Letters to the Editor

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Proposal for a Ferromagnetic Amplifier in the Microwave Range

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IN recent years certain anomalous absorption effects in ferromagnetic insulators at high signal powers[1,2] have received clarification.[3–5] Nonlinear terms in the equations of motion of the magnetization couple the usual uniform precession induced by the rf field to certain pairs of nonuniform modes of motion in a time-varying manner, resulting in unstable growth of these modes at the expense of the uniform precession.

In particular, the subsidiary absorption peak at high powers[1,2,4,5] involves the growth of disturbances ("magnetostatic" modes[6]) whose frequencies ω_1, ω_2 add up to the frequency ω of the uniform precession. (In the plane-wave approximation used in references 4 and 5, $\omega_1=\omega_2=\omega_k=\omega/2$.) The object of this note is to point out that such effects can be turned to account in the construction of a microwave amplifier.

Figure 1 is an almost exact circuit analog of the two nonuniform modes coupled through the uniform precession. The modes are presented by two meshes tuned to ω_1, ω_2, their decay constants by two resistors, and their coupling by the common load $L(t)$ varying at frequency $\omega=\omega_1+\omega_2$, hereafter called the pump frequency. When the depth of modulation of L exceeds a critical value depending on the circuit losses, the meshes break into self-oscillations at their respective frequencies. But, if an input signal and an output load are coupled to mesh 1 (or 2) and the modulation depth (i.e., the pumping power) adjusted so that oscillations would occur without the extra load, but not with it, stable power gain results. That coupling to the "circuits" (that is to the magnetostatic modes) of the ferromagnetic material is possible has been demonstrated experimentally by Dillon[7] and by Solt and White.[8]

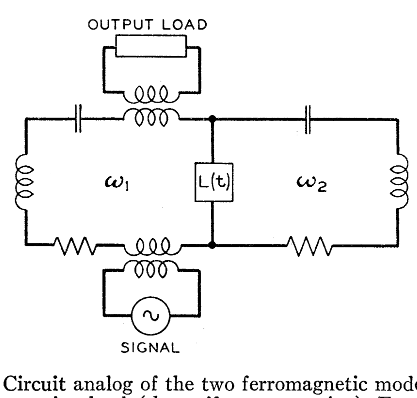

Fig. 1. Circuit analog of the two ferromagnetic modes coupled by a time-varying load (the uniform precession). For details on circuits of this type, see J. M. Manley and H. E. Rowe, Proc. Inst. Radio Engrs. 44, 904 (1956).

In this form, the device may not be practical. There exists an infinity of "magnetostatic" mode pairs whose frequencies add up approximately to ω, whose mutual couplings have comparable strength, and whose instability thresholds are therefore of similar magnitude. The wanted mode pair, on the other hand, has its threshold raised by the presence of the output load. Hence, as the pumping power is raised, unwanted pairs may become unstable before the wanted pair exhibits much gain, if any. The uniform precession then "sticks,"[4] no matter how high the pumping power.

However, so far the sample has served both as doubly resonant system and as coupling element. These functions can be separated, and the difficulty then disappears. All "magnetostatic" mode frequencies are confined to a range from $\gamma(H-4\pi M)$ to $\gamma(H+2\pi M)$, where γ is the gyromagnetic ratio, H the applied dc field, M the saturation magnetization. By adjusting H or ω or both we can assure that there is *no* pair whose frequencies add up to ω. The sample is placed into a microwave structure resonant to ω_1, ω_2 and (for efficiency) to $\omega=\omega_1+\omega_2$. If the sample is positioned where the h lines of one of the cavity modes ω_1, ω_2 have a component along H, the other a component normal to H, then power gain can result as before, the stability threshold on the uniform precession mode now depending on the two cavity Q's rather than on sample losses. In this type of operation (which we call "electromagnetic"), thresholds at least as low as for the previous ("magnetostatic") operation should be attainable.

As a third alternative, the cavity can be made to supply one mode (say ω_2), the sample the other (semistatic operation). The threshold then depends on both cavity and sample losses. In electromagnetic operation, but not always in the other two cases, it is also possible to adjust H so that the sample itself is resonant to ω. The threshold pumping power is then especially low. The threshold θ_{crit} of the uniform precession mode in the three cases are as follows:

Operation	"Magnetostatic"	Semistatic	Electro-magnetic
Threshold θ_{crit}	$\dfrac{1}{F_{\mathrm{ms}}}\left(\dfrac{\Delta H_1}{4\pi M}\right)^{\frac{1}{2}}\left(\dfrac{\Delta H_2}{4\pi M}\right)^{\frac{1}{2}}$	$\left(\dfrac{\Delta H_1}{4\pi M}\right)^{\frac{1}{2}}\dfrac{K}{F_{\mathrm{ss}}(Q_2)^{\frac{1}{2}}}$	$\dfrac{K}{F_{\mathrm{em}}(Q_1Q_2)^{\frac{1}{2}}}$

The threshold pumping field h_{crit} is found by multiplying θ_{crit} by the line width ΔH of the uniform precession, if the sample is resonant to ω, and by $1/\gamma$ times the frequency deviation from resonance when it is not. F is a filling factor measuring the overlap of the component along H of one of the mode-fields in the sample with the transverse component of the other. In principle F can be of order unity. ΔH_1, ΔH_2 are the line widths of magnetostatic modes 1 and 2; Q_1, Q_2 are the Q's of the cavity modes. K is a factor of order (operating frequency/$4\pi\gamma M$).

