Parity Doublets and Hyperfragment Binding*

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The Thomas variation-iteration method is applied to a model of a hypernucleus in which the binding arises from a potential which mixes orbital states of opposite parity, a situation which may prevail if the Λ^0 has the parity doublet structure suggested by Lee and Yang. The P-state part of the ground state increases with the mass of the hyperfragment and may have a probability as large as 20% for mass number 8, with correspondingly large effect upon the decay rate of the fragment and upon the ratio of mesonic to nonmesonic decay.

I. INTRODUCTION

LEE and Yang[1] have suggested that the apparent mass degeneracy of the θ and τ mesons may be an instance of a new symmetry principle for strong interactions called "parity conjugation invariance." Their theory postulates the existence of parity doublets of particles of odd "strangeness", including the Λ^0. The members of this doublet (Λ_1,Λ_2) would then be bound in nuclei with equal strength. As a consequence (as noted by Treiman[2]) a given species of hypernucleus, with definite spin and mass number, will have degenerate ground states of opposite parity:

$$\begin{aligned}\psi_e&=a\varphi_1(\Lambda_1)+b\varphi_2(\Lambda_2),\\ \psi_o&=a\varphi_1(\Lambda_2)+b\varphi_2(\Lambda_1).\end{aligned}\tag{1}$$

The ratio of the amplitudes a and b measures the parity mixing which influences such properties of the hypernucleus as its decay rate[2] (in terms of the free Λ_1 and Λ_2 decay rates) and the ratio of mesonic to nonmesonic decay.[2–4]

While previous authors[5–11] have considered the problem of hypernuclear binding from the points of view of coupling schemes, charge independence, range and spin dependence of $\Lambda-N$ forces, our chief concern here will be the parity mixing of orbital states. To this end we consider a potential of the form

$$\bar{\mathbf{V}}=V_1(r)+(\boldsymbol{\sigma}\cdot\hat{r})C_PV_2(r),\tag{2}$$

where $\hat{r}$ is the unit vector of the Λ^0 relative to the center of mass of the "nuclear core," $\boldsymbol{\sigma}$ is the spin vector of the Λ^0, assumed to be of spin $\frac{1}{2}$, C_P is an operator[1] which changes Λ_1 into Λ_2 and vice versa, and $V_1(r)$, $V_2(r)$ are central scalar attractive potentials. The coefficient of C_P is the only odd-parity term which can be constructed of the vectors $\mathbf{r}$ and $\boldsymbol{\sigma}$. There is no reason, other than simplicity, for excluding forces which depend also on the spins of the nucleons, such as tensor forces. We may therefore expect the present model to apply best to hypernuclei having spherical cores, such as He^5, and to require modification in other cases.

Following other authors, we shall assume that the $\Lambda-N$ force has a range short compared to the nuclear force range and shall consider $V_1(r)$ and $V_2(r)$ to be proportional to the nucleon density. Evidence concerning the consistency of this assumption will be considered in Sec. III. We shall be content to approximate the potential functions by square wells, whose depths will be determined from the observed binding energies by a variation-iteration procedure.

The potential (2) can also represent a particle of zero spin interacting with a core of spin $\frac{1}{2}$, e.g., a K meson bound in a nucleus. For this case, Schwinger's recent suggestion[12] of a direct $(K_1\pi K_2)$ interaction, which has the parity exchange property, leads in the static limit of weak coupling[13] to precisely the second term of (2). While the evidence for such K fragments is scant, some observed decays of heavy hyperfragments can be interpreted in this way.[14]

II. CALCULATION

In this section we will apply the variation-iteration method[15] to obtain an approximation to the ground state solution of the Schrödinger equation with potential (2), letting $V_1(r)=cV_2(r)$ where c is a dimensionless

