the decay of K_+,

$$K_+ \to e^- + \pi^+ + \nu, \tag{A5}$$

into the two different final parity states are proportional to $pf_1 + qg_1^*$ and $pf_2 - qg_2^*$, respectively, while the corresponding elements for

$$K_+ \to e^+ + \pi^- + \bar{\nu} \tag{A6}$$

are proportional to $pg_1 + qf_1^*$ and $pg_2 - qf_2^*$. The branching ratio r for the decay of K_+ into $e^- + \pi^+ + \nu$ and $e^+ + \pi^- + \bar{\nu}$ is, therefore,

$$r = \frac{|pf_1 + qg_1^*|^2 + |pf_2 - qg_2^*|^2}{|pg_1 + qf_1^*|^2 + |pg_2 - qf_2^*|^2}. \tag{A7}$$

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Fermi Decay of Higher Spin Particles*

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The explicit form for the spin projection operators introduced by Fronsdal is calculated for arbitrary spin and applied to first-order processes involving four fermions. The matrix element for the most general nonderivative interaction is found for the special case in which two of the particles have spin $\frac{1}{2}$. The method of relating matrix elements written in different orders is extended to this case.

The theory is applied to the decay of the mu meson, extending the work of Caianiello. It is found that the experimental decay spectrum can be equally well fitted by an assignment of spin $\frac{1}{2}$ or $\frac{3}{2}$. The method is then applied to the Fermi decay of hyperons. Lifetimes are calculated for decays in which the initial particle has a spin of $\frac{1}{2}$ or $\frac{3}{2}$, and the final particles all have spin $\frac{1}{2}$. All the lifetimes are less than 2 orders of magnitude longer than the corresponding observed lifetimes for the normal mode of decay.

The hypothesis of a universal Fermi interaction is extended to include fermions of arbitary spin. Under this hypothesis, the experimental muon spectrum is most closely reproduced with spin $\frac{3}{2}$. The results also indicate that the muon has the same particle-antiparticle character as an electron of the same charge.

INTRODUCTION

RECENT experimental evidence has indicated the possibility that the "strange" particles may have spins larger than unity. Ruderman and Karplus[1] have found, by an analysis of mesonic and nonmesonic decay of hyperfragments, that the spin of the Λ^0 is either $\frac{1}{2}$ or $\frac{3}{2}$. Walker and Shephard[2] analyzed the angular correlations between the planes of production and decay of the Σ and the Λ^0 and found the spins to be $\frac{3}{2}$, $\frac{5}{2}$, or $\frac{7}{2}$. In addition to the strange particles, the long-known mu meson may conceivably have spin higher than $\frac{1}{2}$.

When considering the possibility that some of the hyperons might be fermions with spin higher than $\frac{1}{2}$, we meet a difficulty in that some of them are charged, and so interact with the electromagnetic field. A gauge-invariant way of describing this interaction has been given by Fierz and Pauli.[3] Only very few calculations have been carried out on the electromagnetic properties of particles described by the Fierz-Pauli equation, and the only result of interest to us is that of Mathews,[4] who calculated the Compton scattering cross section and the bremsstrahlung in the case of spin $\frac{3}{2}$. His result definitely rules out the possibility that the muon is such a particle, while the conclusions that can be made with regard to hyperons are less definite.

In the present paper we have calculated the lifetimes and spectra of Fermi decays of higher spin particles to first order, i.e., using the field-free wave functions. This calculation has been applied to the hyperons, the heavy mesons,[5] and the muon. We have included the muon on the basis that the electromagnetic properties of higher spin particles might be different than those predicted by Fierz and Pauli and calculated by Mathews.

FREE FIELDS

The wave function appropriate for describing a free particle of integral spin s is a tensor of rank s, and satisfies the wave equation

$$(p^2 + m^2)\Phi_{\alpha_1 \cdots \alpha_s} = 0, \tag{1a}$$

and the subsidiary conditions

$$\Phi_{\cdots \alpha_i \cdots \alpha_j \cdots} = \Phi_{\cdots \alpha_j \cdots \alpha_i \cdots}, \tag{1b}$$

$$p^{\alpha_1}\Phi_{\alpha_1 \cdots \alpha_s} = 0, \tag{1c}$$

$$g^{\alpha_1\alpha_2}\Phi_{\alpha_1\alpha_2 \cdots \alpha_s} = 0, \tag{1d}$$

where $g^{\mu\nu}$ is the metric tensor.

* This work was supported in part by the National Science Foundation.

[1] M. Ruderman and R. Karplus, Phys. Rev. **102**, 247 (1956).
[2] W. Walker and W. Shephard, Phys. Rev. **101**, 1810 (1956).
[3] M. Fierz and W. Pauli, Proc. Roy. Soc. (London) **A173**, 211 (1939).
[4] J. Mathews, Phys. Rev. **102**, 270 (1956).
[5] Results for the heavy mesons will be given in a separate publication.

For half-odd-integral spin $s=n+\frac{1}{2}$, the wave function is a tensor of rank n, each component of which is a Dirac 4-spinor. Equation (1a) is replaced by

$$(\mathbf{p}+im)\Phi_{\alpha_1\cdots\alpha_n}=0, \tag{1a'}$$

and there is the additional subsidiary condition

$$\gamma^{\alpha_1}\Phi_{\alpha_1\cdots\alpha_n}=0, \tag{1e}$$

where the γ^μ are the Dirac matrices, and $\mathbf{p}\equiv\gamma^\mu p_\mu$.

