = 2.3$  mb, or about twice our value using Irving's wave function for a smaller alpha particle. For a quasi-Yukawa central potential, Levinger finds that  $\sigma_{\text{int}} = 60(1 + 0.84x)$ , in surprisingly good agreement with our Eq. (27) if we put  $y = y' = 0$  and  $x' = x$ .

### V. EXPERIMENTAL DATA

In this section, we shall make a comparison with various experiments on the photodisintegration of the  $\text{He}^4$  nucleus. Experiments on the photodisintegration of alpha particles have been performed by Benedict and Woodward,<sup>19</sup> Halpern *et al.*,<sup>20</sup> Fuller,<sup>6</sup> de Saussure and Osborne,<sup>21</sup> and Smith and Barton.<sup>22</sup>

Benedict and Woodward measured the cross sections for the production of protons of energies from 45 to 120 Mev produced from  $\text{He}^4$  by a bremsstrahlung beam, and have expressed their results in terms of differential cross sections. We calculate the total cross section from their graph by assuming an angular distribution of the form  $(a + b \sin^2\theta)$ . Fuller<sup>6</sup> irradiated helium gas with 26-, 29-, and 40-Mev bremsstrahlung and measured the energy and angular distributions of the protons produced by the photodisintegration of  $\text{He}^4$  using nuclear emulsion techniques. The results of his measurements were read from a curve for  $\sigma$  vs photon energy. Both Woodward and Fuller obtain photon energies by using the assumption that the dominant reaction giving protons is  $\text{He}^4(\gamma, p)\text{He}^3$ . Fuller has checked this assumption in his energy range by use of different betatron energies.

Halpern *et al.*<sup>20</sup> measured the cross section for the photodisintegration of  $\text{He}^4$  by direct detection of the

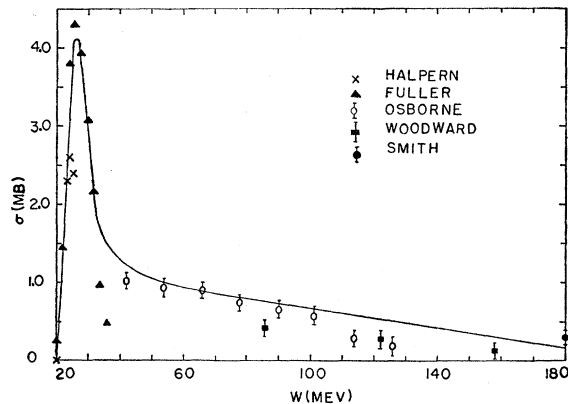


FIG. 1. Experimental measurements of the total cross section, in millibarns, for photodisintegration of the alpha particle vs photon energy  $W$ . The crosses show measurements by Halpern *et al.*<sup>20</sup>; the solid triangles, Fuller<sup>6</sup>; the open circles, Osborne *et al.*<sup>21</sup>; the solid squares, Woodward *et al.*<sup>19</sup>; and the solid circle, Smith *et al.*<sup>22</sup> The curve is used for the numerical calculation of  $\sigma_b$  and  $\sigma_{\text{int}}$ .

<sup>19</sup> T. S. Benedict and W. M. Woodward, *Phys. Rev.* **83**, 1269 (1951).

<sup>20</sup> Ferguson, Halpern, Nathan, and Yergin, *Phys. Rev.* **95**, 776 (1954).

<sup>21</sup> G. de Saussure and L. S. Osborne, *Phys. Rev.* **99**, 843 (1955).

<sup>22</sup> J. H. Smith and M. Q. Barton, *Phys. Rev.* **100**, 1265(A) (1955) and J. H. Smith (private communications).

outgoing neutrons, and analyzed their yield curve by the photon difference method. We have read their cross sections also from their curve of photoneutron cross section for  $\text{He}^4$ . de Saussure and Osborne<sup>21</sup> have investigated the photodisintegration reaction  $\gamma + \text{He}^4 \rightarrow \text{He}^3 + n$  by measuring the energy and angular distribution of the  $\text{He}^3$  recoil nuclei for photon energies between 40 and 120 Mev.

Smith and Barton<sup>22</sup> have measured proton yield and neutron-proton coincidences from alpha particles irradiated by 285-Mev bremsstrahlung. Analysis of the neutron-proton coincidences for 65-Mev protons on the quasi-deuteron model of Levinger gives an alpha-particle cross section of 0.2 mb at a photon energy of about 180 Mev. Since some protons were not in coincidence with neutrons, Smith suggests using a total cross section about 50% larger, i.e., 0.3 mb.

In the following, we shall assume that  $\sigma(\gamma, n) = \sigma(\gamma, p)$  for the photodisintegration of  $\text{He}^4$ . Thus we double the proton cross sections of Woodward and of Fuller<sup>23</sup> and the neutron cross sections of Halpern and of Osborne to find the total cross sections of photodisintegration of the alpha particle.