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956).
[2] S. B. Treiman, Phys. Rev. **104**, 1475 (1956).
[3] H. Primakoff and W. B. Cheston, Phys. Rev. **92**, 1537 (1953).
[4] M. Ruderman and R. Karplus, Phys. Rev. **102**, 247 (1956); T. K. Fowler, Phys. Rev. **102**, 844 (1956).
[5] R. H. Dalitz, Phys. Rev. **99**, 1475 (1955).
[6] K. Nishijima, Progr. Theoret. Phys. Japan **14**, 527 (1955).
[7] J. T. Jones and J. K. Knipp, Nuovo cimento **2**, 857 (1955).
[8] R. Gatto, Nuovo cimento **1**, 372 (1955).
[9] G. Wentzel, Phys. Rev. **101**, 835 (1956).
[10] R. H. Dalitz, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics, Rochester, 1956* (Interscience Publishers, Inc., New York), p. V–40.
[11] D. B. Lichtenberg and M. Ross, Phys. Rev. **103**, 1131 (1956).
[12] J. Schwinger, Phys. Rev. **104**, 1164 (1956); R. Arnowitt and B. Teutsch have considered the decay of charged K mesons assuming a weak parity-mixing interaction.
[13] S. B. Treiman and the author have obtained in this limit the potential

$$V=-(2\pi^2)^{-1}(fg/\mu)(\boldsymbol{\tau}^N\cdot\boldsymbol{\tau}^K)(\boldsymbol{\sigma}^N\cdot\nabla)U(r),$$

where f and g are the coupling constants of nucleon and K meson respectively to the pion field and $U(r)$ is the Yukawa potential. While this potential is singular at the origin, it can be cut off, for example, by use of an extended nucleon source.
[14] Fry, Schneps, and Swami, Phys. Rev. **99**, 1561 (1955).
[15] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951); L. H. Thomas, Phys. Rev. **51**, 202 (1937).

parameter. Introducing the length a, we define $\mathbf{x}=\mathbf{r}/a$ (and corresponding unit vector $\hat{x}$), and additional dimensionless parameters

$$\lambda=(2ma^2/\hbar^2)V_0,\quad \eta^2=(2ma^2/\hbar^2)|E|, \tag{3}$$

where m is the reduced mass of Λ^0 and nuclear core, $-E$ is the binding energy of the hypernucleus, and V_0 is the depth of the parity-mixing potential; that is, $V_2(x)=V_0S(x)$ where $S(x)$ is dimensionless. The corresponding Schrödinger equation is

$$[\nabla_x^2+\lambda(c+C_P\boldsymbol{\sigma}\cdot\hat{x})S(x)-\eta^2]\psi(\mathbf{x})=0. \tag{4}$$

For the ground state of this system, having a definite parity $\pi=\pm1$ and $m_J=\frac{1}{2}$, we may write

$$x\psi(\mathbf{x})=f(x,\pi)Y_0(\theta,\varphi)\chi^{\frac{1}{2}} +\beta g(x,-\pi)3^{-\frac{1}{2}}[Y_1^0(\theta,\varphi)\chi^{\frac{1}{2}}-2^{\frac{1}{2}}Y_1^1(\theta,\varphi)\chi^{-\frac{1}{2}}]. \tag{5}$$

C_P operates on the arguments $\pm\pi$ in Eq. (5), which refer to the intrinsic parity of the Λ^0. Thus $C_P\boldsymbol{\sigma}\cdot\hat{x}$ changes the S-state part of (5) into a P state of the same over-all parity, as

$$\boldsymbol{\sigma}\cdot\hat{x}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\cos\varphi\\ \sin\varphi e^{i\varphi}\end{pmatrix}. \tag{6}$$

It must then change the P state into an S state of the same parity, as the square of $C_P\boldsymbol{\sigma}\cdot\hat{x}$ is unity. Similarly for $m_J=-\frac{1}{2}$, parity π, we have

$$x\psi(\mathbf{x})=f(x,\pi)Y_0\chi^{-\frac{1}{2}} -\beta g(x,-\pi)3^{-\frac{1}{2}}[Y_1^0\chi^{-\frac{1}{2}}-2^{\frac{1}{2}}Y_1^{-1}\chi^{\frac{1}{2}}], \tag{7}$$

with the Condon and Shortley definition[16] of the angular functions. After separation of the radial part of Eq. (4), we can suppress the variable π.