Following Fronsdal,[6] we now introduce an orthogonal projection operator Θ with the following properties

$$\Theta_{\cdots\alpha_i\cdots\alpha_j\cdots}^{\beta_1\cdots}=\Theta_{\cdots\alpha_j\cdots\alpha_i\cdots}^{\beta_1\cdots} \tag{2a}$$

$$p^{\alpha_1}\Theta_{\alpha_1\cdots}^{\beta_1\cdots}=0, \tag{2b}$$

$$g^{\alpha_1\alpha_2}\Theta_{\alpha_1\alpha_2\cdots}^{\beta_1\cdots}=0, \tag{2c}$$

$$\Theta_{\alpha_1\cdots}^{\beta_1\cdots}\Theta_{\beta_1\cdots}^{\epsilon_1\cdots}=\Theta_{\alpha_1\cdots}^{\epsilon_1\cdots}, \tag{2d}$$

and for half-integral spin

$$\gamma^{\alpha_1}\Theta_{\alpha_1\cdots}^{\beta_1\cdots}=0. \tag{2e}$$

In addition $\Theta_{\alpha_1\cdots}^{\beta_1\cdots}$ commutes with $\mathbf{p}$.

This projection operator derives its importance from the fact that if $\Phi_{\alpha_1\cdots}$ is any solution of (1a) or (1'a), then the wave function

$$\Psi_{\alpha_1\cdots}=\Theta_{\alpha_1\cdots}^{\beta_1\cdots}\Phi_{\beta_1\cdots} \tag{3}$$

satisfies all the subsidiary conditions as well as the wave equation, i.e., is that part of $\Phi_{\alpha_1\cdots}$ that describes a particle of unique spin. It is now possible to show[6] that

$$\sum_{\text{spin}}\Psi_{\alpha_1\cdots}\bar{\Psi}^{\beta_1\cdots}=\Theta_{\alpha_1\cdots}^{\beta_1\cdots}\Lambda^\pm=\Lambda^\pm\Theta_{\alpha_1\cdots}^{\beta_1\cdots}, \tag{4}$$

where the $\Lambda^\pm$ are the usual energy projection operators of the spin-$\frac{1}{2}$ theory.

Explicit construction of the Θ operator is carried out as follows. First note that Θ must be expressed solely in terms of the momentum four-vector, the metric tensor and, in the case of half-odd integral spin, the γ matrices. By going to the center-of-mass system Eq. (2b) simply reduces Θ to a three-dimensional tensor, Eq. (2a) is taken into account by symmetrization, and Eq. (2d) amounts to fixing a constant. Thus, only Eqs. (2c) and (2e) remain to be accounted for. As an illustration, we shall calculate Θ for the spin $\frac{3}{2}$ case.

In the rest system we must have

$$\Theta_\mu{}^\nu=g_\mu{}^\nu+a\gamma_\mu\gamma^\nu,$$

where the indices run from 1 to 3, while all components of $\Theta_\mu{}^\nu$ with either μ or ν or both equal to 4, are zero. We determine the constant a by means of Eq. (2e):

$$\gamma^\nu+3a\gamma^\nu=0.$$

Thus, in the rest system

$$\Theta_\mu{}^\nu=g_\mu{}^\nu-\tfrac{1}{3}\gamma_\mu\gamma^\nu. \tag{5}$$

The generalization to an arbitrary frame is found by noting that in the rest system

$$\begin{gathered}(\boldsymbol{\gamma},0)=\left(g_\mu{}^\nu-\frac{p_\mu p^\nu}{p^2}\right)\gamma_\nu;\\ \begin{bmatrix}1&&&\\&1&&\\&&1&\\&&&0\end{bmatrix}=\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\end{gathered} \tag{6}$$

so that the covariant form of Θ is (with μ, ν running from 1 to 4)

$$\Theta_\mu{}^\nu=\left(g_\mu{}^\nu-\frac{p_\mu p^\nu}{p^2}\right)-\frac{1}{3}\left(g_\mu{}^\epsilon-\frac{p_\mu p^\epsilon}{p^2}\right)\times\left(g_\sigma{}^\nu-\frac{p_\sigma p^\nu}{p^2}\right)\gamma_\epsilon\gamma^\sigma \tag{7a}$$

$$=g_\mu{}^\nu-\tfrac{1}{3}\gamma_\mu\gamma^\nu-\frac{1}{3p^2}(\mathbf{p}\gamma_\mu p^\nu+p_\mu\gamma^\nu\mathbf{p}). \tag{7b}$$

The general expressions for Θ for arbitrary spin have been calculated in the Appendix. For integral spin

$$\Theta_{\alpha_1\cdots\alpha_s}^{\beta_1\cdots\beta_s}=\left(\frac{1}{s!}\right)^2\sum_{\substack{P(\alpha)\\P(\beta)}}\left[\prod_{i=1}^{s}\Theta_{\alpha_i}{}^{\beta_i}+a_1\Theta_{\alpha_1\alpha_2}\Theta^{\beta_1\beta_2}\prod_{i=3}^{s}\Theta_{\alpha_i}{}^{\beta_i}+\cdots+\begin{cases}a_{s/2}\Theta_{\alpha_1\alpha_2}\Theta^{\beta_1\beta_2}\cdots\Theta_{\alpha_{s-1}\alpha_s}\Theta^{\beta_{s-1}\beta_s}, & \text{for even } s\\ a_{(s-1)/2}\Theta_{\alpha_1\alpha_2}\cdots\Theta^{\beta_{s-2}\beta_{s-1}}\Theta_{\alpha_s}{}^{\beta_s}, & \text{for odd } s\end{cases}\right] \tag{8}$$

where the sum is over all permutations of α and β, and

$$a_r{}^{(s)}=(-\tfrac{1}{2})^r\frac{s!}{r!(s-2r)!}\cdot\frac{1}{(2s-1)(2s-3)\cdots(2s-2r+1)},\quad \Theta_\alpha{}^\beta=g_\alpha{}^\beta-\frac{p_\alpha p^\beta}{p^2}. \tag{9}$$

[6] C. Fronsdal (to be published).

