Possible Tests of Time Reversal Invariance in Beta Decay

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Noninvariance under space reflection and charge conjugation has now been established for beta decay processes. Invariance under time reversal remains an open question, however. We discuss here several possible tests for the validity of this symmetry operation. General expressions are given for the distribution function in three experimental situations, which have the possibility of detecting terms in allowed beta decay that are not invariant under time reversal: (a) experiments in which the nuclei are oriented and electron and neutrino momenta are measured; (b) experiments in which the nuclei are not oriented, but the recoil momentum and electron momentum and polarization are observed; (c) experiments in which the nuclei are oriented and the electron momentum and polarization are measured. The distribution functions obtained omit Coulomb distortion effects and relativistic corrections for the nucleons, but are otherwise complete. Such experiments should permit, in addition to the detection of terms which are not invariant under time reversal, the beginnings of a determination of the ten complex coupling constants which now characterize beta decay. An additional, somewhat surprising, result is found. If the two-component neutrino theory of Lee and Yang is correct, and if certain perhaps reasonable assumptions concerning the relative magnitudes of the various coupling constants are valid, then the longitudinal polarization of electrons in allowed beta decay even from unoriented nuclei should be almost complete (specifically, equal to v/c).

I. INTRODUCTION

THE question has been raised recently whether weak interactions, e.g., beta decay, are invariant under space inversion, charge conjugation, and time reversal.[1-3] Lee and Yang[1] have proposed a number of experiments to test the possibility that parity conservation is violated in weak interactions. In particular, they have pointed out that in beta decay from oriented nuclei, an asymmetry in electron intensity with respect to the nuclear polarization direction would immediately imply parity nonconservation. Experiments along these lines have now been carried out. Wu, Ambler, Hayward, Hoppes, and Hudson[4] in fact find a large asymmetry effect in the beta decay of Co^{60}. In addition to this, Garwin, Lederman, and Weinrich[5] and Friedman and Telegdi[5] find that parity is not conserved in π- and μ-meson decay.

It is an immediate consequence of a theorem of Lüders and Pauli[6] that noninvariance under any one of the operations space inversion, charge conjugation, and time reversal implies, at least for a theory invariant under the proper Lorentz group, noninvariance under at least one of the other operations. It is thus of considerable interest to find whether charge conjugation invariance or time reversal invariance or both are violated together with parity conservation. The experiment of Wu et al.[4] indicates that charge conjugation invariance is violated, but it is not yet clear from this experiment whether time reversal invariance must also be abandoned. The situation is essentially this. If one does *not* measure either the nuclear recoil or the polarization of the electrons, the only terms which can appear in the electron distribution function, which unambiguously indicate noninvariance with respect to time reversal, come about because of Coulomb distortion of the electron wave function. In principle these terms can be separated experimentally from those which do not depend on Coulomb distortion, because the two types of terms have different momentum dependences. However, these terms are reduced by a factor of $Z/137$ and for this and other reasons they may be difficult to detect experimentally.

Effects which indicate noninvariance with respect to time reversal and which do not depend on Coulomb distortion can, however, appear in experiments in which either the nuclear recoil or the electron polarization are measured. There are four vector quantities which conceivably could be measured in a beta-decay experiment: $\langle \mathbf{J} \rangle$, the polarization of the decaying nucleus; $\boldsymbol{\sigma}$, the polarization direction of the electron; $\mathbf{p}_e$, the electron momentum; and $\mathbf{p}_\nu$, the neutrino momentum. Since all four of these vectors change sign under time reversal, the scalar triple product of any three of them gives a term invariant under rotations but noninvariant under time reversal. Hence the detection of such a term in a beta-decay experiment would indicate noninvariance under time reversal.[6a]

We consider in this paper three types of experiments in which such terms might appear. In Sec. II the distribution function is given for the allowed beta decay of *oriented* nuclei in which both the *electron* and *recoil* momenta are observed. In Sec. III we give the distribution function for the allowed beta decay of *nonoriented* nuclei in which the *polarization* of the electrons is

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[2] Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).
[3] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[4] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
[5] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681(L) (1957).
[6] G. Lüders, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 5 (1954); W. Pauli, *Niels Bohr and the Development of Physics* (Pergamon Press, London, 1955). See also reference 2.