The *ansatz* (5) gives the radial equations

$$\begin{aligned}[][d^2/dx^2+\lambda cS(x)-\eta^2]f(x)&=-\lambda S(x)\beta g(x),\\ [d^2/dx^2-2/x^2+\lambda cS(x)-\eta^2]\beta g(x)&=-\lambda S(x)f(x).\end{aligned} \tag{8}$$

The constant β has been introduced for later convenience as a measure of the mixing of S and P states.

Following Feshbach and Schwinger,[15] we introduce

$$\mathbf{A}=\begin{pmatrix}-d^2/dx^2+\eta^2 & 0\\ 0 & -d^2/dx^2+2/x^2+\eta^2\end{pmatrix},$$

$$\mathbf{B}=\begin{pmatrix}cS(x) & S(x)\\ S(x) & cS(x)\end{pmatrix},\quad \boldsymbol{\varphi}=\begin{pmatrix}f(x)\\ \beta g(x)\end{pmatrix},$$

and write (8) as

$$\mathbf{A}\boldsymbol{\varphi}=\lambda\mathbf{B}\boldsymbol{\varphi}, \tag{9}$$

or

$$\boldsymbol{\varphi}=\lambda\mathbf{A}^{-1}\mathbf{B}\boldsymbol{\varphi}. \tag{10}$$

[16] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Macmillan Company, New York, 1935), p. 52.

Equations (9) and (10) can now be expressed as variational principles for the smallest value λ_0 of the well depth parameter λ:

$$\lambda_0=\text{Ext}\frac{(\boldsymbol{\varphi},\mathbf{A}\boldsymbol{\varphi})}{(\boldsymbol{\varphi},\mathbf{B}\boldsymbol{\varphi})}, \tag{11}$$

$$\lambda_0=\text{Ext}\frac{(\boldsymbol{\varphi},\mathbf{B}\boldsymbol{\varphi})}{(\boldsymbol{\varphi},\mathbf{B}\mathbf{A}^{-1}\mathbf{B}\boldsymbol{\varphi})}, \tag{12}$$

where "Ext" means extremum.

We use Eqs. (11) and (12) in the following way: With trial functions f_0 and g_0, we use (11) to determine $\beta^{(0)}$, the best value of β, and a corresponding $\lambda_0^{(0)}$. We use the same trial functions and carry out the same procedure in evaluating a $\beta^{(\frac{1}{2})}$, and $\lambda_0^{(\frac{1}{2})}$ from (12). Equation (12) will provide a better, i.e. smaller, value of λ_0 than Eq. (11) since it contains the iterated matrix[17]

$$\begin{pmatrix}f_1(x)\\ \beta g_1(x)\end{pmatrix}=\mathbf{A}^{-1}\mathbf{B}\begin{pmatrix}f_0(x)\\ \beta g_0(x)\end{pmatrix}. \tag{13}$$

Comparison of the two results will provide an indication of the degree of convergence. We may point out that in our case the convergence is more regular than in the tensor force problem treated by Feshbach and Schwinger since

$$\text{Det}|\mathbf{B}|=(c^2-1)S^2(x)$$

is definite for given c.

The Green's functions for $\mathbf{A}$, used in obtaining the first iteration, are

$$G_\eta(x,x')=\frac{1}{\eta}\sinh\eta x_<\exp(-\eta x_>),\quad \text{for}\quad l=0,$$

$$G_{\ }(x,x')=\frac{1}{\eta}\left(\cosh\eta x_<-\frac{\sinh\eta x_<}{\eta x_<}\right)\times\left(1+\frac{1}{\eta x_>}\right)\exp(-\eta x_>),\quad \text{for}\quad l=1, \tag{14}$$

where $x_<$ is the smaller and $x_>$ the larger of x, x'.

