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Experimental Study of Plasmoids

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A plasma source can be used to project ionized matter across a magnetic field. The configuration of plasma observed when an electromagnetic braking action is produced by the presence of low-pressure gas ($\sim 1\ \mu$) in the vacuum chamber provides an insight into the manner in which magnetic-field lines can be dragged and twisted. By firing several sources simultaneously, it is possible to simulate in geometrical form the production of spiral galaxies and barred spirals.

I. PRODUCTION OF A PLASMOID IN FIELD-FREE SPACE

1. π_1-Plasmoid

It has not only been demonstrated[1,2] that plasma can be projected from a plasma gun or plasma source with speeds up to 2×10^7 cm/sec, but the experimental observations suggest that the plasma travels (even in field-free space) not as an amorphous blob, but as a structure (called a plasmoid) whose form is determined by the magnetic field it carries along with itself. The mechanism whereby the plasma is propelled from the source has already been outlined,[1] and the hypothesis that the plasma travels in field-free space in the form of a torus has been supported with a Kerr-cell picture of the plasma leaving the source in a toroidal form.

The probe traces shown in Figs. 1 and 2 are further evidence that the projected plasma has structure. The exact structure of the plasma cannot be accurately delineated from these probe traces, but at least the probe traces are consistent with the hypothesis that the plasma is in the form of a torus. It is unfortunate, from the observational point of view, that these plasmoids move so rapidly. If they could be produced so that their center of mass was stationary in the laboratory, they could be photographed much more easily. (Later in this paper there will be described stationary plasmoids formed in a magnetic field which definitely exhibit toroidal structure.)

It has been possible with a magnetic coupling loop to pick up signals that are believed to be associated with the magnetic fields trapped by the plasmoid of the type shown in Figs. 2, 3, and 4 of reference 1. (Note that no external dc magnetic field is employed here.) Examples of such signals are shown in Fig. 3. Although the structure of these signals is too complex for analysis,

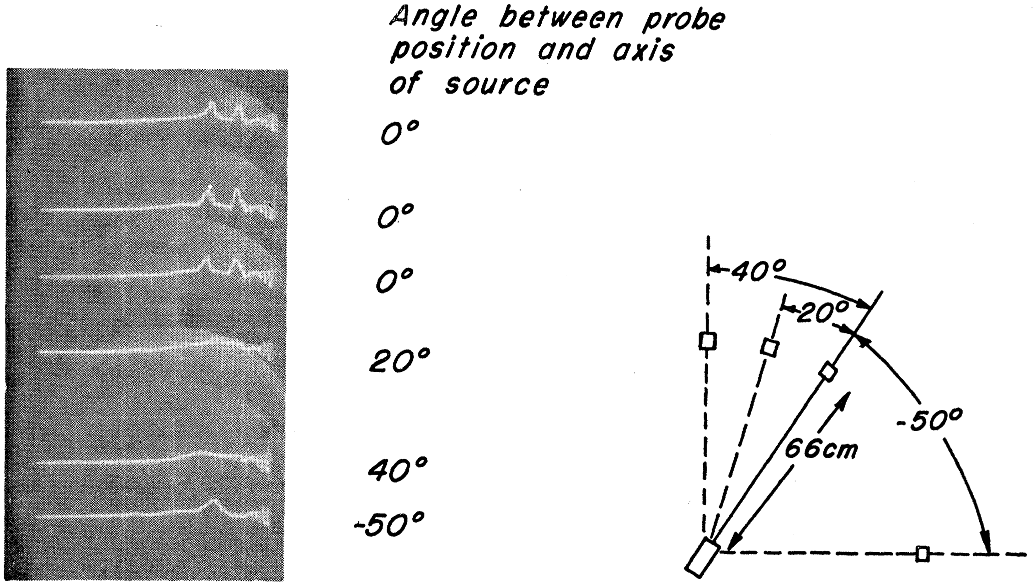

FIG. 1. Probe traces taken with alnico bar-magnet probes at 66 cm from the source of plasma at the various angles shown. The peak current in the source is about 3000 amperes. The sweep speed is 5 μsec/cm. Time goes from right to left and the source is fired at the beginning of the trace. The alnico bar-magnet is $1\times1\times2$ cm.

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[1] W. H. Bostick, Phys. Rev. **104**, 292 (1956).

[2] Harris, Theus, and Bostick, Phys. Rev. **105**, 46 (1957).

these signals are nevertheless experimentally identified with the magnetic fields carried by the plasmoid.

In the magnetic coupling loop signals, as with the probe signals, the outstanding feature is the steep leading edge, which can in no way be associated with an ordinary shock wave. The steepness of the leading edge of the plasmoid may possibly explain the abrupt onset of magnetic storms approximately 24 hours after a solar disturbance. It is entirely possible that ions and electrons ejected from the sun come to the earth in the form of a plasmoid.

In fact if a probe is immersed in a local magnetic field (e.g., due to a bar magnet) and a plasma source is fired at the probe at 1-meter distance, the signal from the probe not only has a steep leading edge, but it exhibits large irregular oscillations that indicate rapidly varying ion densities and electric fields produced by the plasma encountering the stationary magnetic field.

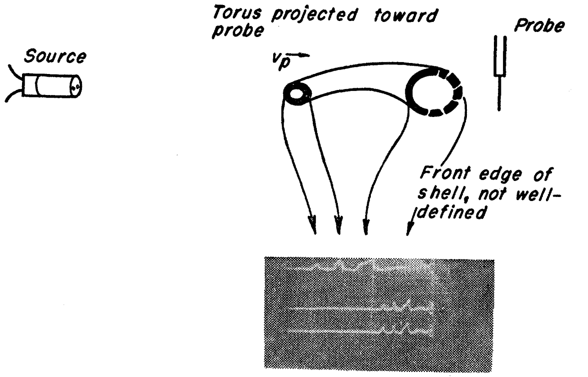

FIG. 2. Probe traces taken at a distance of 10 cm from the source at sweep speeds of 0.5 μsec/cm and 1.0 μsec/cm. Sensitivity is 15 v/cm. The signal is developed across a 50-ohm resistance to ground. The probe is 1 mm in diam and 2 cm long. Time goes from right to left and the trace starts at the firing of the source. An attempt has been made to identify the various parts of the hypothesized torus with the portions of the 0.5-μsec/cm trace.

The type of plasmoid (to be designated the π_1-plasmoid, because it is unstable) diagrammed in Fig. 4 of reference 1 is expected to expand in directions of both increasing R and r, which are respectively the large and small radii of the torus. If m_T is the total mass of the plasmoid in grams and I is the total circulating current in amperes, an approximate differential equation (neglecting the logarithmic term) is

$$m_T d^2R/dt^2 = 8I^2/50\pi. \tag{1}$$

If at $t=0$ the original flux $\phi_0 \cong 2\pi^2 I_0 R_0/5$, where I_0 and R_0 are the initial current and radius, and $dR/dt=0$, the solution is given by

$$\begin{aligned} &R_0[R(1+R/R_0)]^{\frac{1}{2}} \\ &\quad -\tfrac{1}{2}R_0^{\frac{3}{2}} \ln\{[(1+R/R_0)^{\frac{1}{2}}+(R/R_0)^{\frac{1}{2}}]/\sqrt{R_0}\} \\ &\qquad -R_0^{\frac{3}{2}}\{\sqrt{2}-\tfrac{1}{2}\ln[(\sqrt{2}+1)/\sqrt{R_0}]\} \cong (2\gamma)^{\frac{1}{2}}t, \end{aligned} \tag{2}$$