[6a] Of course terms similar in form, though with different momentum dependence, can appear even if time reversal invariance is valid, when Coulomb effects are taken into account. See the note added in proof at the end of the paper.

observed, as well as the *electron* and *recoil* momenta. Section IV contains the distribution function for the allowed beta decay of *oriented* nuclei in which the *electron momentum* and *polarization* are detected, but the recoil is not observed.

The calculations are based on the interaction Hamiltonian density

$$H_{\text{int}} = (\bar{\psi}_p \psi_n)(C_S \bar{\psi}_e \psi_\nu + C_S' \bar{\psi}_e \gamma_5 \psi_\nu)$$
$$+ (\bar{\psi}_p \gamma_\mu \psi_n)(C_V \bar{\psi}_e \gamma_\mu \psi_\nu + C_V' \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu)$$
$$+ \tfrac{1}{2}(\bar{\psi}_p \sigma_{\lambda\mu} \psi_n)(C_T \bar{\psi}_e \sigma_{\lambda\mu} \psi_\nu + C_T' \bar{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_\nu)$$
$$- (\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n)(C_A \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu + C_A' \bar{\psi}_e \gamma_\mu \psi_\nu)$$
$$+ (\bar{\psi}_p \gamma_5 \psi_n)(C_P \bar{\psi}_e \gamma_5 \psi_\nu + C_P' \bar{\psi}_e \psi_\nu) \qquad (1)$$
$$+ \text{Hermitian conjugate},$$

as given by Yang and Lee.[1] The terms with primed coefficients are the parity-nonconserving interactions introduced by these authors. Time reversal invariance would require that all ten coefficients C, C' be real. If this invariance does not hold, then beta decay is characterized by ten complex coupling constants, i.e., twenty parameters. Aside from testing whether or not time reversal invariance is preserved in beta decay one will ultimately want to determine the values of all coupling constants. In the following sections, we give general expressions for the electron distribution functions for arbitrary allowed beta decay. The Coulomb distortions are neglected, however, since the terms in which we are interested already appear in lowest order. Also, since the nucleons are treated nonrelativistically, the pseudoscalar couplings are omitted.

Since parity is not conserved in beta decay, there can exist a polarization of the electrons along their line of flight, which polarization is uncorrelated with the nuclear recoil or orientation of the initial nucleus. Such effects arise from terms of the form $\boldsymbol{\sigma} \cdot \mathbf{p}_e$, which violate parity conservation. This effect is discussed in Sec. V. It is found, surprisingly, that under certain conditions which may well be met, this polarization is nearly complete.

II. RECOIL EXPERIMENTS WITH ORIENTED NUCLEI

We give here the distribution in electron and neutrino directions and electron energy for an allowed transition $(J \rightarrow J' = J, J \pm 1$, no nuclear parity change) from an oriented nucleus. If $\langle \mathbf{J} \rangle$ is the expectation value of the vector angular momentum of the original nucleus, $\mathbf{j}$ a *unit* vector in the direction of $\langle \mathbf{J} \rangle$ and m the electron mass, we find

$$\omega(\langle J \rangle | E_e, \Omega_e, \Omega_\nu) dE_e d\Omega_e d\Omega_\nu$$
$$= \frac{1}{(2\pi)^5} p_e E_e (E^0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu \xi \left\{ 1 + a\frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e} \right.$$
$$+ c\left[\frac{1}{3}\frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} - \frac{(\mathbf{p}_e \cdot \mathbf{j})(\mathbf{p}_\nu \cdot \mathbf{j})}{E_e E_\nu} \right]\left[\frac{J(J+1) - 3\langle (\mathbf{J} \cdot \mathbf{j})^2 \rangle}{J(2J-1)} \right]$$
$$\left. + \frac{\langle \mathbf{J} \rangle}{J} \cdot \left[A\frac{\mathbf{p}_e}{E_e} + B\frac{\mathbf{p}_\nu}{E_\nu} + D\frac{\mathbf{p}_e \times \mathbf{p}_\nu}{E_e E_\nu} \right] \right\}. \qquad (2)$$

The coefficients ξ, a, b, c, A, B, D, which depend on eight of the complex coupling constants C, C', are given in the appendix, Eqs. (A3)–(A9). We have used natural units such that $c = \hbar = 1$.