We let

$$\begin{aligned}S(x)&=1,\quad x<1\\ &=0,\quad x>1\end{aligned} \tag{15}$$

and choose as trial functions in the interior region ($x<1$):

$$\begin{aligned}f_0(x)&=x\exp(-\gamma x/2),\\ g_0(x)&=x^2\exp(-\delta x/2).\end{aligned} \tag{16}$$

[17] Note that λ is absorbed in the definitions of f_1 and g_1. For a complete discussion of the variation-iteration method see P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953). The significance of the notations $\lambda_0^{(0)}$ and $\lambda_0^{(\frac{1}{2})}$ is there made clear.

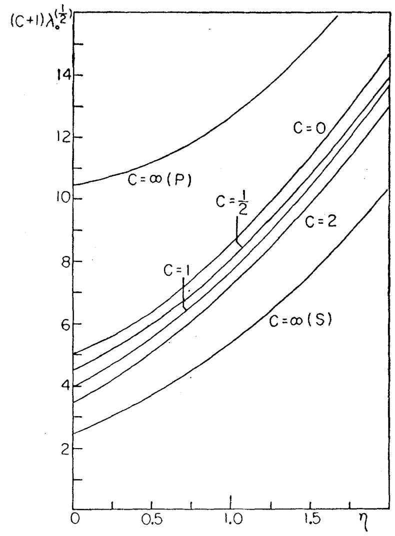

FIG. 1. "Effective" well depth parameter $(c+1)\lambda_0^{(\frac{1}{2})}$ as a function of binding energy parameter η for various values of the ratio c of ordinary to parity-mixing potential. The curves labeled $c=\infty$ (S) and $c=\infty$ (P) correspond to pure S and P states, respectively, bound in an ordinary potential.

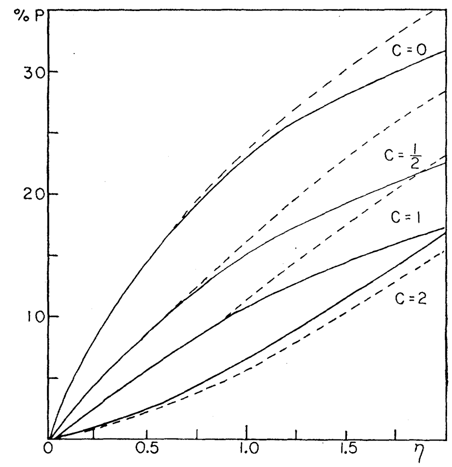

FIG. 2. Probability of the Λ^0 to occupy a P state, expressed in percent. Dashed curves: lowest approximation. Solid curves: first iteration.

γ and δ are determined by matching the logarithmic derivatives at $x=1$ to the exact S and P functions for the exterior region. Matching amplitudes as well, we obtain

$$f_0(x)=x\exp[-(1+\eta)x],\quad x<1$$
$$=\exp[-(1+\eta x)],\quad x>1, \tag{17}$$

$$g_0(x)=x^2\exp[-(3+\kappa\eta)x],\quad x<1$$
$$=\kappa(1+1/\eta x)\exp[-(3-\kappa+\eta x)],\quad x>1, \tag{18}$$

with $\kappa=\eta(1+\eta)^{-1}$.

Evaluation of the matrix elements $(\boldsymbol{\varphi},\mathbf{A}\boldsymbol{\varphi})$, etc., as explicit functions of β is then a straightforward job. Each matrix element is a quadratic function of β with coefficients which are elementary functions of η. The quantity β is obtained finally as a solution of a quadratic equation, the appropriate branch of the solution being fixed by the requirement that λ_0 be positive. For all the cases studied this yields a positive value for β. Values of $\lambda_0^{(0)}$ and $\lambda_0^{(\frac{1}{2})}$ obtained in this way are given in Table I as a function of η and may be compared for convergence. A more intuitively valuable quantity, namely $(c+1)\lambda_0^{(\frac{1}{2})}$, is plotted in Fig. 1.