Combining all the measurements, we have the total cross section from threshold to 150 Mev, with a standard error of about 10% for the better measurements. As shown in Fig. 1, the measurements are not completely consistent. More experimental measurements are in progress.<sup>24</sup> We have calculated experimental values of  $\sigma_b$  and  $\sigma_{\text{int}}$  by numerical integration to 150 Mev using the preliminary curve shown in Fig. 1. Table I shows a comparison between theory and experiments. Three standard central-force mixtures are:

$$\text{Rosenfeld}^{25}: 0.93P^M - 0.13 - 0.26P^H + 0.46P^B, \\ \text{giving } x + \frac{1}{2}y = 0.80;$$

TABLE I. Integrated cross sections for photodisintegration of the helium nucleus.

	$\sigma_b = \int (\sigma/W) dW$	$\sigma_{\text{int}} = \int \sigma dW$
Experiment <sup>a</sup>	2.7 mb	124 Mev-mb
Calculation:		
Central forces	0.8 mb	$60[1 + 0.97(x + \frac{1}{2}y)]$
Central plus tensor	1.23 mb <sup>b</sup>	$60[1 + 0.695(x + \frac{1}{2}y) + 0.182(x' + \frac{1}{2}y')]^c$

<sup>a</sup> See Fig. 1.

<sup>b</sup> See Sec. III.

<sup>c</sup> See Sec. IV.  $x$  and  $x'$  are the fractions of central and tensor potentials, respectively, that have Majorana exchange character;  $y$  and  $y'$  denote similar quantities for Heisenberg forces.

<sup>23</sup> Fuller's published cross sections have been increased by roughly 20% due to recalibration of the photon monitor (private communication).

<sup>24</sup> E. L. Goldwasser (private communication).

<sup>25</sup> L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1948).

Inglis<sup>26</sup>:  $0.8P^M + 0.2P^B$ , giving  $x + \frac{1}{2}y = 0.80$ ;

and Serber<sup>27</sup>:  $0.5 + 0.5P^M$ , giving  $x + \frac{1}{2}y = 0.50$ .

For central forces and Irving's central wave function  $\sigma_{\text{int}}$  is 89 Mev-mb for a Serber mixture and 106 Mev-mb for a Rosenfeld or Inglis mixture. If we use the same mixture parameter for central and tensor forces in the Pease-Feshbach potential ( $x = x'$  and  $y = y'$ ), a Serber mixture gives 86 Mev-mb, while a Rosenfeld or Inglis mixture gives 102 Mev-mb.

## VI. DISCUSSION

A comparison between theory and experiment shows a serious discrepancy between theory and experiment for  $\sigma_b$  though there is a fairly good agreement between the two for  $\sigma_{\text{int}}$  for Inglis or Rosenfeld mixtures. The large disagreement between theory and experiment for  $\sigma_b$  may be due to the small rms value of the alpha particle given by Irving's wave functions. (Calculations of  $\sigma_b$  usually provide a good check on the ground-state wave function used.)

As discussed above, the interpretation of the alpha particle's rms radius is not completely clear at present. McIntyre<sup>11</sup> has recently analyzed the electron-deuteron scattering successfully by treating the proton as a spread-out charge distribution and the neutron as a point charge. If we start from Hofstadter's measured rms radius of  $b = 1.61 \times 10^{-13}$  cm for the alpha particle, and, following McIntyre, subtract the contribution due to the proton's radius, we obtain an alpha rms radius of about  $b = [(1.61)^2 - (0.77)^2]^{\frac{1}{2}} = 1.40 \times 10^{-13}$  cm. (The subtraction of mean square radius is justified for Gaussian charge distributions, which are not in disagreement with Hofstadter's measurements.)

<sup>26</sup> D. R. Inglis, *Revs. Modern Phys.* **25**, 390 (1953).

<sup>27</sup> R. Serber, *Phys. Rev.* **72**, 1114 (1947).

For central forces,

$$\sigma_b = \frac{4\pi^2}{3} \left( \frac{e^2}{\hbar c} \right) \frac{4}{3} b^2. \quad (29)$$

[See Eqs. (11) and (15).] Equation (29) is a good approximation for tensor forces. [*Note added in proof.*—We are grateful to L. L. Foldy (private communication and preprint, "Photodisintegration of the Lightest Nuclei,")] for the following: (1) only the independence of the two charge distributions is needed to justify our subtraction of the proton's mean square radius; (2) Equation (29) can be generalized for  $A \leq 4$ , using only the assumption of a space-symmetric wave function.] Using the experimental value  $\sigma_b = 2.7$  mb in this equation, we find  $b = 1.44 \times 10^{-13}$  cm which supports Hofstadter's value of  $1.40 \times 10^{-13}$  cm obtained by taking account of the proton size.

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## APPENDIX

In Sec. IV, the following integrals are used:

$$E(a) = \int_0^1 \frac{(1-q^2)^2 q^3 dq}{(a+q)^{10}} = \frac{21a^3 + 19a^2 + 7a + 1}{504a^9(a+1)^7},$$

$$D(a) = \int_0^1 \frac{(1-q^2)^2 q^5 dq}{(a+q)^{12}} = \frac{231a^3 + 159a^2 + 45a + 5}{13860a^8(a+1)^9},$$

$$F(a) = \int_0^1 \frac{(1-q^2)^3 q^5 dq}{(a+q)^{14}} = \frac{3003a^4 + 9630a^3 + 4710a^2 + 350a + 35}{360360a^8(a+1)^{10}}.$$