The first three terms in Eq. (2) are the usual ones for beta decay from unpolarized nuclei. The fourth term arises when the initial nucleus is oriented [for a non-oriented nucleus $3\langle (\mathbf{J} \cdot \mathbf{j})^2 \rangle = J(J+1)$]. This term has no implications for parity conservation, charge conjugation invariance, or time reversal invariance. The terms containing $\langle \mathbf{J} \rangle \cdot \mathbf{p}_e$ and $\langle \mathbf{J} \rangle \cdot \mathbf{p}_\nu$ can arise only if parity is not conserved.[7] The last term, which involves $\langle \mathbf{J} \rangle \cdot (\mathbf{p}_e \times \mathbf{p}_\nu)$, can occur only if invariance with respect to time reversal is violated.

This $\langle \mathbf{J} \rangle \cdot (\mathbf{p}_e \times \mathbf{p}_\nu)$ term can readily be distinguished experimentally from the other two terms. For example, suppose the nuclei are polarized along the z axis. The coincidence counting rate for recoil nuclei traveling in the x direction and electrons in the y direction should differ from the rate for recoil nuclei in the x direction and electrons in the $(-y)$ direction only if the term $\langle \mathbf{J} \rangle \cdot (\mathbf{p}_e \times \mathbf{p}_\nu)$ is present (notice that $\mathbf{p}_\nu = -\mathbf{p}_e - \mathbf{p}_N$, where $\mathbf{p}_N$ is the recoil momentum). One can imagine other geometrical arrangements which could be used to distinguish separately the various terms in Eq. (2). The feasibility of experiments along these lines is being studied at Princeton University by G. E. Schrank and T. R. Carver, who suggest that "optical pumping" could be used to align the nuclei in a dilute gas.

Another possibility is to study the beta decay of free polarized neutrons.[8] In this case the following simplifications obtain in the expressions of the preceding section. The term containing the factor c in Eq. (2) vanishes, since for a spin $\tfrac{1}{2}$ system $3\langle (\mathbf{J} \cdot \mathbf{j})^2 \rangle = J(J+1)$. Furthermore, here $|M_{GT}|^2 = 3$ and $|M_F|^2 = 1$ so that one has (since $\Delta J = 0$)

$$\lambda_{J'J} |M_{GT}|^2 = 2,$$

$$\left(\frac{J}{J+1} \right)^{\tfrac{1}{2}} |M_F| |M_{GT}| = 1.$$

These values are to be used in Eqs. (A3)–(A9) in the appendix.

It is to be noted that D, the coefficient of the term $\langle \mathbf{J} \rangle \cdot (\mathbf{p}_e \times \mathbf{p}_\nu)$, involves the imaginary part of cross-product terms between the scalar and tensor coupling constants. It is known that $|C_S|^2$ and $|C_T|^2$ are large. If the imaginary parts of C_S and C_T are also large, this term should be easy to detect.

III. ELECTRON POLARIZATION IN RECOIL EXPERIMENT

In this section we give the distribution function in electron and neutrino directions, electron energy, and electron polarization for allowed beta decay from a non-

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[7] It is easy to show that if the neutrino direction is integrated out, the terms in Eq. (2) proportional to a, c, B, and D vanish. For $\Delta J = \pm 1$ the remaining expression is equivalent to Eq. (A6) of the appendix of Lee and Yang,[1] when one omits terms of order $Z/137$.