To obtain the fraction of P state present, let $\beta=\mathfrak{N}\beta'$, where $\mathfrak{N}$ (real and positive) is chosen such that

$$\int_0^\infty f^2dx=\mathfrak{N}^2\int_0^\infty g^2dx. \tag{19}$$

Then the fraction of P state is

$$\beta'^2\mathfrak{N}^2\int_0^\infty g^2dx\Big/\int_0^\infty(f^2+\beta'^2\mathfrak{N}^2g^2)dx=\beta'^2(1+\beta'^2)^{-1}. \tag{20}$$

This fraction, expressed as a percentage, is plotted for various values of c and η in Fig. 2. It is given for both the (0) and $(\frac{1}{2})$ approximations. For the former, we use for normalization

$$\int_0^\infty[f_0^2+(\beta^{(0)})^2g_0^2]dx;$$

and for the latter,

$$\int_0^\infty[f_0f_1+(\beta^{(\frac{1}{2})})^2g_0g_1]dx.$$

TABLE I. Depth parameter λ_0 as a function of binding energy parameter η for various ratios c of ordinary to parity-mixing potential. Results are given for the original trial functions and for the first iteration to enable comparison for convergence.

	$c=0$		$c=\frac{1}{2}$		$c=1$		$c=2$		$c=\infty$		
η	$\lambda_0^{(0)}$	$\lambda_0^{(1/2)}$	$\lambda_0^{(0)}$	$\lambda_0^{(1/2)}$	$\lambda_0^{(0)}$	$\lambda_0^{(1/2)}$	$\lambda_0^{(0)}$	$\lambda_0^{(1/2)}$	$c\lambda_0^{(0)}$	$c\lambda_0^{(1/2)}$	$c\lambda_0$[a]
0	5.80	5.05	3.32	2.98	2.20	2.01	1.26	1.16	2.67	2.49	2.48
0.25	6.55	5.75	3.91	3.46	2.67	2.38	1.56	1.40	3.38	3.05	3.00
0.5	7.73	6.23	4.65	3.99	3.23	2.79	1.93	1.67	4.25	3.71	3.63
1	10.3	8.39	6.53	5.29	4.66	3.79	2.88	2.34	6.54	5.36	5.25
1.5	13.8	11.1	8.92	⋯	6.50	⋯	4.11	⋯	9.70	7.57	6.95
2	18.3	14.7	12.2	9.24	8.96	6.79	5.68	4.32	13.8	10.4	9.25

[a] Exact solution.

TABLE II. Values of $\eta=(2ma^2/\hbar^2)^{\frac{1}{2}}|E|^{\frac{1}{2}}$ and effective well depth $(c+1)V_0$ for observed binding energy $|E|$, assuming two different values for the nuclear radius. V_0 and $|E|$ are given in Mev.

		$a=1.2(A-1)^{1/3}\times10^{-13}$ cm			$a=[1.2(A-1)^{1/3}+0.7]\times10^{-13}$ cm		
A	$\|E\|$	η	$cV_0(c=\infty)$	$V_0(c=0)$	η	$cV_0(c=\infty)$	$V_0(c=0)$
3	0.20	0.128	33	65	0.193	15	29
	0.60	0.222	36	67	0.333	17	31
4	1.45	0.419	28	50	0.591	16	28
5	2.50	0.630	25	43	0.865	16	26
6	3.50	0.823	24	40	1.10	17	26
7	4.50	1.01	24	38	1.33	17	26
8	5.45	1.18	24	37	1.55	18	27
9	6.35	1.35	24	37	1.74	18	27

A comparison here gives directly a measure of the convergence of the wave function, which is of course not as good as for λ_0; but it is reasonable, especially for $\eta<1.5$, corresponding to the range of binding energies which have been observed.