For half-odd integral spin

$$\Theta_{\alpha_1\cdots\alpha_{s-\frac12}}^{\beta_1\cdots\beta_{s-\frac12}}(s)=\frac{s+\frac12}{2s+2}\gamma^{\alpha}\gamma_{\beta}\Theta_{\alpha\alpha_1\cdots\alpha_{s-\frac12}}^{\beta\beta_1\cdots\beta_{s-\frac12}}(s+\tfrac12). \quad (10)$$

MATRIX ELEMENTS

We now consider the decay of a fermion of mass m_1, spin s_1, into 3 other fermions of masses m_2, m_3, m_4 and spins s_2, s_3, s_4. If any one of the decay products is a neutrino we can only consider the spin value $\frac12$. If the neutrino mass is zero, the above theory for higher spins does not apply, and the assumption of a small but finite mass m leads to lifetimes[7] which are proportional to m^{2s-1}. Because most decays involve the emission of either two neutrinos or an electron and a neutrino, we shall limit ourselves to the case $s_3=s_4=\frac12$. The extension of the following discussion to the most general case is quite straightforward. We further assume only direct coupling, which limits the value of $|s_1-s_2|$ to 0 or 1.

Case 1, $s_1=s_2=s$. There are two possible sets of invariants[8]

$$F^{\sigma}(s,s)=(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\Gamma^{\sigma}u^{\alpha_1\cdots\alpha_n}(1))(\bar{u}(4)\Gamma^{\sigma}u(3)), \quad (11a)$$

$$F'^{\sigma}(s,s)=(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\Gamma^{\sigma}\mu^{\alpha_1'\alpha_2\cdots\alpha_n}(1))\times(\bar{u}(4)\Gamma^{\sigma}\gamma^{\alpha_1}\gamma_{\alpha_1'}u(3)), \quad (11b)$$

where $n=s-\frac12$. Invariants with more than two γ^{α}'s are zero or reduce to (11a) or (11b) by the subsidiary condition (1*e*). No new invariants are obtained by changing the order of the wave functions, or by shifting the γ^{α_1} and $\gamma_{\alpha_1'}$ in (11b). Consider first (11a). We introduce the notation

$$F^{\sigma}=F^{\sigma}(abcd)=(\tilde{a}C^{-1}\Gamma^{\sigma}b)(\tilde{c}C^{-1}\Gamma^{\sigma}d),$$

where C is the charge conjugation operator, and study the effect of permuting two of the variables a, b, c, d. The general method[9] developed for the case $s=\frac12$, may be applied with no modification. Therefore, the F^{σ} form a basis for a representation of the permutation group on 4 objects. For example:

$$P_{ac}F^{\sigma'}(abcd)=(P_{ac})_{\sigma}{}^{\sigma'}F^{\sigma}(abcd).$$

The explicit form of the permutation matrices[10] are reproduced here for convenience:

$$P_{cd}=\begin{pmatrix}-1&&&&\\&1&&&\\&&1&&\\&&&-1&\\&&&&-1\end{pmatrix},$$

$$P_{ad}=\frac14\begin{pmatrix}1&-4&-6&4&1\\-1&-2&0&-2&1\\-1&0&-2&0&-1\\1&-2&0&-2&-1\\1&-4&6&-4&1\end{pmatrix}, \quad (12)$$

$$P_{ac}=P_{cd}P_{ad}P_{cd}.$$

In order to deal with the invariants of the form (11b) we introduce an additional operator, defined as follows:

$$Q(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\Gamma^{\sigma}u^{\alpha_1'\cdots\alpha_n}(1))(\bar{u}(4)\Gamma^{\sigma}\gamma^{\alpha_1}\gamma_{\alpha_1'}u(3))$$
$$=(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\Gamma^{\sigma}u^{\alpha_1'\cdots\alpha_n}(1))(\bar{u}(4)\tilde{\gamma}^{\alpha_1}C^{-1}\Gamma^{\sigma}\gamma_{\alpha_1'}u(3)).$$

By virtue of the relation

$$C^{-1}\gamma_{\alpha}=-\tilde{\gamma}_{\alpha}C^{-1},$$

we have, for example, $Q=-1$ for the scalar interaction. In this manner the explicit representation of Q is found to be (when the subsidiary conditions are taken into account):

$$Q=\begin{pmatrix}-1&&2&&\\&1&&&\\&&-1&&\\&&&-1&\\&&&&1\end{pmatrix},$$

$$Q^{-1}=\begin{pmatrix}-1&&-2&&\\&1&&&\\&&-1&&\\&&&-1&\\&&&&1\end{pmatrix}. \quad (13)$$

By the definitions of P_{cd} and of Q, or by actual multiplication, we verify that

$$QP_{cd}Q=P_{cd}.$$

The fact that Q as well as P_{cd} and P_{ad} are nonsingular proves that no additional invariants may be obtained from (11b) by a different ordering of factors.

Applying the operator $-P_{ad}Q$ to (11b), one obtains

$$F''^{\sigma}(s,s)=(\bar{u}(3)\gamma_{\alpha_1'}\Gamma^{\sigma}u^{\alpha_1'\alpha_2\cdots\alpha_n}(1))\times(\bar{u}(4)\gamma^{\alpha_1}\Gamma^{\sigma}u_{\alpha_1\cdots\alpha_n}(2)).$$

This is readily seen to be zero or to reduce to invariants of the form (11a), except when $\sigma=2$, in which case we obtain an invariant that we shall designate $F^5(s,s)$:

$$F^5(s,s)=(\bar{u}(3)\gamma^{\alpha_1}u^{\alpha_1'\alpha_2\cdots\alpha_n}(1))\times(\bar{u}(4)\gamma_{\alpha_1'}u_{\alpha_1\cdots\alpha_n}(2)). \quad (11c)$$

[7] S. Kusaka, Phys. Rev. **60**, 61 (1941).

[8] We define $\Gamma^0=1$, $\Gamma^1=\gamma_\mu$, $\Gamma^2=i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2\sqrt{2}$, $\Gamma^3=i\gamma_5\gamma_\mu$, $\Gamma^4=\gamma_5$, where $\gamma_i^{\dagger}=\gamma_i$, $i=1\cdots5$.