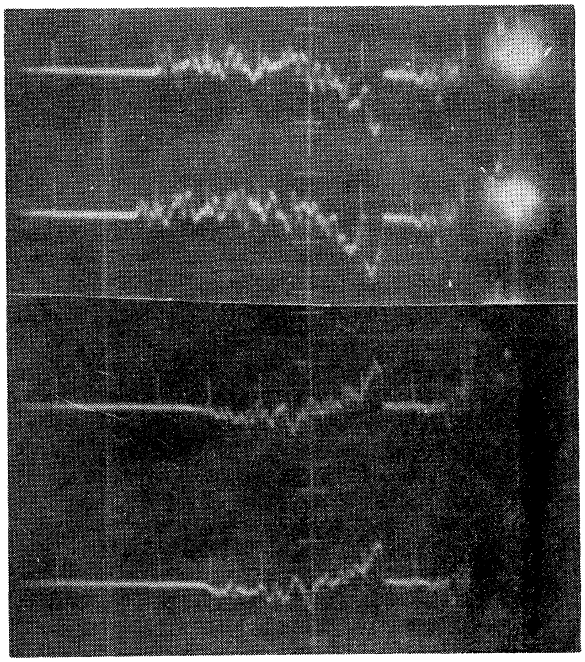

FIG. 3. Signals from a 1-turn, 0.64-cm-diam coupling loop terminated with 50 ohms, where there is no external magnetic field applied. The coil is oriented in the plane of the paper, with respect to the source shown in Fig. 2, reference 1, and located 10 cm directly in front of the source. The sensitivity is 0.5 v/cm. The sweep speed is 0.5 μsec/cm, with time going from right to left. The first two traces represent the current in one direction in the source; the second two traces represent the current in the opposite direction. The true plasma signals arrive at the loop at a time of about 1.3 μsec after the firing of the source that triggers the sweep. These signals are an indication of trapped magnetic fields within the plasma.

where

$$\gamma \equiv 8I_0^2 R_0^2/50\pi m_T.$$

From this solution it can be seen that when $R/R_0 \gg 1$, $R \sim t$. The expected expansion of the π_1-plasmoid may thus be thought of as a magnetic explosion where most of the outward velocity is picked up when the plasmoid is small.

2. S-Plasmoid

It is possible to conceive of a plasmoid which at first sight seems to be more stable than the π_1-plasmoid. This plasmoid, which we shall designate the S-plasmoid, is diagrammed in Fig. 4. Conceivably this plasmoid could be produced by winding a thin metallic ribbon

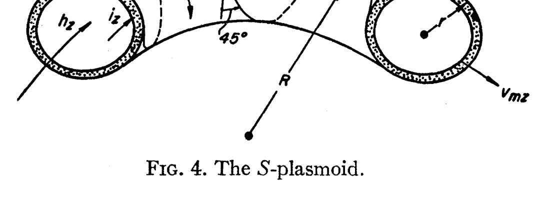

FIG. 4. The S-plasmoid.

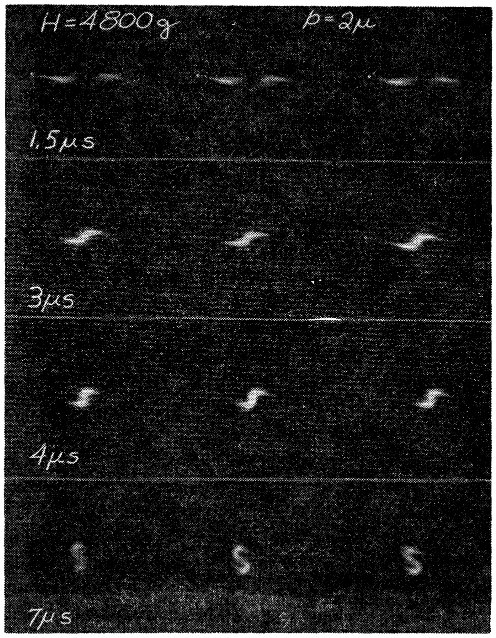

FIG. 5. A sequential study of barred spirals which are produced by firing two plasmoids from sources 10 cm apart at one another simultaneously across a magnetic field of 4800 gauss. The Kerr-cell exposure times are 2 μsec and the various delay times of the sequence are indicated in μsec. The pressure in the chamber is 2μ. The plasmoids and their trajectories are rendered luminous primarily by the recombination light of the titanium and deuterium ions that come from the plasma source. The photographs on the left are the left stereoscopic photographs and those on the right, the right stereoscopic photographs, with an angle between the two views of 10°. The middle photograph is taken straight ahead along the direction of the magnetic field. The current (3000 amperes for 0.4 sec) through the source produces a magnetic field which opposes the dc field and diminishes the velocity of projection of the plasmoid across the magnetic field.

into a helix and then bending the helix into a torus. If the metallic ribbon is then suddenly energized with a current so that it is vaporized and ionized, there presumably would be formed the S-plasmoid. Though this method of producing the S-plasmoid in field-free space would be tedious, it might nevertheless be fruitful and should eventually be tried.

From the theoretical point of view let us briefly examine the S-plasmoid in field-free space (i.e., no external dc magnetic field) and see if it is stable.

If we neglect the effects of nkT and centripetal acceleration due to any rotary motions of the mass of plasma, we note that equilibrium about r (see Fig. 4) requires that

$$h_z = h_\theta,$$

or that

$$i_z = i_\theta.$$

Then since

$$I_z = 2\pi r i_z \quad \text{and} \quad I_\theta = 2\pi R i_\theta,$$

we have

$$I_\theta / I_z = R/r.$$

Let us assume that at time $t=0$, the initial current $I_z = I_0$, and that the plasma is a good-enough conductor so that for all subsequent time the initial magnetic fluxes ϕ_z and ϕ_θ will be preserved. For purposes of simplification, electromagnetic units are used here. Then

$$\begin{aligned}\phi_z &= L_z I_z = 4\pi R[\ln(8R/r)-2]I_z\\ &= L_{z0} I_0 = 4\pi R[\ln(8R_0/r_0)-2]I_0 \end{aligned} \tag{3}$$

and

$$W_z = \tfrac{1}{2}L_z I_z^2 = 2\pi R[\ln(8R/r)-2]I_z^2. \tag{4}$$

Also,

$$\phi_\theta = L_\theta I_\theta = 2\pi r^2 I_\theta/R = 2\pi r^2 R I_z/Rr = 2\pi r I_z \tag{5}$$

$$= L_{\theta 0} I_{\theta 0} = 2\pi r_0 I_0 \tag{6}$$

and

$$W_\theta = \tfrac{1}{2}L_\theta I_\theta^2 = \tfrac{1}{2}(2\pi r^2 I_\theta^2/R) = \pi R I_z^2. \tag{7}$$

Now let us note that if ϕ_z and ϕ_θ are constant, $\alpha \equiv \phi_z/\phi_\theta$ must remain constant, and

$$\phi_z/\phi_\theta = (2R/r)[\ln(8R/r)-2].$$

Hence, preservation of flux ϕ_z and ϕ_θ requires that R/r be a constant.

The total energy is

$$\begin{aligned} W_T &= W_z + W_\theta\\ &= \tfrac{1}{2}\phi_z^2/L_z + \tfrac{1}{2}\phi_\theta^2/L_\theta = \tfrac{1}{2}\phi_z^2[1/L_z + 1/(\alpha^2 L_\theta)]\\ &= \tfrac{1}{2}\phi_z^2\{(1/4\pi R)[\ln(8R/r)-2]^{-1} + R/(2\pi r^2\alpha^2)\}\\ &= (\phi_z^2/4\pi r)\{(r/2R)[\ln(8R/r)-2]^{-1} + R/(r\alpha^2)\}. \end{aligned} \tag{8}$$

Now the force which will expand the S-plasmoid, and yet preserve the fluxes ϕ_z and ϕ_θ and hence R/r, is

$$\begin{aligned} &-(\partial W_T/\partial r)_{R/r}\\ &\quad = (\phi_z^2/4\pi r^2)\{(r/2R)[\ln(8R/r)-2]^{-1} + R/(r\alpha^2)\}. \end{aligned} \tag{9}$$

This force is to be compared with the force which expands only the π_1-plasmoid:

$$-(\partial W_z/\partial r)_{R/r} = (\phi_z^2/8\pi r R)[\ln(8R/r)-2]^{-1}. \tag{10}$$

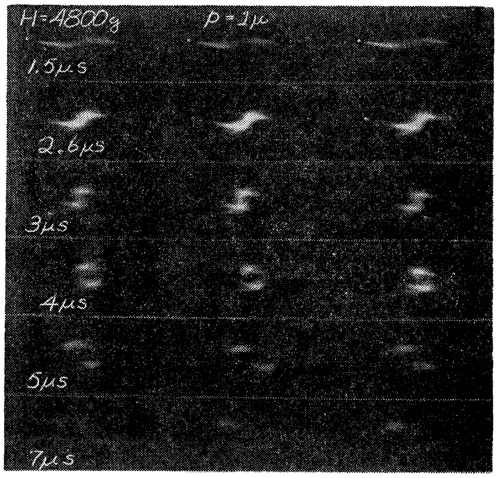

FIG. 6. The same as Fig. 5 except that the pressure in the chamber is 1 μ.