III. DISCUSSION

We shall confine our observations mainly to the dependence of orbital mixing on the binding energy of hyperfragments. The effects which an admixture of P state would have on the theory of the ratio of mesonic to nonmesonic decay rates and on the hyperfragment lifetimes have been previously discussed by Treiman.[2]

While the binding energy appears to be a smooth function of the mass number (see Fig. 3) within the rather large experimental error, one can reasonably expect that further refinement of the measurements will show deviations from this smooth behavior depending on the structure of the nuclear core, including its spin. In view of the experimental and theoretical uncertainties, we have thought it best to use a smoothed nuclear core radius in attempting a comparison with experiment.

In Table II are given values of η calculated from the observed binding energies for the various hypernuclei, assuming two different radii for the nuclear core. Any apparent dependence of the data on charge of the core has been ignored. We have also tabulated the effective well depth $(c+1)V_0$ in Mev for the cases of pure ordinary $(c=\infty)$ and pure parity-mixing $(c=0)$ potentials.

Certain conclusions can now be drawn:

(a) Since the calculated values of η cover the range $\eta\lesssim1.5$, it will be seen, referring to Fig. 2, that a rather large admixture of P state results even for $c=2$, corresponding to "twice as much" ordinary as parity-mixing potential. The admixture increases rapidly with binding energy, so that a hyperfragment would have to be light to exhibit the lifetime of the hypothetical long-lived Λ^0.

(b) From Fig. 1 it can be seen that our parity-mixing potential is less effective in binding than the ordinary potential (as would be expected from the introduction of a centrifugal barrier). For small binding energies, pure parity-mixing requires a well depth about twice

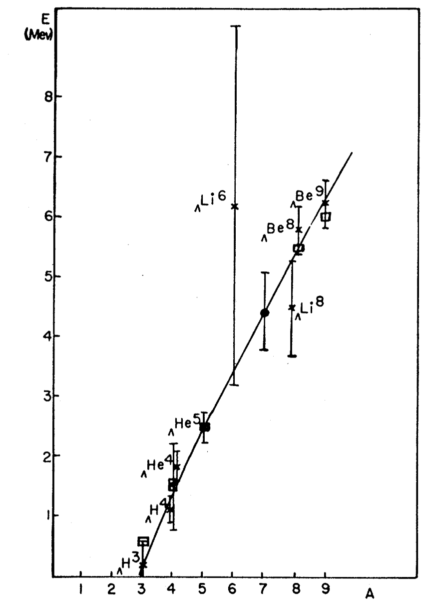

FIG. 3. Binding energy of hyperfragments as a function of mass number. The data in this table represent by no means a complete survey on the part of the author. The data are obtained from the following sources: The crosses represent the data of J. Schneps [Ph.D. Dissertation, University of Wisconsin, 1956 (unpublished)] and of Slater, Silverstein, Levi-Setti, and Telegdi [Bull. Am. Phys. Soc. Ser. II, **1**, 319 (1956)] combined with weights reflecting the probable errors assigned by the investigators. The solid circles are the data of Gilbert, Violet, and White [Phys. Rev. **103**, 248 (1956)]. The squares are the result of a "world average" prepared at the University of Chicago, and very kindly supplied to the author by Professor Telegdi. It will be seen that these points sufficiently determine the curve for our present purposes.

the depth to bind the first S state and about one-half the depth to bind the first P state in an ordinary potential of the same range.[18]

(c) One might expect that c and V_0 would characterize the $\Lambda-N$ force and thus be relatively constant as a function of mass number. If we choose a value of c, it can be seen from Table II that V_0 is nearly constant for $A\geqslant6$, especially for the larger radius a. Between $A=3$ and $A=6$ there appears to be a definite decrease in V_0. We shall not attempt here an interpretation of this apparent decrease. It should be noted that V_0a^2 is relatively insensitive to a, as is well known for short-range potentials, and consequently V_0 *is* sensitive. For the lightest hyperfragments, it is probably not sufficiently accurate to use the square well. Instead one should use a potential determined by the nucleon density distribution, as was done by Dalitz.[10]

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[18] There is no necessity for the ordinary potential to be attractive. The case of a repulsive ordinary potential $(c<0)$ is being studied.