[8] J. M. Robson, Phys. Rev. **100**, 933 (1955).

oriented nucleus,

$$\omega(\sigma | E_e, \Omega_e, \Omega_\nu) dE_e d\Omega_e d\Omega_\nu$$

$$= \frac{1}{(2\pi)^5} p_e E_e (E^0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu$$

$$\times \frac{1}{2}\xi\left\{ 1 + a\frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e} + \sigma \cdot \left[G\frac{\mathbf{p}_e}{E_e} + H\frac{\mathbf{p}_\nu}{E_\nu} \right.\right.$$

$$\left.\left. + K\frac{\mathbf{p}_e}{E_e + m}\left(\frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu}\right) + L\frac{\mathbf{p}_e \times \mathbf{p}_\nu}{E_e E_\nu} \right] \right\}. \quad (3)$$

The coefficients G, H, K, L are given in the Appendix, Eqs. (A10)–(A13). If one uses for σ in Eq. (3) the 2×2 Pauli spin matrix, the expression (3) gives the density matrix for spin states referred to the rest system of the moving electron. On the other hand, if one sets σ equal to a unit vector $\mathbf{n}$, $\omega(\mathbf{n}|E_e,\Omega_e,\Omega_\nu)$ and $\omega(-\mathbf{n}|E_e,\Omega_e,\Omega_\nu)$ give the probabilities of emission of electrons whose spins are in the directions $\mathbf{n}$ and $-\mathbf{n}$ in the rest system of the electron. The polarization in the direction $\mathbf{n}$ is then defined by[9]

$$P(\mathbf{n}) = \frac{\omega(\mathbf{n}|\cdots) - \omega(-\mathbf{n}|\cdots)}{\omega(\mathbf{n}|\cdots) + \omega(-\mathbf{n}|\cdots)}. \quad (4)$$

The terms in Eq. (3) proportional to $\sigma \cdot \mathbf{p}_e$, $\sigma \cdot \mathbf{p}_\nu$, and $(\sigma \cdot \mathbf{p}_e)(\mathbf{p}_e \cdot \mathbf{p}_\nu)$ violate parity conservation. The term proportional to $\sigma \cdot (\mathbf{p}_e \times \mathbf{p}_\nu)$ violates time reversal invariance.

Polarized electrons can be detected experimentally by atomic scattering experiments, where the spin-orbit coupling term gives rise to angular asymmetries. Tolhoek and de Groot[10] show that the most direct experiment is a measurement of a polarization transverse to the direction of motion of the electron by observing the left-right asymmetry. If we take the unit vector $\mathbf{n}$ in the z direction and $\mathbf{p}_e$ in the x direction, coincidence counting of electrons with recoils moving in the y direction will give a polarization

$$P = L\frac{(\mathbf{p}_e \times \mathbf{p}_\nu)_z}{E_e E_\nu} \bigg/ \left(1 + a\frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} + b\frac{m}{E_e}\right), \quad (5)$$

according to Eqs. (3) and (4). If the electrons are subsequently scattered by atoms and detected to the left and to the right in the x–y plane ("left" is the direction defined by $\mathbf{n} \times \mathbf{p}_e$), the polarization is given by

$$P_{\exp} = \frac{1}{|a|}\left(\frac{\mathfrak{R} - \mathfrak{L}}{\mathfrak{R} + \mathfrak{L}}\right),$$

where $\mathfrak{R}$ and $\mathfrak{L}$ are the counting rates to the right and left, and a is the asymmetry parameter defined by Eq. (30) and Fig. 5 of the paper by Tolhoek and de Groot.[10]

As is seen from Eq. (A13), L depends on the imaginary part of cross products of the scalar and vector coupling constants and the tensor and axial vector coupling constants. If $|C_A|^2$ and $|C_V|^2$ are small,[8,11] this term may be more difficult to detect than the analogous term discussed in Sec. II.