Equations (9) and (10) give the somewhat unexpected result that for all values of $R/r \geq 1$, $|\partial W_T/\partial r| > |\partial W_z/\partial r|$, and hence the *S*-plasmoid should expand actually faster than the π_1-plasmoid.

It is interesting to note that for $W_z = W_\theta$, $\phi_z = \phi_\theta$ (i.e., where stability might possibly occur), the corresponding value of R/r is 1.0. Unfortunately, the formulas used for L_z, ϕ_z, and W_z do not hold below $R/r \geq 2.5$. It is further interesting to note that for $R/r = n$, where n is an integer, a current streamline will coincide cyclically with itself after one complete revolution of the plasma. And since R/r is a constant, the preservation of $R/r = n$ amounts to a kind of macroscopic quantum condition.

It is possible to plot W_T as a function of R for various values of r. The minima of these curves represents the situation where $W_\theta = W_z$, $\phi_\theta = \phi_z$, $\alpha = 1.0$, and $R/r = 1.0$.

The expansion of the *S*-plasmoid will presumably occur along the minima of the curves and in the direction of increasing r. However, the portion of the curves to the left of minima have no meaning because here $R/r < 1$. Hence, any realizable conditions involve a trajectory considerably to the right of the minima. It will be necessary to make the computation with expressions for L_z and L_θ which hold for values of R/r that are close to 1.

Furthermore, the stability relationships should be examined for configurations (like a muff) where the cross section of the *S*-plasmoid is not circular, but oblong.

This brief analysis suggests that the *S*-plasmoid is unstable, but it cannot be definitely stated that all shapes of *S*-plasmoids in field-free space are unstable. There is a very real possibility that the *S*-plasmoid immersed in an external dc magnetic field would be stable.

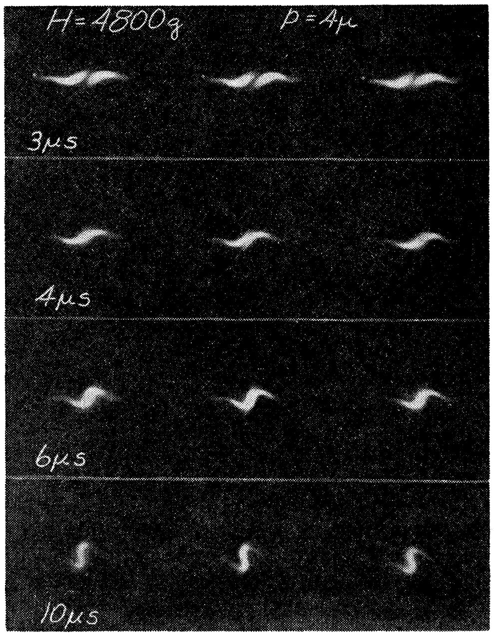

FIG. 7. The same as Fig. 5 except that the pressure is 4 μ.

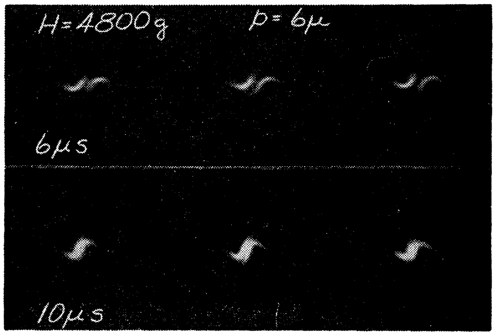

FIG. 8. The same as Fig. 5 except that the pressure is 6 μ.

II. PRODUCTION OF PLASMOIDS IN A MAGNETIC FIELD

1. Introduction

It has already been demonstrated[1,2] that plasmoids can be projected across a magnetic field. It is quite possible that ionized material ejected from the surface of the sun proceeds and escapes across the magnetic field of the sun in the same manner that laboratory-produced plasmoids cross a magnetic field.

Plasmoids produced in a good vacuum are elongated cylinders[1,2] that travel as a wave front of constant velocity across the magnetic field. These plasmoids bear very little resemblance to the *S*-plasmoid and move so rapidly as to make instantaneous photography very difficult if not impossible. However, these plasmoids leave a wake or track[1] which enables us to photograph their path quite easily.

It is further observed[1] that these plasmoids experience an electromagnetic braking action which decelerates and deflects them when they encounter one another or when they travel through gas at a pressure of about one micron. Indeed, several of these plasmoids can be made to spiral[1] in consort to produce a ring of plasma. The organic relation between this laboratory-observed process and the evolution of spiral galaxies and stars has already been suggested. More recent measurements[3] with a Kerr cell portray a time-sequence in the formation of this torus or ring, and show that not only is the torus produced automatically, but also that it is stationary (i.e., with regard to translation) and stable over a period of at least 30 μsec. During this time the torus appears to retain a circular form of about the same large and small radii.

There is some temptation to identify this observed[3] torus with the *S*-plasmoid immersed in a magnetic field. Before succumbing to this temptation, let us examine the results of some more recent measurements which

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[3] W. H. Bostick and O. A. Twite, Nature **179**, 214 (1957).

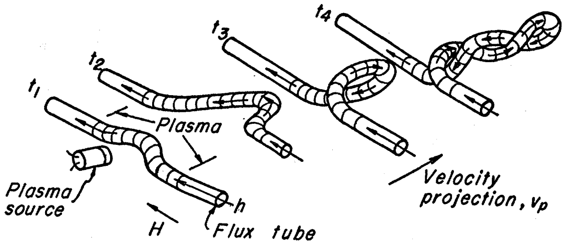

FIG. 9. Suggested configuration of plasma and magnetic field of a single plasmoid projected across a magnetic field when an electromagnetic braking action occurs because of a pressure of about 1 μ in the vacuum chamber. The tightly twisted configuration will itself assume a general helical configuration (try twisting two strands of wire, rope, or rubber tightly). This helical configuration is seen especially clearly in Figs. 7 and 8.

teach us the prudent lesson that somehow we must learn to understand the way in which magnetic-field lines are dragged, spun, and interwoven, if we are to give an adequate description of the resultant plasma configurations.

2. Barred Spirals

One of the relatively simplest results that must be understood is the "barred spiral" which is produced[3] by firing two sources at one another across a magnetic field. Over a wide range of variation of parameters the two plasmoids seem to seek each other out unerringly. The sequential stereoscopic photographs of Fig. 5 show the process of production of these "barred spiral" structures at a pressure of 2 μ.

It can be seen from Fig. 5 that the leading edges of the two plasmoids seem to seek, and attach themselves to, each other. The same process can be observed in Fig. 6, where the pressure is 1 μ, where the bond between the two plasmoids apparently does not hold as well as in Fig. 5. The photographs of Fig. 6 also show the interesting feature that the tails of spiral arms become forked.

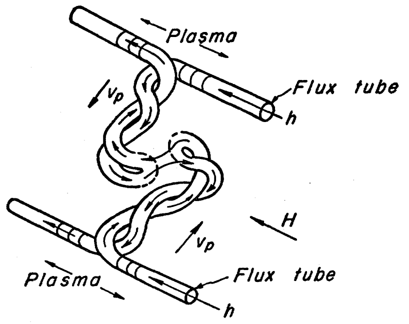

FIG. 10. Suggested configurations of plasma and magnetic fields existing in the formation of a barred spiral. The stagnation point produced when the two leading loops of the plasmoids approach one another permits the lines of force to leap from one plasmoid to another carrying plasma across and tying the two plasmoids together.

Figure 7 shows a sequence of photographs where the pressure is 4 μ and where the leading edges of the plasmoids are positioned by the twisting of the plasmoids so that they are in no position to conveniently attach to one another. Under these circumstances the two plasmoids press tightly against one another but remain separated for at least 6 μsec. In Fig. 8 where the pressure is 6 μ, the plasmoids remain separated for at least 10 μsec. Furthermore, the stereoscopic photographs of Figs. 5, 6, 7, and 8 show that the plasmoid in proceeding across the magnetic field at these fairly high pressures assumes the form of a helix of progressively increasing diameter. These particular helixes are all left-handed screws because the dc magnetic field was always in the same direction.

The photographs of Figs. 5, 6, 7, and 8 give us enough information to suggest that the configuration of plasma and magnetic field, when one plasma source is fired across a magnetic field, is that shown in Fig. 9. Eventually it may be possible to analyze this process quantitatively. For the moment, a description by a drawing will have to suffice.