[9] M. Fierz, Z. Physik **104**, 553 (1937); E. R. Caianiello, Nuovo cimento **10**, 43 (1953); R. J. Finkelstein, Nuovo cimento **1**, 1104 (1955).

[10] See for example, R. Finkelstein and P. Kaus, Phys. Rev. **92**, 1316 (1953).

The complete set of matrix elements for this case is thus $F^\sigma(s,s)$, $\sigma=0\cdots 5$, as given by (11a) and (11c).

Case 2, $s_1=s_2+1=s$. The possible invariants are

$$F'^\sigma(s,s-1)=(\bar{u}_{\alpha_2\cdots\alpha_n}(2)\Gamma^\sigma u^{\alpha_1\cdots\alpha_n}(1))\times(\bar{u}(4)\Gamma^\sigma\gamma_{\alpha_1}u(3)). \quad (14)$$

Invariants with more than one γ_α reduce to zero or (14) by the subsidiary condition (1e). Applying $-P_{ad}$, we obtain

$$F^\sigma(s,s-1)=(\bar{u}(3)\gamma_{\alpha_1}\Gamma^\sigma u^{\alpha_1\cdots\alpha_n}(1))\times(\bar{u}(4)\Gamma^\sigma u_{\alpha_2\cdots\alpha_n}(2)). \quad (15)$$

The subsidiary conditions give

$$F^0=F^4=0,\quad F^2=2Q^{-1}P_{ad}QF'^1,$$
$$F^1=2Q^{-1}P_{ad}QF'^0,\quad F^3=-2Q^{-1}P_{ad}QF'^4. \quad (16)$$

The complete set of matrix elements for this case is thus

$$F=\sum_{\sigma=0,1,4} g_\sigma(\bar{u}_{\alpha_2\cdots\alpha_n}(2)\Gamma^\sigma u^{\alpha_1\cdots\alpha_n}(1))\times(\bar{u}(4)\Gamma^\sigma\gamma_{\alpha_1}u(3)). \quad (14')$$

We emphasize that the preference for one representation over any other is one of convenience in deducing identities and calculating matrix elements, since the corresponding results in any other representation may be easily obtained with the help of the permutation matrices.

TRANSITION PROBABILITIES

In this section, we shall again restrict ourselves to the special cases for which the matrix elements have been discussed in detail above, $s_3=s_4=\frac{1}{2}$. If $m_3=m_4=0$, the differential transition probability may be written

$$\mathcal{P}d^3p_2=\frac{d^3p_2}{16(2\pi)^3m_1E_2(2s+1)}\times\int\frac{E_4{}^2d\Omega}{p_4\cdot(p_2-p_1)}\sum g_\sigma g_{\sigma'}F_\sigma F_{\sigma'}{}^\dagger,$$

where F_σ is given by (11a) and (11c) in the case $s_1=s_2=s$ and by (14′) in the case $s_1=s_2+1=s$.

As an illustration of the calculational advantage gained in choosing a certain representation, we shall discuss the case $s_1=s_2+1=s$. If we write the matrix element in the form of (14′), then

$$|M|^2=\sum_{\text{spin}}\sum_{\sigma=0}^{4} g_\sigma g_{\sigma'}F_\sigma F_{\sigma'}{}^\dagger=\sum_{\text{spin}}\sum_{\sigma=0,1,4} g_\sigma{}^2|F_\sigma|^2,$$

i.e., there are no cross terms between tensors of different ranks, and we see that the σ sum runs over the simplest of the tensors. By using Eq. (4), we may sum over the energy and spin states, obtaining

$$|M|^2=\sum_{\sigma=0,1,4} g_\sigma{}^2\Big\{\mathrm{Tr}(\mathbf{p}_2-im_2)\Theta_{\alpha_2\cdots\alpha_n}^{\beta_2\cdots\beta_n}(p_2)\times\Gamma^\sigma(\mathbf{p}_1-im_1)\Theta_{\beta_1\cdots\beta_n}^{\alpha_1\cdots\alpha_n}(p_1)\Gamma^\sigma\Big\}\{\mathrm{Tr}\ \mathbf{p}_4\Gamma^\sigma\gamma_{\alpha_1}\mathbf{p}_3\gamma^{\beta_1}\Gamma^\sigma\}.$$

The Θ's appearing here correspond to the half-odd-integer spin values s and $s-1$. We may now use Eq. (10) to express these Θ's in terms of the Θ's corresponding to the integral spin values $s+\frac{1}{2}=n+1$ and $s-\frac{1}{2}=n$. In terms of the latter,

$$|M|^2=\sum_{\sigma=0,1,4} g_\sigma{}^2\frac{4s^2-1}{16s(s+1)}\Theta_{\alpha'\alpha_2\cdots\alpha_n}^{\beta'\beta_2\cdots\beta_n}(p_2)\Theta_{\beta\beta_1\cdots\beta_n}^{\alpha\alpha_1\cdots\alpha_n}(p_1)\times\{\mathrm{Tr}(\mathbf{p}_2-im_2)\gamma^{\alpha'}\gamma_{\beta'}\Gamma^\sigma(\mathbf{p}_1-im_1)\gamma^\beta\gamma_\alpha\Gamma^\sigma\}\times\{\mathrm{Tr}\ \mathbf{p}_4\Gamma^\sigma\gamma_{\alpha_1}\mathbf{p}_3\gamma^{\beta_1}\Gamma^\sigma\}. \quad (17)$$