IV. ELECTRON POLARIZATION IN DECAY OF ORIENTED NUCLEI

We give here the distribution function in electron energy and angle and electron polarization for allowed beta decay from oriented nuclei. The neutrino angle has been integrated out. One finds

$$\omega(\langle\mathbf{J}\rangle,\sigma|E_e,\Omega_e)dE_e d\Omega_e$$

$$= \frac{1}{(2\pi)^4} p_e E_e (E^0 - E_e)^2 dE_e d\Omega_e$$

$$\times \xi\left\{ 1 + b\frac{m}{E_e} + \left(A\frac{\langle\mathbf{J}\rangle}{J} + G\sigma\right)\cdot\frac{\mathbf{p}_e}{E_e} + \sigma\cdot\left[N\frac{\langle\mathbf{J}\rangle}{J}\right.\right.$$

$$\left.\left. + Q\frac{\mathbf{p}_e}{E_e + m}\left(\frac{\langle\mathbf{J}\rangle}{J}\cdot\frac{\mathbf{p}_e}{E_e}\right) + R\frac{\langle\mathbf{J}\rangle}{J}\times\frac{\mathbf{p}_e}{E_e} \right] \right\}. \quad (6)$$

N, Q, and R are given by Eqs. (A14)–(A16). σ has the same significance as in Sec. III.

The terms involving A and G in Eq. (6) have already appeared in Secs. II and III. The terms involving N and Q violate neither parity conservation nor time reversal invariance and are of no particular interest to us here.[12] The last term, involving $\sigma \cdot (\langle\mathbf{J}\rangle \times \mathbf{p}_e)$ violates both parity conservation and time reversal invariance.

This term could be detected by orienting the nuclei in the y direction and observing electrons moving in the x direction. These electrons will have a polarization in the z direction given by

$$P = R\left(\frac{\langle\mathbf{J}\rangle}{J}\times\frac{\mathbf{p}_e}{E_e}\right)_z \bigg/ \left(1 + b\frac{m}{E_e}\right), \quad (7)$$

according to Eqs. (4) and (6). This polarization could be detected by the right-left asymmetry in a subsequent atomic scattering in the (x,y) plane, just as in Sec. III.

The expression for R given by Eq. (A16) shows that the transverse polarization will be small if $|C_A|^2$ and $|C_V|^2$ are small. For $\Delta J = \pm 1$, R involves the same combination of coupling constants as the $(Z/137)$ term in the coefficient of $\langle\mathbf{J}\rangle\cdot\mathbf{p}_e$ of Lee and Yang.[1] If R is small because of the smallness of $|C_A|^2$ and $|C_V|^2$, the Coulomb distortion term in the Wu experiment will be correspondingly small, and it may be difficult to make a binding conclusion about time reversal invariance or noninvariance.

[9] The polarization as defined by Eq. (4) is equivalent to the "orientation coefficient" of Tolhoek (see reference 10).
[10] H. A. Tolhoek and S. R. de Groot, Physica **17**, 1 (1951).

[11] B. M. Rustad and S. L. Ruby, Phys. Rev. **97**, 991 (1955).
[12] N and Q, as well as c in Eq. (2), are special cases of results already obtained by H. A. Tolhoek and S. R. de Groot, Physica **17**, 81 (1951), when the coupling constants are real and all C' vanish.

V. POLARIZATION OF ELECTRONS IN BETA DECAY

If one integrates over the neutrino directions in Eq. (3) or averages over nuclear orientation directions in Eq. (6), one is led to the distribution function

$$\omega(\sigma|E_e,\Omega_e)dE_e d\Omega_e = \frac{1}{(2\pi)^4}p_e E_e(E^0-E_e)^2 dE_e d\Omega_e$$
$$\times \xi\left\{1+b\frac{m}{E_e}+G\frac{\sigma\cdot p_e}{E_e}\right\}. \quad (8)$$

The term $\sigma\cdot p_e$ gives rise to polarization of the electron along its line of flight. According to Eq. (4), the polarization of the electron in a direction n parallel to p_e is given by

$$P=G\frac{p_e}{E_e}\bigg/\left(1+b\frac{m}{E_e}\right). \quad (9)$$

According to the work of Sherr and Miller,[13] $b\ll1$, at least for $(\Delta J=\pm1,$ no) transitions. If Eq. (A10) for G is specialized to the two-component theory of the neutrino[3] $(C=-C')$, and if $|C_V|^2$ and $|C_A|^2$ are neglected relative to $|C_S|^2$ and $|C_T|^2$, respectively, then the longitudinal polarization is

$$P=\mp v_e/c, \quad (10)$$

irrespective of the type of allowed transition. Thus, if these assumptions should turn out to be valid, relativistic beta-decay electrons would be almost completely polarized. Alternatively, a measurement of the polarization of electrons emitted in beta decay provides a check on these assumptions.