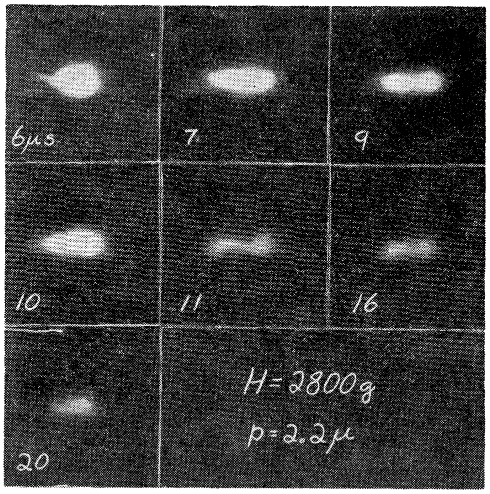

FIG. 11. Sequence of photographs of formation of a flattened ring which flips into a "figure 8." The ring is formed by firing two sources across a magnetic field (into the paper) of 2800 gauss with a pressure of 2.2 μ in the chamber. The exposure time is 2 μsec and the various delay times are indicated in μsec. The current in the source is in such a direction as to diminish the velocity of propagation in the magnetic field.

It is now possible to see how two plasmoids fired at one another across a magnetic field can attach themselves to each other, as shown in Fig. 10. Such a plasma-magnetic field configuration can also explain the forked tail on each plasmoid seen very clearly in Fig. 6. Apparently if the leading loops of the two plasmoids have twisted into such a position that no stagnation point is reached, the plasmoids studiously avoid one another as shown in Figs. 7 and 8.

After the union of the two plasmoids has been accomplished, as in Figs. 5 and 6, the angular momentum will wind them up into a spiral, to a certain extent, until the angular momentum has been brought to zero by the stretching of the field lines. The resultant plasma and magnetic configuration then seems to be stable. The barred spirals have been followed in time out to 15 seconds, at which they still preserve their shapes with well-defined boundaries. Furthermore the plasma does not seem to migrate in the direction of the original dc magnetic field. It is rather astonishing that such a bizarre configuration of plasma and magnetic field should appear to be stable. No theoretician known to the author has *a priori* dreamed of such a configuration, to say nothing of contemplating its stability.

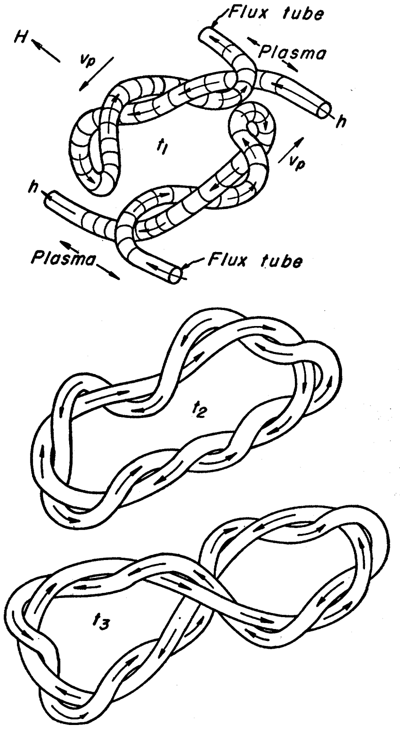

FIG. 12. Suggested sequence of plasma and magnetic field configurations to explain the flattened ring and "figure 8" observed in Fig. 11. The velocity of projection v_p of the original plasmoids is indicated.

3. Production of Rings

We must now try to understand, at least in a qualitative way, how rings or tori can be produced by plasmoids. Measurements already reported[3] show that with four plasma sources a ring can be produced which apparently maintains its shape for at least 30 seconds. Moreover, it is observed that this ring does not move or stretch appreciably in the direction of the dc magnetic field during this time interval. The magnetic-field configuration in the ring must, therefore, be such as to confine the plasma in this fairly stable ring.

It has been possible to produce a ring with only two

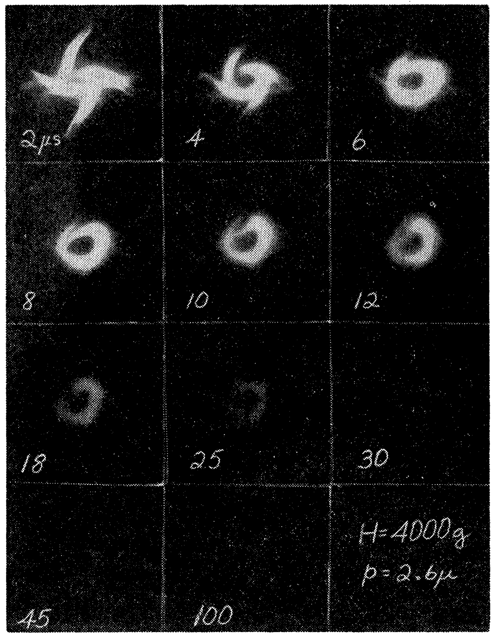

FIG. 13. An example of the formation of a ring by firing four sources across a magnetic field of 4000 gauss (into the paper) at a pressure of 2.6 μ. The exposure time is 2 μsec and the various delay times are indicated in μsec. The current in the sources is in such a direction that the velocity of projection of the plasma is retarded by the dc magnetic field. The 100-μsec delay photograph actually shows a faint ring on the original.

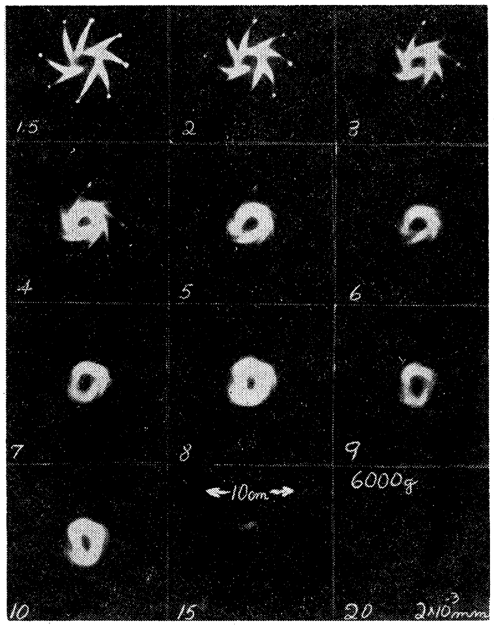

FIG. 14. Photographic sequence of the formation of a ring from the plasmoids from eight sources fired across a magnetic field of 6000 gauss, into the paper. The pressure is 2 μ. The exposure time is 2 μsec, and the delay times are indicated in μsec. The current in the sources is in the direction to diminish the velocity of projection of the plasmoids across the dc magnetic field. Note that at 15-μsec delay, the ring has grown an ear.

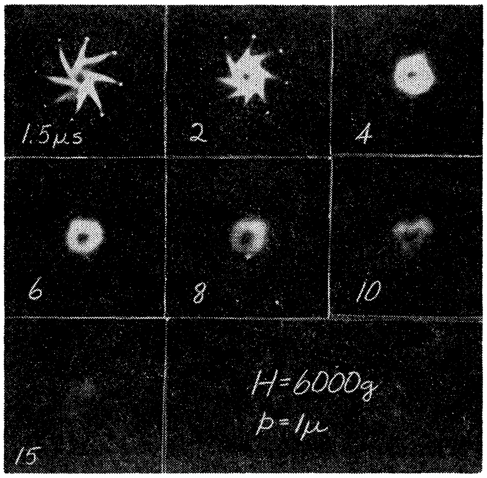

FIG. 15. Same as Fig. 14 except that the sources are oriented more symmetrically and the pressure is 1 μ.

sources (see Fig. 11), but the ring produced is flattened. In fact, as time goes on the ring configuration develops a constriction and flips over into a "figure 8." Before attempting to understand the rings that are produced with four or more plasma sources, let us first try to understand this flattened ring which is produced by two sources. Figure 12 suggests a plasma-magnetic field configuration to explain the ring shown in Fig. 11. It is readily understandable that a flattened ring formed by tightly twisted strands will constrict in the center and form a "figure 8." It can be seen from the hypothesis of Fig. 12 that we might actually expect two rings, one formed from each strand, but that they are topologically intertwined.