The traces may now be evaluated separately from the product of the Θ's. We have listed, in the Appendix, those formulas which are useful in calculating this product, and which lead to a straightforward but tedious evaluation of $|M|^2$ for arbitrary spin s. We have completed this calculation for $s=\frac{3}{2}$ with the following result for the differential transition probability

$$\mathcal{P}d^3p_2=\frac{m_1{}^2m_2{}^2d^3p_2}{3(2\pi)^4E_1E_2}\{(K_1'\cosh\theta+K_3')\times(\cosh\omega-\cosh\theta+\tfrac{1}{6}e^{-\omega}\sinh^2\theta)+\tfrac{1}{3}K_2'(1+\tfrac{1}{2}e^{-\omega}\cosh\theta)\sinh^2\theta\}, \quad (18)$$

where

$$K_1'=g_0{}^2+g_1{}^2+g_4{}^2,\quad K_2'=g_1{}^2,\quad K_3'=g_0{}^2-g_4{}^2,$$
$$\cosh\theta=-p_1\cdot p_2/m_1m_2,\quad \omega=\ln(m_1/m_2).$$

This agrees with the result of Caianiello.[11] It may be compared with the expression given by Michel,[12] who considered the case $s=\frac{1}{2}$. Michel's formula may be written

$$\mathcal{P}d^3p_2=\frac{m_1{}^2m_2{}^2d^3p_2}{2(2\pi)^4E_1E_2}\{(K_1\cosh\theta+K_3)(\cosh\omega-\cosh\theta)+\tfrac{2}{3}K_2\sinh^2\theta\}, \quad (19)$$

where

$$K_1=g_0{}^2+2(g_1{}^2+g_2{}^2+g_3{}^2)+g_4{}^2,$$
$$K_2=g_1{}^2+2g_2{}^2+g_3{}^2,$$
$$K_3=g_0{}^2-2g_1{}^2+2g_3{}^2-g_4{}^2.$$

[11] E. R. Caianiello, Phys. Rev. **83**, 735 (1951).
[12] L. Michel, Proc. Phys. Soc. (London) **A63**, 514 (1949).

MUON DECAY

It is generally assumed that the mu meson decays, through weak coupling, in the following manner:

$$\mu \rightarrow e + \nu + \nu.$$

Depending on the value of the spin assigned to the muon, $\frac{1}{2}$ or $\frac{3}{2}$, the spectrum of the emitted electron is given by either Eq. (19) or (18). However, the mass of the electron is so small compared with the energies involved over most of the spectrum, that we may, advantageously, introduce the approximation of neglecting terms proportional to the electron mass. The resulting spin $\frac{1}{2}$ transition probability is given by

$$\tau\mathcal{P}_{\frac{1}{2}}(\eta)d\eta \simeq 4\eta^2 d\eta[3(1-\eta)+2\rho(\tfrac{4}{3}\eta-1)], \qquad (20)$$

where

$$\rho=\frac{3K_2}{K_1+2K_2}, \quad \tau \equiv \text{lifetime}=\frac{6(2\pi)^3}{(E_{2\,\max})^2(K_1+2K_2)},$$

$$\eta=\frac{E_2}{E_{2\,\max}}\simeq\frac{2E_2}{m_1}.$$

For the spin $\frac{3}{2}$ case, we have

$$\tau'\mathcal{P}_{\frac{3}{2}}(\eta)d\eta \simeq 10\eta^2 d\eta[1-\eta+\tfrac{1}{12}\eta^2+\rho'(\tfrac{4}{3}\eta-1)], \qquad (21)$$

where

$$\rho'=\frac{K_2'}{K_1'+K_2'}, \quad \tau'=\frac{(15/2)(2\pi)^3}{(E_{2\,\max})^2(K_1'+K_2')}.$$

We emphasize that these are the only two possible spin assignments for the muon, namely $\frac{1}{2}$ or $\frac{3}{2}$, since all the decay products have spin $\frac{1}{2}$.

In Fig. 1, we have plotted $\mathcal{P}_{\frac{3}{2}}(\eta)$ for the extremal ρ' values 0 and $\frac{1}{2}$. In Fig. 2, the spectra given by (20) is shown for the experimental value of Lederman *et al.*,[13] $\rho=0.64$, and for that of Crowe *et al.*,[14] $\rho=0.50$, as well as the spectrum given by (21) for $\rho'=0.25$. We thus see that by properly choosing the coupling constants g_σ, the experimental results can be equally well reproduced by a spin assignment of either $\frac{1}{2}$ or $\frac{3}{2}$.

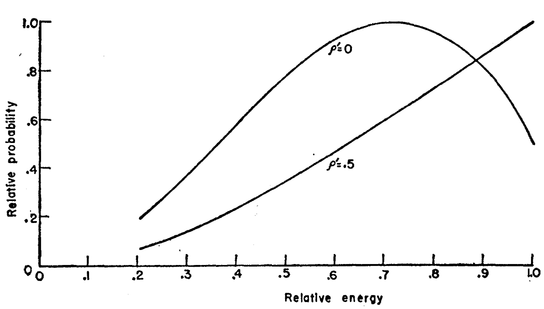

FIG. 1. The energy spectrum $\mathcal{P}_{\frac{3}{2}}$ for extremal values of ρ'.

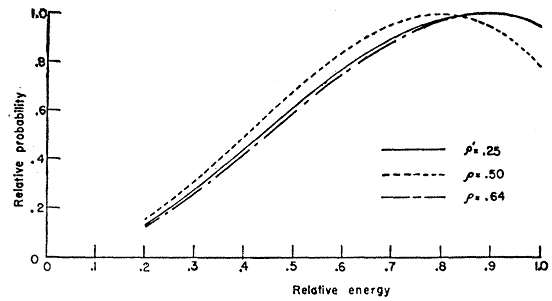

FIG. 2. The energy spectra measured by Lederman *et al.* ($\rho=0.64$) and by Crowe *et al.* ($\rho=0.50$), and the spectrum $\mathcal{P}_{\frac{3}{2}}$ predicted by the hypothesis of a universal Fermi interaction ($\rho'=0.25$).