In spite of superficial similarities between this device and the three-level maser proposed by Bloembergen,[9] the present device operates quite differently; in somewhat oversimplified terms it relies on modulation of the real part of a susceptibility, rather than on reversal of the normal populations of two levels. However, in common with the three-level maser, the present device should have a low noise figure.

Finally, we stress that the same principle of amplification will apply to any system in which appropriate parameters can be varied, be it through anharmonic behavior of the physical system or otherwise. Since there are many such systems (e.g., anharmonically bound molecules) there may be a great many frequency ranges where this principle might find application.

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[3] P. W. Anderson and H. Suhl, Phys. Rev. **100**, 1788 (1955).
[4] H. Suhl, Proc. Inst. Radio Engrs. **44**, 1270 (1956).
[5] H. Suhl, J. Phys. Chem. Solids **1**, 209 (1957).
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Some Consequences of *TCP*-Invariance

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RECENT experiments[1] have shown that parity (P) in the usual sense is not conserved in some weak interactions. There are strong indications that charge conjugation (C) invariance is also violated. According to a general theorem,[2] invariance with respect to the product TCP follows for a wide class of field theories from invariance with respect to the proper Lorentz group alone. Here T denotes the anti-unitary operator of Wigner time reversal. It is therefore important to investigate which connections between properties of particles and antiparticles follow from this general invariance and which can only be deduced from more severe invariance requirements (e.g., CP).

First we show that masses and (for unstable particles) also lifetimes of particles and antiparticles are equal as a consequence of TCP. The validity of this statement is not based on any perturbation expansion. We write the Hamiltonian as

$$H=H_S+H_W, \qquad (1)$$

where H_S contains the free-field part and the strong interactions, while H_W represents the weak interactions. Mass and lifetime of a particle can be obtained from an investigation of the following expectation value:

$$(\psi, (\lambda-H)^{-1}\psi), \qquad (2)$$

regarded as a function of the complex variable λ. Here ψ is an eigenstate of H_S, which represents one particle with momentum zero. For $H_W=0$, the mass of this particle corresponds to a singularity of (2) on the real axis. Under the influence of H_W the singularity shifts and, for an unstable particle, moves off the real axis. For particles with a simple exponential decay, mass and lifetime are given by the real and imaginary parts of this singularity.

Using the symbol Θ for the product TCP, one notices that $\Theta\psi$ describes one antiparticle at rest. From the general theorem it then follows that

$$(\Theta\psi, (\lambda-H)^{-1}\Theta\psi)=(\psi, (\lambda-H)^{-1}\psi), \qquad (3)$$

so that the two expressions have the same singularities. To show (3), we go through the following steps:

$$\begin{aligned}(\Theta\psi, (\lambda-H)^{-1}\Theta\psi)&=(\Theta^{-1}(\lambda-H)^{-1}\Theta\psi, \psi)\\ &=((\lambda^*-H)^{-1}\psi, \psi)=(\psi, (\lambda-H)^{-1}\psi), \end{aligned}\qquad (4)$$

where proper use has been made of the anti-unitarity of Θ. The equality of the masses of stable particles and of the lifetimes to first order in H_W had previously been stated by Lee, Oehme, and Yang.[3]

Second, we investigate under what circumstances the equality of branching ratios for the decay of particle and antiparticle into corresponding channels also can be concluded from the general invariance. For the sake of brevity we consider H_W only to first order. The branching ratios are essentially obtained from $|(\varphi^{\text{in}},H_W\psi)|^2$, where ψ is the same state as in (2) and φ^{in} is an incoming eigenstate of H_S representing the decay products. Since Θ transforms φ^{in} into an outgoing state of the corresponding antiparticles, the equality of branching ratios cannot be concluded in general. It can be shown to hold, however, if the scattering processes induced by H_S do not involve transitions between different decay channels.

The previous remarks can be applied to the decay of charged K mesons if we assume that τ, τ', and θ represent decay modes of the same particle. The experimental equality of masses and lifetimes of K^+ and K^- would not insure invariance of H with respect to C or CP. Also the branching ratios for the decays into two and three π mesons would be equal for either charge if the spin of the K meson were zero. In that case these channels would have opposite parity and could not be mixed by a parity-conserving H_S. But even for higher spin these branching ratios could be expected to be equal to a high accuracy if only the general invariance would hold, because H_S is very likely to produce practically no transitions between states of two and