As has already been reported[1,3] it is possible to form rings by firing four sources. It is believed that these rings have essentially the same structure as the ring shown in Figs. 11 and 12, except that they are initially circular instead of flattened, and they have more angular momentum. Therefore, we may expect the rings formed by four sources to have a tendency to preserve their circular shape instead of flipping into a "figure 8." An

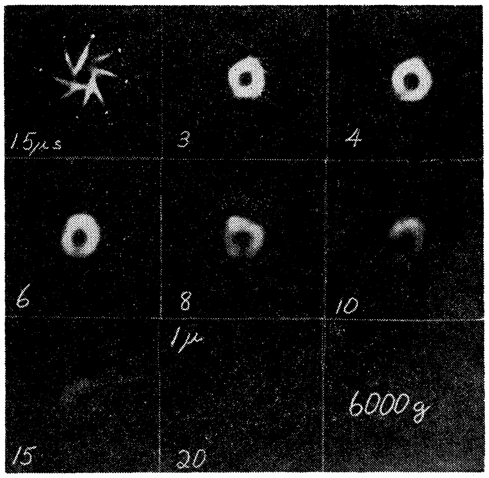

FIG. 16. Same as Fig. 14 except that the sources are aimed so as to produce a smaller diameter ring and the pressure is 1 μ.

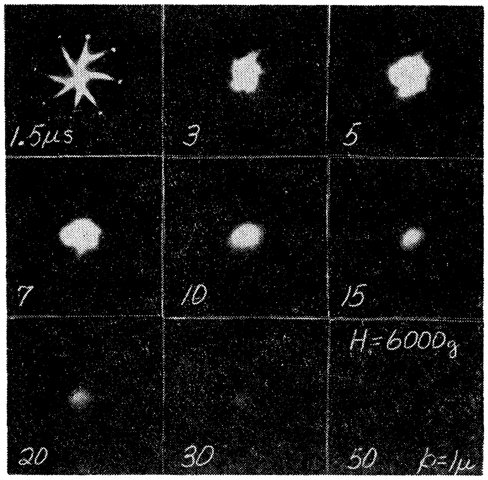

FIG. 17. Same as Fig. 16 except that the sources are aimed to produce an even smaller ring. The original photograph shows an illuminated blob of plasma lasting out to 50 μsec.

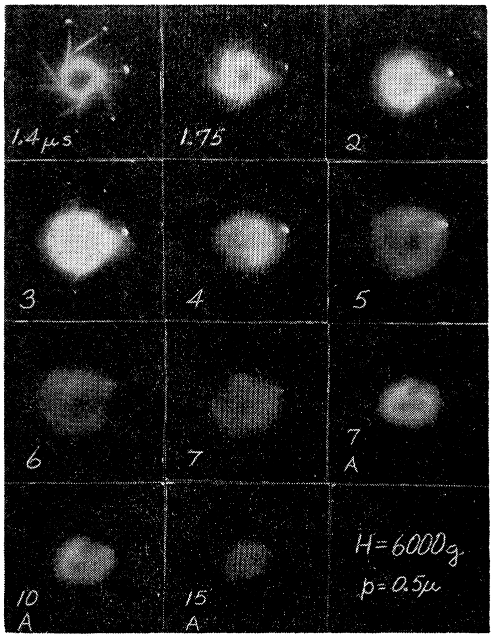

FIG. 18. Sequence of photographs taken in a manner similar to those of Fig. 14 except that the source current is in the direction which *aids* the velocity of propagation of the plasmoid across the magnetic field, and the pressure is 0.5 μ. The whirl-ring that is formed gets progressively out of focus as it apparently moves toward the camera, and at 7-μsec delay, it is badly blurred. The delay times "7*A*, 10*A*, and 15*A*" corresponds to moving the camera back 15 cm which, at least for 7*A*, sharpens up the picture. It is believed that for delays 10*A* and 15*A*, and perhaps for 7*A*, the ring has traveled until it has encountered the Lucite window of the vacuum system which is 20 cm from the position where the ring is formed.

example of a photographic sequence of the formation of a ring from four sources is given in Fig. 13.

It is possible to produce rings somewhat similar to those of Fig. 13 by firing eight sources. Various examples of photographic sequences with eight sources are given in Figs. 14, 15, 16, and 17. Very likely, the rings formed by eight sources have higher peripheral velocity and hence more angular momentum than those formed from four sources. This higher peripheral velocity may account for the deformations of the ring that are observed in Figs. 14, 15, and 16 at the later times.

It is to be emphasized that these rings, which have been produced by having the source current in the direction so as to diminish the velocity of propagation of the original plasmoids across the dc magnetic field, remain in focus up to at least 30 μsec, and in some cases up to 100 μsec. Therefore, we can say that the ring does not move or stretch appreciably in the direction of the magnetic field.

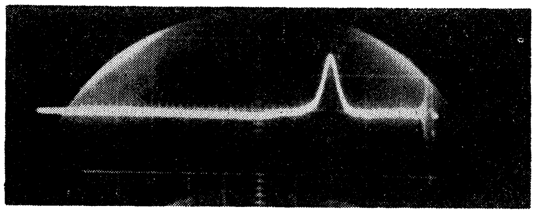

FIG. 19. Signal obtained by probe 1 mm in diameter, 0.5 cm long placed 1 cm from a grounded probe with 50 ohms connecting them. The probe assembly is placed 30-cm distance, axially (away from the camera) down the solenoid from the position of formation of the ring. The sensitivity is 2 v/cm and the sweep speed is 2 μsec/cm with time going from right to left. Probe assembly is placed laterally off the axis at the approximate radius of the plasma ring that is formed. The solenoid is 44 cm long and 13 cm in diameter. The pressure in the chamber was 2 μ. Much more work is necessary in the study of this type of signal. We tentatively identify these signals with a whirl-ring moving away from the camera.

The situation is quite different when the source current is in the direction so as to increase the velocity of propagation across the dc magnetic field. Under these circumstances, it has been observed[1,2] that the initial velocity of propagation across the field is greater and the initial diameter of the plasmoid is greater. A sequence of photographs taken under these circumstances with eight sources firing simultaneously is shown in Fig. 18, where the ring that is produced now apparently moves along the lines of force toward the camera. Probe traces (see Fig. 19) taken at a position 30 cm behind the source suggest that there is also plasma (presumably in the form of a ring) traveling along the lines of force in the opposite direction with a velocity $\leq 3\times 10^6$ cm/sec. Here now is a situation that appears to be the simultaneous production of two whirl-rings which move away from one another along the dc magnetic field lines. Figure 20 shows stereoscopic photographs taken of the ring that moves toward the camera. The ring appears to have a helical twist which probably represents the direction of magnetic field and motion of the plasma within the ring. A suggested

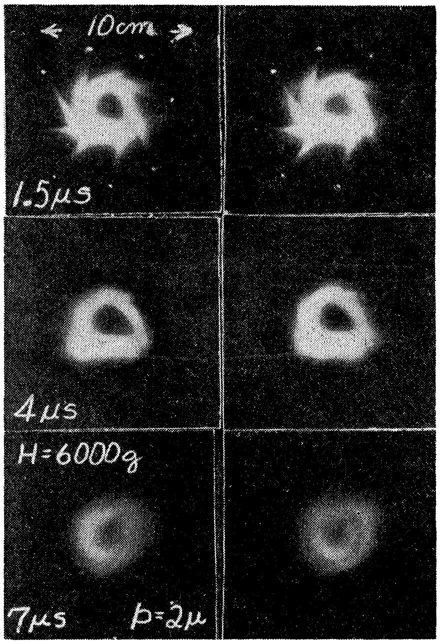

FIG. 20. A sequence of stereo photos (20° between the two) of the rings produced, as in Fig. 18, with the pressure equal to 2 μ. For the 7-μsec delay the camera was refocused for an object distance closer by about 15 cm, to compensate for the fact that the ring apparently moves toward the camera. The 4-μsec delay photograph, especially, suggests that the ring is constructed of material that has a helical twist.