UNIVERSAL FERMI INTERACTION

As a criterion for choosing some combinations of coupling constants, we shall investigate the consequences of accepting the validity of the hypothesis of the universal Fermi interaction. In the case of 4 interacting spin-$\frac{1}{2}$ particles the most general matrix element may be written

$$F(abcd)=\sum_{\sigma=0}^{4} g_\sigma(\bar{a}\Gamma^\sigma b)(\bar{c}\Gamma^\sigma d)$$
$$=g_0S+g_1V+g_2T+g_3A+g_4P. \qquad (22)$$

The hypothesis of a universal Fermi interaction may be expressed in the form suggested by Finkelstein and Kaus,[10] by (i) replacing Eq. (22) by

$$F=\sum_{abcd} F(abcd), \qquad (22')$$

where the sum is over all modes of all spin-$\frac{1}{2}$ particles; and (ii) by imposing the Jordan-Wigner anticommutation relations on all creation and absorption operators, whether belonging to different fermions or to different modes of the same fermion. The latter requirement is based on the assumption that all fermions are modes of one fundamental field. This effectively replaces $F(abcd)$ in (22) by

$$\tfrac{1}{2}(1-P)F(abcd), \qquad (23)$$

where P is the permutation operator that interchanges the wave functions of either the two particles or the two antiparticles in $F(abcd)$. If the order of wave function in $F(abcd)$ has been so chosen that a and c have the same particle-antiparticle character (pac), one finds that the most general matrix element of the form (23) may be written

$$a(S-T+P)+b(V-A)+c(S-A-P). \qquad (24)$$

If, on the other hand, a and d have the same pac, one obtains

$$aS+bA+cP.$$

We propose that the hypothesis of the universal Fermi interaction be extended to encompass fermions of

[13] Sargent, Rinehart, Lederman, and Rogers, Phys. Rev. **99**, 885 (1955).
[14] Crowe, Helm, and Tautfest, Phys. Rev. **99**, 872 (1955).

different spins, assuming *all* fermions to be modes of the same fundamental field.

We shall again confine our attention to the interaction of two higher spin and two spin-$\frac{1}{2}$ particles, as given by the matrix elements (11a), (11c), and (14′). Four cases must be considered:

Case 1a, $s_1=s_2=s$, and the particles 1 and 2 have the same pac.—The most general matrix element consistent with the hypothesis of the universal Fermi interaction is:

$$F=\sum_{\sigma,\sigma'=0}^{4} g_{\sigma'}\tfrac{1}{2}(1-P_{cd})_{\sigma}{}^{\sigma'}(\bar{u}(4)\Gamma^{\sigma}u^{\alpha_1\cdots\alpha_n}(1))$$
$$\times(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\Gamma^{\sigma}u(3))+\sum_{\sigma'=0}^{4} g_{\sigma'}\tfrac{1}{2}(1-P_{cd}')_0{}^{\sigma'}$$
$$\times(\bar{u}(4)\gamma^{\alpha_1}u^{\alpha_1'\alpha_2\cdots\alpha_n}(1))(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\gamma_{\alpha_1'}u(3)),$$

where

$$P_{cd}'(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\gamma_{\alpha_1'}u(3))\equiv(\bar{u}(3)\gamma_{\alpha_1'}u_{\alpha_1\cdots\alpha_n}(2))$$
$$=Q^{-1}P_{cd}(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\gamma_{\alpha_1'}u(3)).$$

But

$$(P_{cd}')_0{}^{\sigma'}=(Q^{-1}P_{cd})_0{}^{\sigma'}=\delta_0{}^{\sigma'},$$

so that $(1-P_{cd}')_0{}^{\sigma'}=0$ and there remains

$$F=\sum_{\sigma=0,3,4} g_{\sigma}(\bar{u}(4)\Gamma^{\sigma}u^{\alpha_1\cdots\alpha_n}(1))(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\Gamma^{\sigma}u(3)).\quad(25)$$

Thus of the six invariants, only three remain.

Case 1b, $s_1=s_2=s$, and the particles 1 and 2 have opposite pac.—We now have

$$F=\sum_{\sigma,\sigma'=0}^{4} g_{\sigma'}\tfrac{1}{2}(1-P_{cd})_{\sigma}{}^{\sigma'}(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\Gamma^{\sigma}u^{\alpha_1\cdots\alpha_n}(1))$$
$$\times(\bar{u}(4)\Gamma^{\sigma}u(3))+\sum_{\sigma'=0}^{4} g_{\sigma'}\tfrac{1}{2}(1-P_{cd}')_0{}^{\sigma'}$$
$$\times(\bar{u}_{\alpha_1\cdots\alpha_n}(2)u^{\alpha_1'\cdots\alpha_n}(1))(\bar{u}(4)\gamma^{\alpha_1}\gamma_{\alpha_1'}u(3)),\quad(26)$$

where

$$P_{cd}'(\bar{u}(4)\gamma^{\alpha_1}\gamma_{\alpha_1'}u(3))\equiv(\bar{u}(3)\gamma^{\alpha_1}\gamma_{\alpha_1'}u(4))$$
$$=(\tilde{u}(3)\tilde{\gamma}^{\alpha_1}\tilde{\gamma}_{\alpha_1'}C^{-1}u(4))=-P_{cd}(\bar{u}(4)\gamma^{\alpha_1}\gamma_{\alpha_1'}u(3))$$
$$+2\delta_{\alpha_1'}{}^{\alpha_1}(\bar{u}(4)u(3)).$$

The first term gives zero as in case 1a, while the second term is contained in the first part of (26). Therefore, in this case as well, only three invariants consistent with the hypothesis of the universal Fermi interaction exist:

$$F=\sum_{\sigma=0,3,4} g_{\sigma}(\bar{u}_{\alpha_1\cdots\alpha_n}(2)\Gamma^{\sigma}u^{\alpha_1\cdots\alpha_n}(1))(\bar{u}(4)\Gamma^{\sigma}u(3)).\quad(27)$$

Note that if (27) is expressed in terms of the invariants used in (25), and vice versa, one obtains the familiar forms (24). From a computational viewpoint, (25) and (27) are the most convenient representations in the respective cases.