Note added in proof.—After the above report was written we recalculated all the expressions for allowed beta decay taking Coulomb effects fully into account. This work will be published at a later time. The essential results involving time reversal invariance may, however, be summarized as follows: The coefficients ξa, ξc, ξA, and ξG all receive Coulomb corrections proportional to $(Z/137p_e)$ times beta-decay coupling constants in combinations which can occur only if time reversal invariance is violated. But all of these terms would be small if $|C_v|$, $|C_v'|$, $|C_A|$, $|C_A'|$ are small. On the other hand, the coefficients ξD, ξL, and ξR receive corrections proportional to $(Z/137p_e)$ times beta-decay constants in combinations which can occur even if time reversal invariance is valid. The important point, however, is that for ξD these Coulomb corrections would be small if $|C_v|$, $|C_v'|$, $|C_A|$ and $|C_A'|$ are small. From the smallness of the Fierz interference terms in allowed beta decay we can conclude either that the vector and axial vector coupling constants are small or else that time reversal invariance is violated. In either case, the detection of an appreciable $\langle J\rangle\cdot p_e\times p_\nu$ term in Eq. (2) would therefore constitute a proof that time reversal invariance is violated.

APPENDIX

In the following formulas the upper signs refer to electron decay and the lower signs to positron decay. $|M_F|^2$ is the conventional Fermi nuclear matrix element with selection rules $\Delta J=0$, no nuclear parity change; and $|M_{GT}|^2$ is the Gamow-Teller matrix element with selection rule $\Delta J=0$, ±1, no nuclear parity change, $J=0\to J=0$ forbidden. J and J' are the angular momenta of the original and final nuclei, $\delta_{J'J}$ is the Kronecker delta symbol, and

$$\lambda_{J'J}=\begin{cases} 1, & J\to J'=J-1 \\ 1/(J+1), & J\to J'=J \\ -J/(J+1), & J\to J'=J+1; \end{cases} \quad (A1)$$

$$\Lambda_{J'J}=\begin{cases} 1, & J\to J'=J-1 \\ -(2J-1)/(J+1), & J\to J'=J \\ J(2J-1)/[(J+1)(2J+3)], & J\to J'=J+1. \end{cases} \quad (A2)$$

We find

$$\xi=|M_F|^2(|C_S|^2+|C_V|^2+|C_S'|^2+|C_V'|^2)+|M_{GT}|^2(|C_T|^2+|C_A|^2+|C_T'|^2+|C_A'|^2), \quad (A3)$$

$$a\xi=|M_F|^2(-|C_S|^2+|C_V|^2-|C_S'|^2+|C_V'|^2)+\frac{|M_{GT}|^2}{3}(|C_T|^2-|C_A|^2+|C_T'|^2-|C_A'|^2), \quad (A4)$$

$$b\xi=\pm2\text{ Re}[|M_F|^2(C_S C_V{}^*+C_S'C_V'{}^*)+|M_{GT}|^2(C_T C_A{}^*+C_T'C_A'{}^*)], \quad (A5)$$

$$c\xi=|M_{GT}|^2\Lambda_{J'J}(|C_T|^2-|C_A|^2+|C_T'|^2-|C_A'|^2), \quad (A6)$$

[13] R. Sherr and R. H. Miller, Phys. Rev. **93**, 1076 (1954).