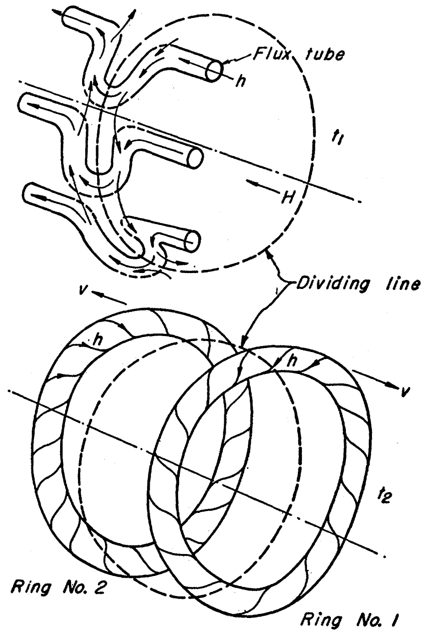

FIG. 21. Suggested description of formation of two whirl-rings that move away from one another, as observed in Figs. 19 and 20,

description of the process of formation of these two rings is given in Fig. 21, where it can be seen that the two rings that are now produced are not topologically entangled but are free to move away from one another. It is suggested that each of these rings is similar to the S-plasmoid of Fig. 4, except that they exist in a dc magnetic field, and the h_z and h_θ fields have time to penetrate the plasma and add vectorially to produce a helical magnetic field configuration within the ring. The measurements as yet do not yield quantitative information on whether the rings maintain their diameter as they move along the dc magnetic field lines. It will be necessary to construct a longer solenoid for the magnetic field than the 44-cm solenoid that was used in these experiments, in order to examine the radial stability of these rings and to measure their velocity along the field. It will also be necessary to devise a suitable technique for exploring the magnetic fields that are trapped in these rings.

CONCLUSION

By firing simultaneously two or more plasmoids across a magnetic field it has been possible to produce cooperative phenomena which, in geometrical form, suggest the simulation of the production of spiral galaxies and astronomical barred spirals. There is hence some promise that it will be possible to study these astronomical processes in the laboratory. Furthermore, the plasma and magnetic field configurations produced in the laboratory are, of themselves, of considerable physical importance. Hypotheses to explain the experimental effects have been advanced in outline. Accurate quantitative work and detailed theoretical analysis should now be undertaken.

ACKNOWLEDGMENTS

The author wishes to acknowledge the fine experimental work of Orrin A. Twite, who assisted in the work and took many of the photographs, and to V. G. McIntosh, who constructed the plasma sources. Gratitude is due to W. R. Baker, O. A. Anderson, Jack Reidel, and N. J. Norris for the loan of the Kerr-cell equipment. The author wishes to thank S. A. Colgate and C. M. Van Atta for their support, encouragement, and advice in this work, and Allen Kaufman for his suggestions concerning the analysis.

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Connection between Pair Density and Pressure for a Bose Gas Consisting of Rigid Spherical Atoms*

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It is shown that, for a Bose gas of rigid spherical atoms of radius a, the pressure p is given by the pair-density $g(r)$ and the energy per atom u by the formula

$$p/\rho T=\tfrac{2}{3}u/T+\tfrac{2}{3}\pi a^3\cdot\tfrac{1}{2}mT\cdot g''(a).$$

THE virial theorem, for a classical gas of rigid spherical atoms of radius a, leads to the well-known expression for the pressure p:

$$p/\rho T=\tfrac{2}{3}u/T+\tfrac{2}{3}\pi a^3 g(a).$$

(Boltzmann's constant is put equal to unity; ρ is the particle-density, u the energy per particle, and $g(r)$ the pair-density.)

We shall prove the corresponding formula for a Bose gas to be

$$p/\rho T=\tfrac{2}{3}u/T+\tfrac{2}{3}\pi a^3\cdot\tfrac{1}{2}mT\cdot g''(a). \tag{I}$$

(Planck's constant is put equal to unity.) This we will derive first from the virial theorem by a limiting process. As this derivation, though rather simple, is not quite satisfactory, we shall give a second proof where no limiting process is needed.

(1) The virial theorem reads

$$p/\rho=\tfrac{2}{3}u-\frac{4\pi}{6}\int_0^\infty g(r)r\frac{dV}{dr}r^2dr. \tag{1}$$

We assume the potential V to be of the following form:

$$\begin{aligned} V(r)&=0 \quad \text{when} \quad r>a,\\ V(r)&=V \quad \text{when} \quad r<a. \end{aligned} \tag{2}$$

For the integral in (1) this leads to

$$+\tfrac{2}{3}\pi a^3 g(a)V. \tag{3}$$

One has to study the limit of (3) for $V\rightarrow\infty$. Now for very big values of V, such that $a(Vm)^{\frac{1}{2}}\gg 1$ and the

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* The problem treated in the present note is due to George Placzek. In the limit of small densities he showed Eq. (I) of this paper to be valid. During a visit in Princeton (February, 1955) we discussed the general proof, and we hoped to publish our result together. George Placzek's untimely death has made this impossible.

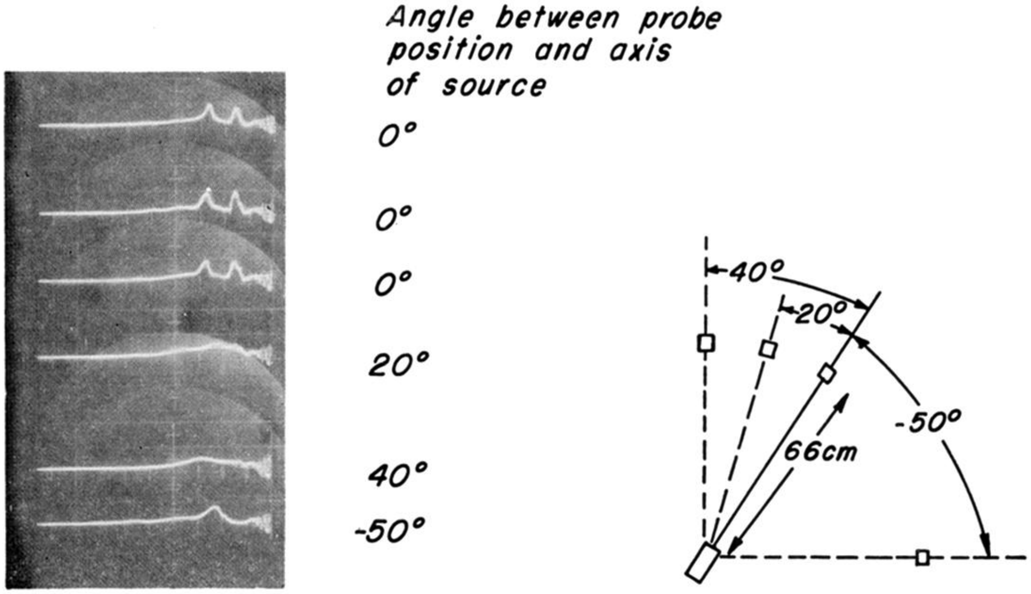

FIG. 1. Probe traces taken with alnico bar-magnet probes at 66 cm from the source of plasma at the various angles shown. The peak current in the source is about 3000 amperes. The sweep speed is 5 μsec/cm. Time goes from right to left and the source is fired at the beginning of the trace. The alnico bar-magnet is $1\times1\times2$ cm.

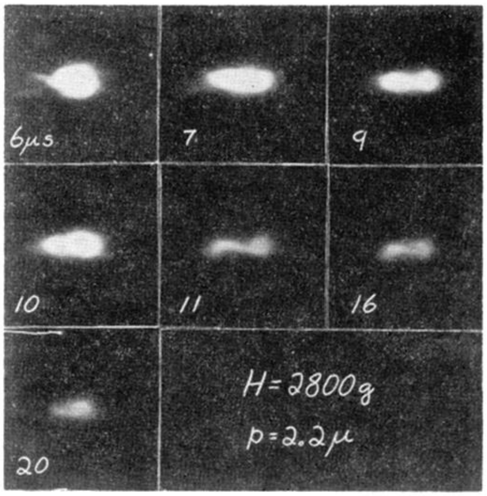

FIG. 11. Sequence of photographs of formation of a flattened ring which flips into a "figure 8." The ring is formed by firing two sources across a magnetic field (into the paper) of 2800 gauss with a pressure of 2.2 μ in the chamber. The exposure time is 2 μsec and the various delay times are indicated in μsec. The current in the source is in such a direction as to diminish the velocity of propagation in the magnetic field.