Case 2a, $s_1=s_2+1=s$, and particles 1 and 2 have the same pac.—The matrix elements may be written in the form (15):

$$F=\sum_{\sigma=0}^{4} g_{\sigma}\tfrac{1}{2}(1-P_{cd})(\bar{u}(3)\gamma_{\alpha_1}\Gamma^{\sigma}u^{\alpha_1\cdots\alpha_n}(1))$$
$$\times(\bar{u}(4)\Gamma^{\sigma}u_{\alpha_2\cdots\alpha_n}(2))$$
$$=\sum_{\sigma=0,3,4} g_{\sigma}F^{\sigma}.$$

By (16) this reduces to the *unique* matrix element

$$F=g_3(\bar{u}(3)\gamma_5u^{\alpha_1\cdots\alpha_n}(1))(\bar{u}(4)\gamma_5\gamma_{\alpha_1}u_{\alpha_2\cdots\alpha_n}(2)).\quad(28)$$

Written in the representation (14), this is the Wigner-Critchfield interaction $S'-A'-P'$. The relations $F^0=F^4=0$ becomes $S'-T'+P'=V'-A'=0$, in that representation.

Case 2b, $s_1=s_2+1=s$, and particles 1 and 2 have opposite particle-antiparticle character.—In this case we write

$$F=\sum_{\sigma=0}^{4} g_{\sigma}\tfrac{1}{2}(1-P_{cd})(\bar{u}_{\alpha_2\cdots\alpha_n}(2)\gamma_{\alpha_1}\Gamma^{\sigma}u^{\alpha_1\cdots\alpha_n}(1))$$
$$\times(\bar{u}(4)\Gamma^{\sigma}u(3)),$$

which reduces to the *unique* matrix element

$$F=g_3(\bar{u}_{\alpha_2\cdots\alpha_n}(2)\gamma_5u^{\alpha_1\cdots\alpha_n}(1))(\bar{u}(4)\gamma_5\gamma_{\alpha_1}u(3)).\quad(29)$$

EFFECT OF THE UNIVERSAL FERMI INTERACTION ON MUON-DECAY

These results may be applied to the decay of a muon. With reference to the spin and pac of the mu meson, the following four possibilities exist:

(i) *Spin $\frac{1}{2}$, same particle-antiparticle character as electron.*—The matrix elements (25) written in charge-retention order will have the form $S-T+P$, $V-A$, and $S-A-P$, corresponding, respectively, to $\rho=0.75$, 0.75, and 0.50. The results of neutron decay seem to favor $S-T+P$, which, when electromagnetic corrections are included,[15] is consistent with the experimental result of Lederman. The experiment of Crowe, on the other hand, seems to favor $S-A-P$.

(ii) *Spin $\frac{1}{2}$, opposite particle-antiparticle character to electron.*—The matrix elements have the form $S\pm P$ and $S-A-P$, corresponding, respectively to $\rho=0$, 0.50. The value $\rho=0$ is in definite disagreement with both experimental results.

(iii) *Spin $\frac{3}{2}$, same particle-antiparticle character as electron.*—The matrix element (28), written in charge-retention order has the unique form $S-A-P$ corresponding to $\rho'=0.25$. The spectrum is plotted in Fig. 2 for this value.

(iv) *Spin $\frac{3}{2}$, opposite particle-antiparticle character to electron.*—The matrix element (29) has the unique form

[15] Behrends, Finkelstein, and Sirlin, Phys. Rev. **100**, 1809(A) (1955).

(29) corresponding to $\rho'=0$. This is in disagreement with both experimental results.

We note that in cases (i) and (ii) Crowe's results contradict the assumption that the muon and neutron decay by the same interaction. On the other hand, there is no basis for such an assumption if the muon is assigned the spin-value $\frac{3}{2}$. If there is no systematic error in either experimental result, we are led to favor (iii). The experimental lifetime gives a value of g^2 of the same order of magnitude as the spin-$\frac{1}{2}$ theory.

FERMI-DECAY OF HYPERONS

It has been suggested[16] that by considering the Λ^0 as an excited state of the neutron, one should expect the Λ^0 to have an analogous mode of decay, i.e.,

$$\Lambda^0 \to \mathfrak{N} + e + \nu$$

in addition to its usual mode

$$\Lambda^0 \to \mathfrak{N} + \pi.$$

By drawing an analogy to pion decay through a virtual nucleon-antinucleon state, Costa and Dallaporta[17] have shown that some of the branching ratios for the various modes of K-meson decay might possibly be explained by a virtual Λ^0-antinucleon state. An actual experimental indication of the beta decay of the Σ is reported by Hornbostel and Salant,[18] who interpret their event 4 as

$$\Sigma \to \mathfrak{N} + e + \nu.$$

It is apparent that these examples are special cases of what is to be expected if the hypothesis of the universal Fermi interaction is correct. We again state the general rule: Under the hypothesis of the universal Fermi interaction, *all* spinor particles should decay by (22′), provided the usual conservation laws are fulfilled.[19] The experimental observation of the allowed processes, of course, will depend upon their lifetimes. We list here the decay schemes to be expected according to various spin assignments, and then calculate some of the lifetimes

$$\Lambda_s \to \mathfrak{N} + \nu + \nu, \quad s = \tfrac{1}{2}, \tfrac{3}{2}, \tag{30a}$$

$$\Sigma_s \to \mathfrak{N} + \nu + \nu, \quad s = \tfrac{1}{2}, \tfrac{3}{2}, \tag{30b}$$

$$\Sigma_{s1} \to \Lambda_{s2} + \nu + \nu, \quad |s_1 - s_2| = 0, 1, \tag{30c}$$

$$\Xi_s \to \mathfrak{N} + \nu + \nu, \quad s = \tfrac{1}{2}, \tfrac{3}{2}, \tag{30d}$$

$$\Xi_{s1} \to \Lambda_{s2} + \nu + \nu, \quad |s_1 - s_2| = 0, 1, \tag{30e}$$

$$\Xi_{s1} \to \Sigma_{s2} + \nu + \nu, \quad |s_1 - s_2| = 0, 1, \tag{30f}$$

where $\mathfrak{N}$ is a nucleon, and ν is a lepton. Any new hyperons will be connected by the proposed generalization of the decay interaction (22′) to the Ξ, Σ, Λ, and $\mathfrak{N}$.