$$A\xi = 2\,\mathrm{Re}\Bigg[\pm|M_{GT}|^2\lambda_{J'J}(C_TC_T'^* - C_AC_A'^*)$$

$$+\delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}(C_SC_T'^* + C_S'C_T^* - C_VC_A'^* - C_V'C_A^*)\Bigg],\quad\text{(A7)}$$

$$B\xi = 2\,\mathrm{Re}\Bigg\{|M_{GT}|^2\lambda_{J'J}\left[\frac{m}{E_e}(C_TC_A'^* + C_T'C_A^*)\pm(C_TC_T'^* + C_AC_A'^*)\right] - \delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}$$

$$\times\left[(C_SC_T'^* + C_S'C_T^* + C_VC_A'^* + C_V'C_A^*)\pm\frac{m}{E_e}(C_SC_A'^* + C_S'C_A^* + C_VC_T'^* + C_V'C_T^*)\right]\Bigg\},\quad\text{(A8)}$$

$$D\xi = 2\,\mathrm{Im}\Bigg\{\delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}(C_SC_T^* - C_VC_A^* + C_S'C_T'^* - C_V'C_A'^*)\Bigg\},\quad\text{(A9)}$$

$$G\xi = \pm2\,\mathrm{Re}\{|M_F|^2(C_SC_S'^* - C_VC_V'^*) + |M_{GT}|^2(C_TC_T'^* - C_AC_A'^*)\},\quad\text{(A10)}$$

$$H\xi = 2\,\mathrm{Re}\Bigg\{|M_F|^2\left[-(C_SC_V'^* + C_S'C_V^*)\mp\frac{m}{E_e}(C_SC_S'^* + C_VC_V'^*)\right]$$

$$+\tfrac{1}{3}|M_{GT}|^2\left[C_TC_A'^* + C_T'C_A^*\pm\frac{m}{E_e}(C_TC_T'^* + C_AC_A'^*)\right]\Bigg\},\quad\text{(A11)}$$

$$K\xi = 2\,\mathrm{Re}\{|M_F|^2(\mp C_SC_S'^*\mp C_VC_V'^* + C_SC_V'^* + C_S'C_V^*)$$

$$+\tfrac{1}{3}|M_{GT}|^2(\pm C_TC_T'^*\pm C_AC_A'^* - C_TC_A'^* - C_T'C_A^*)\},\quad\text{(A12)}$$

$$L\xi = 2\,\mathrm{Im}\{|M_F|^2(C_SC_V^* + C_S'C_V'^*) - \tfrac{1}{3}|M_{GT}|^2(C_TC_A^* + C_T'C_A'^*)\},\quad\text{(A13)}$$

$$N\xi = 2\,\mathrm{Re}\Bigg\{|M_{GT}|^2\lambda_{J'J}\left[\frac{1}{2}\frac{m}{E_e}(|C_T|^2 + |C_A|^2 + |C_T'|^2 + |C_A'|^2)\pm(C_TC_A^* + C_T'C_A'^*)\right] + \delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}$$

$$\times\left[(C_SC_A^* + C_VC_T^* + C_S'C_A'^* + C_V'C_T'^*)\pm\frac{m}{E_e}(C_SC_T^* + C_VC_A^* + C_S'C_T'^* + C_V'C_A'^*)\right]\Bigg\},\quad\text{(A14)}$$

$$Q\xi = 2\,\mathrm{Re}\Bigg\{|M_{GT}|^2\lambda_{J'J}\left[\tfrac{1}{2}(|C_T|^2 + |C_A|^2 + |C_T'|^2 + |C_A'|^2)\mp(C_TC_A^* + C_T'C_A'^*)\right] - \delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}$$

$$\times[(C_SC_A^* + C_VC_T^* + C_S'C_A'^* + C_V'C_T'^*)\mp(C_SC_T^* + C_VC_A^* + C_S'C_T'^* + C_V'C_A'^*)]\Bigg\},\quad\text{(A15)}$$

$$R\xi = 2\,\mathrm{Im}\Bigg\{\pm|M_{GT}|^2\lambda_{J'J}(C_TC_A'^* + C_T'C_A^*)$$

$$+\delta_{J'J}|M_F||M_{GT}|\left(\frac{J}{J+1}\right)^{\frac{1}{2}}(C_SC_A'^* + C_S'C_A^* - C_VC_T'^* - C_V'C_T^*)\Bigg\}.\quad\text{(A16)}$$