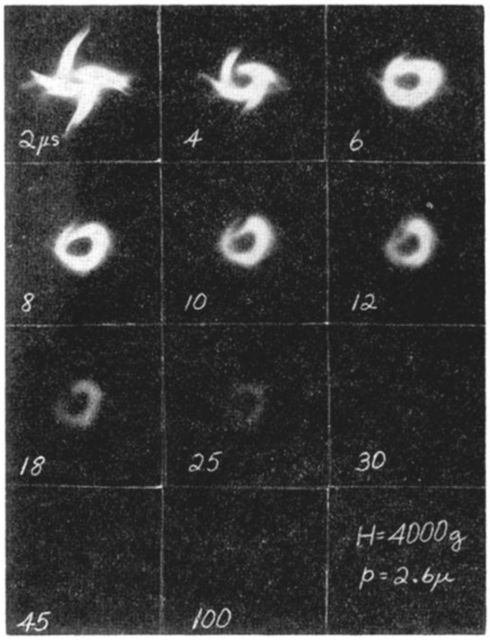

FIG. 13. An example of the formation of a ring by firing four sources across a magnetic field of 4000 gauss (into the paper) at a pressure of $2.6\,\mu$. The exposure time is $2\,\mu$sec and the various delay times are indicated in μsec. The current in the sources is in such a direction that the velocity of projection of the plasma is retarded by the dc magnetic field. The 100-μsec delay photograph actually shows a faint ring on the original.

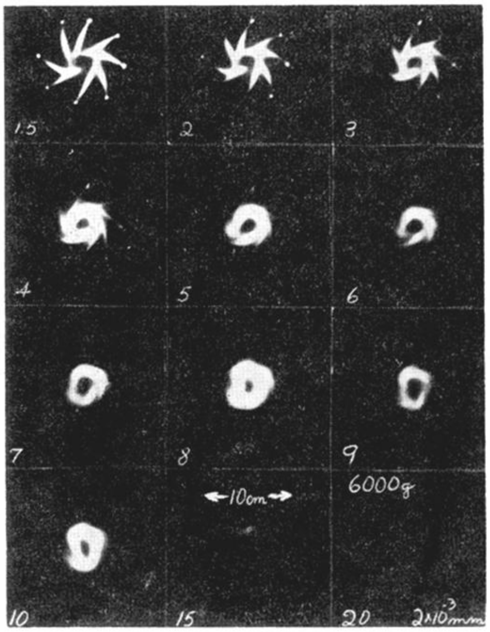

FIG. 14. Photographic sequence of the formation of a ring from the plasmoids from eight sources fired across a magnetic field of 6000 gauss, into the paper. The pressure is 2 μ. The exposure time is 2 μsec, and the delay times are indicated in μsec. The current in the sources is in the direction to diminish the velocity of projection of the plasmoids across the dc magnetic field. Note that at 15-μsec delay, the ring has grown an ear.

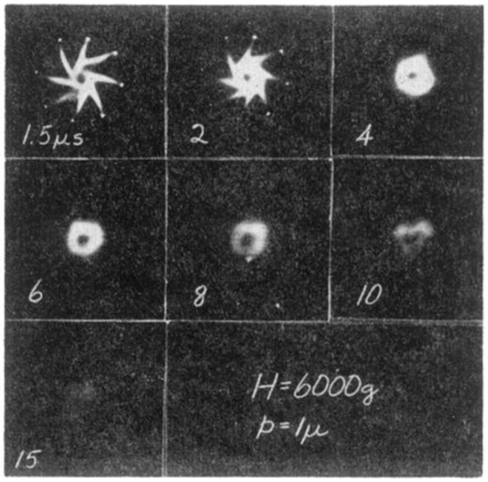

FIG. 15. Same as Fig. 14 except that the sources are oriented more symmetrically and the pressure is 1 μ.

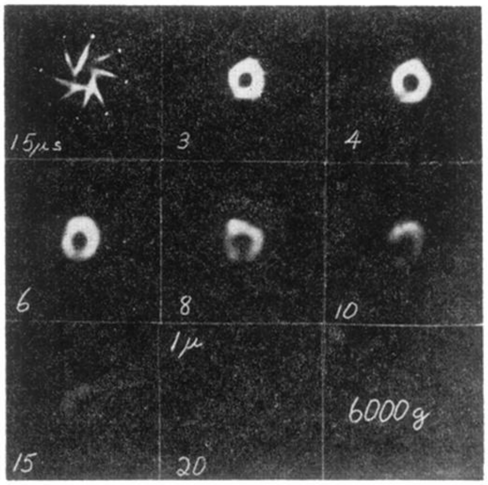

FIG. 16. Same as Fig. 14 except that the sources are aimed so as to produce a smaller diameter ring and the pressure is 1 μ.

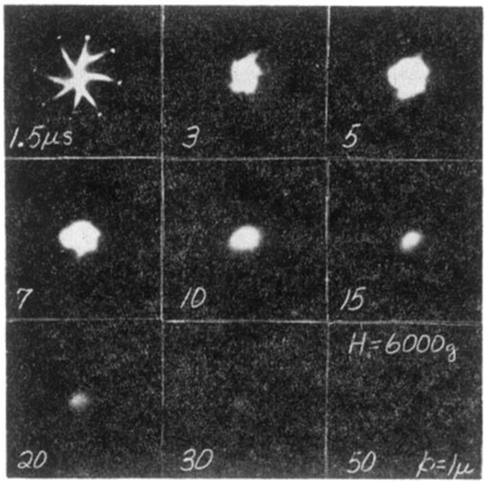

FIG. 17. Same as Fig. 16 except that the sources are aimed to produce an even smaller ring. The original photograph shows an illuminated blob of plasma lasting out to 50 μsec.

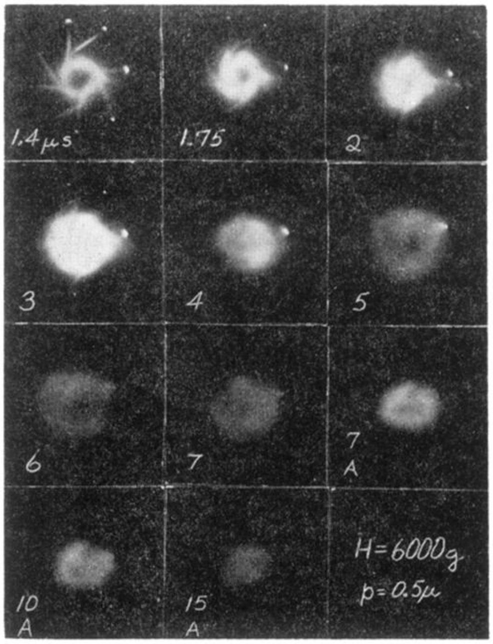

FIG. 18. Sequence of photographs taken in a manner similar to those of Fig. 14 except that the source current is in the direction which *aids* the velocity of propagation of the plasmoid across the magnetic field, and the pressure is 0.5 μ. The whirl-ring that is formed gets progressively out of focus as it apparently moves toward the camera, and at 7-μsec delay, it is badly blurred. The delay times "$7A$, $10A$, and $15A$" corresponds to moving the camera back 15 cm which, at least for $7A$, sharpens up the picture. It is believed that for delays $10A$ and $15A$, and perhaps for $7A$, the ring has traveled until it has encountered the Lucite window of the vacuum system which is 20 cm from the position where the ring is formed.

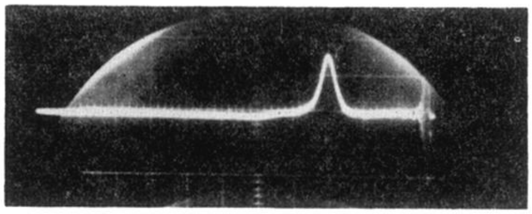

Fig. 19. Signal obtained by probe 1 mm in diameter, 0.5 cm long placed 1 cm from a grounded probe with 50 ohms connecting them. The probe assembly is placed 30-cm distance, axially (away from the camera) down the solenoid from the position of formation of the ring. The sensitivity is 2 v/cm and the sweep speed is 2 μsec/cm with time going from right to left. Probe assembly is placed laterally off the axis at the approximate radius of the plasma ring that is formed. The solenoid is 44 cm long and 13 cm in diameter. The pressure in the chamber was 2 μ. Much more work is necessary in the study of this type of signal. We tentatively identify these signals with a whirl-ring moving away from the camera.

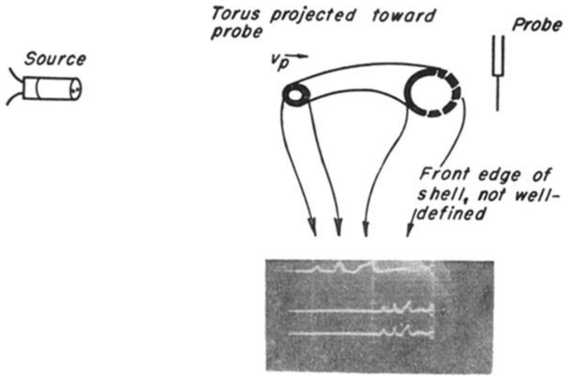

FIG. 2. Probe traces taken at a distance of 10 cm from the source at sweep speeds of 0.5 μsec/cm and 1.0 μsec/cm. Sensitivity is 15 v/cm. The signal is developed across a 50-ohm resistance to ground. The probe is 1 mm in diam and 2 cm long. Time goes from right to left and the trace starts at the firing of the source. An attempt has been made to identify the various parts of the hypothesized torus with the portions of the 0.5-μsec/cm trace.

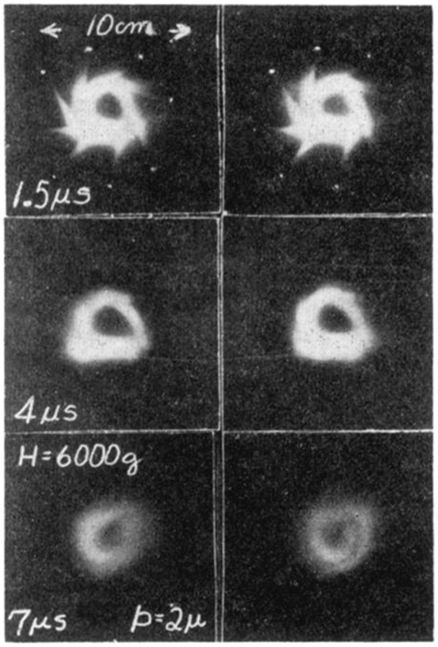

FIG. 20. A sequence of stereo photos (20° between the two) of the rings produced, as in Fig. 18, with the pressure equal to 2 μ. For the 7-μsec delay the camera was refocused for an object distance closer by about 15 cm, to compensate for the fact that the ring apparently moves toward the camera. The 4-μsec delay photograph, especially, suggests that the ring is constructed of material that has a helical twist.

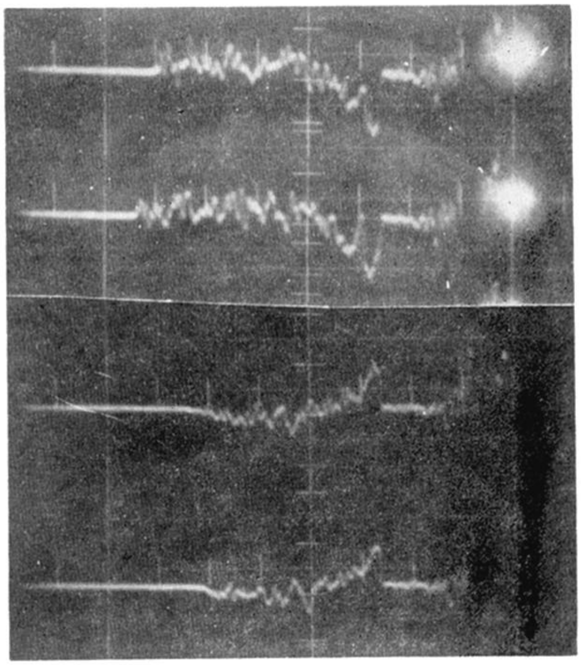

FIG. 3. Signals from a 1-turn, 0.64-cm-diam coupling loop terminated with 50 ohms, where there is no external magnetic field applied. The coil is oriented in the plane of the paper, with respect to the source shown in Fig. 2, reference 1, and located 10 cm directly in front of the source. The sensitivity is 0.5 v/cm. The sweep speed is 0.5 μsec/cm, with time going from right to left. The first two traces represent the current in one direction in the source; the second two traces represent the current in the opposite direction. The true plasma signals arrive at the loop at a time of about 1.3 μsec after the firing of the source that triggers the sweep. These signals are an indication of trapped magnetic fields within the plasma.

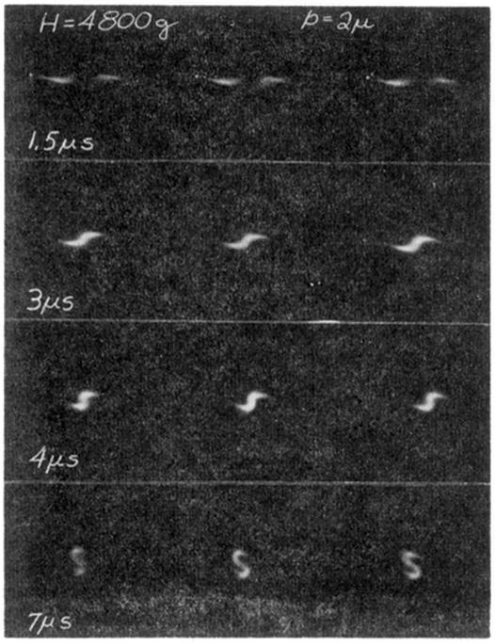

Fig. 5. A sequential study of barred spirals which are produced by firing two plasmoids from sources 10 cm apart at one another simultaneously across a magnetic field of 4800 gauss. The Kerr-cell exposure times are 2 μsec and the various delay times of the sequence are indicated in μsec. The pressure in the chamber is 2μ. The plasmoids and their trajectories are rendered luminous primarily by the recombination light of the titanium and deuterium ions that come from the plasma source. The photographs on the left are the left stereoscopic photographs and those on the right, the right stereoscopic photographs, with an angle between the two views of 10°. The middle photograph is taken straight ahead along the direction of the magnetic field. The current (3000 amperes for 0.4 sec) through the source produces a magnetic field which opposes the dc field and diminishes the velocity of projection of the plasmoid across the magnetic field.

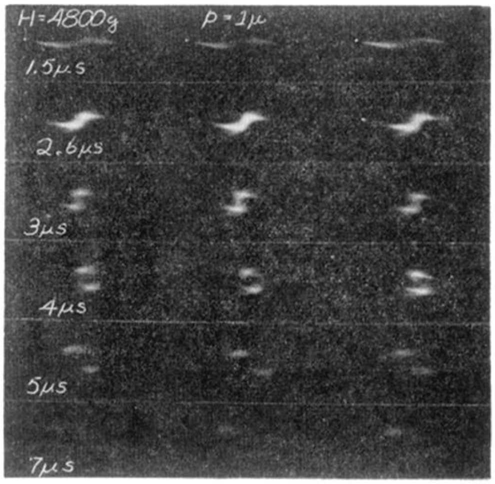

FIG. 6. The same as Fig. 5 except that the pressure in the chamber is 1 μ.

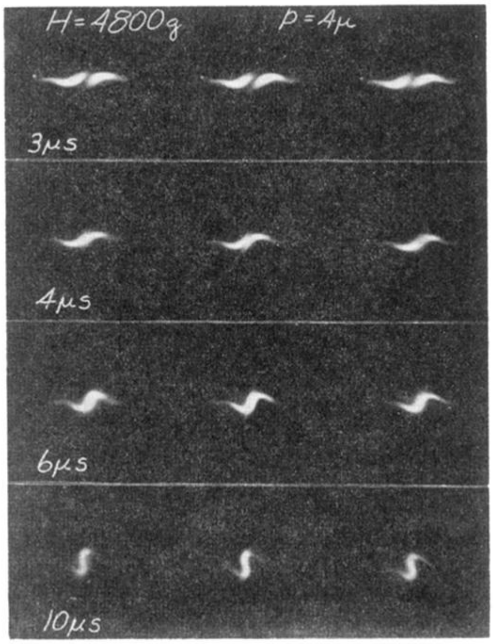

FIG. 7. The same as Fig. 5 except that the pressure is 4 μ.

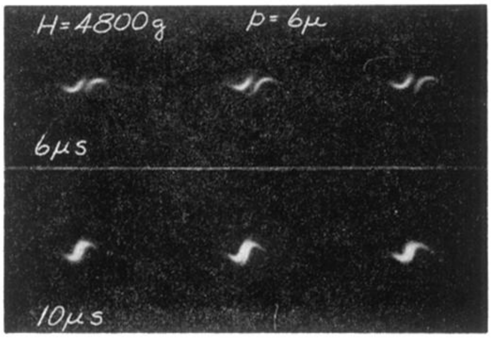

FIG. 8. The same as Fig. 5 except that the pressure is 6 μ.