We have calculated the lifetimes in the special case in which the heavier hyperon has spin $\frac{1}{2}$ or $\frac{3}{2}$ and all the decay products have spin $\frac{1}{2}$. When two of the latter are leptons we may, to a very good approximation, neglect their masses. The differential transition probabilities (18) and (19) are then readily integrated to give

$$\begin{aligned}\frac{1}{\tau_{\frac{3}{2}}} = \frac{2m_1m_2^4}{3(2\pi)^3}\{&\tfrac{1}{8}(K_1' + K_2' + \tfrac{1}{2}K_3'e^{-\omega}) \\ &\times[\tfrac{1}{3}\eta_0(2\eta_0^2-5)(\eta_0^2-1)^{\frac{1}{2}} + \cosh^{-1}\eta_0 \\ &+ (4/15)e^{-\omega}(\eta_0^2-1)^{\frac{5}{2}}] + \tfrac{1}{2}K_3'[\tfrac{1}{3}(\eta_0^2+2)(\eta_0^2-1)^{\frac{3}{2}} \\ &- \eta_0\cosh^{-1}\eta_0 - (1/30)e^{-2\omega}(\eta_0^2-1)^{\frac{5}{2}}]\},\end{aligned} \tag{31}$$

$$\begin{aligned}\frac{1}{\tau_{\frac{1}{2}}} = \frac{m_1m_2^4}{(2\pi)^3}\{&\tfrac{1}{8}(K_1 + 2K_2)[\tfrac{1}{3}\eta_0(2\eta_0^2-5)(\eta_0^2-1)^{\frac{1}{2}} \\ &+ \cosh^{-1}\eta_0] + \tfrac{1}{2}K_3[\tfrac{1}{3}(\eta_0^2+2)(\eta_0^2-1)^{\frac{1}{2}} \\ &- \eta_0\cosh^{-1}\eta_0]\}.\end{aligned} \tag{32}$$

The coupling constant is a matter of some uncertainty. We have used the value $g = 1.374 \times 10^{-49}$ erg cm^3, which is the experimentally determined value of the Gamow-Teller coupling constant in neutron decay.[20] Justified only on the basis of simplicity, we have assumed that the coupling constant is independent of the spins of the various particles.

The inverse lifetimes of the decays to which (31) or (32) could apply have been calculated, and given in Table I, for each of the 8 possible interactions. In Table II we have given the lifetimes that would be predicted on the basis of the hypothesis of a universal Fermi interaction for the various assignments of spin. Because of the law of conservation of heavy particles, the hyperons must have the same particle-antiparticle character. The values of the masses used are as follows:

$$m_{\mathfrak{N}} = 1836\ m_e,$$
$$m_\Lambda = 2181\ m_e,\text{[21]}$$
$$m_\Sigma = 2327\ m_e,\text{[22]}$$
$$m_\Xi = 2582\ m_e.\text{[23]}$$

Processes involving a muon (in place of one of the leptons) give differential transition-probabilities which are considerably more complex than (18) or (19), which are valid when $m_3 = m_4 = 0$. We have completed the calculation for the decay $\Lambda_{\frac{1}{2}} \to \mathfrak{N} + \mu_{\frac{1}{2}} + \nu$ with scalar

[16] R. J. Finkelstein, Phys. Rev. **88**, 555 (1952); M. Markov and V. Stakhanov, J. Exptl. Theoret. Phys. U.S.S.R. **28**, 740 (1955) [Soviet Phys. JETP **1**, 593 (1956)].

[17] G. Costa and N. Dallaporta, Nuovo cimento **2**, 519 (1955).

[18] J. Hornbostel and E. Salant, Phys. Rev. **102**, 502 (1956).

[19] We include the law of conservation of heavy particles in the word "usual."

[20] J. Gerhart, Phys. Rev. **95**, 288 (1954).

[21] Friedlander, Keefe, Menon, and Merlin, Phil. Mag. **45**, 533 (1954).

[22] Fry, Schneps, Snow, and Swami, Phys. Rev. **103**, 226 (1956).

[23] Proceedings of the International Conference on Elementary Particles, Pisa, 1955, Nuovo cimento (to be published).

TABLE I. Calculated values of reciprocal lifetime for some Fermi modes of decay of hyperons for the 8 interactions possible with spin assignments of $\frac{1}{2}$ or $\frac{3}{2}$.

Spin	1/2					3/2		
Interaction	S	V	A	T	P	S	V	P
$\Lambda_s^0\to\mathfrak{N}+\nu+\nu$	0.134×10^8	0.136×10^8	0.404×10^8	0.405×10^8	0.437×10^5	0.128×10^8	0.128×10^8	0.492×10^5
$\Sigma_s^+\to\mathfrak{N}+\nu+\nu$	0.716×10^8	0.728×10^8	0.215×10^9	0.216×10^9	0.428×10^6	0.668×10^8	0.673×10^8	0.477×10^6
$\Sigma_s^+\to\Lambda_{\frac{1}{2}}^0+\nu+\nu$	0.209×10^6	0.233×10^6	0.635×10^6	0.651×10^6	0.807×10^4	0.207×10^6	0.212×10^6	0.492×10^4
$\Xi_s\to\mathfrak{N}+\nu+\nu$	0.501×10^9	0.519×10^9	0.151×10^{10}	0.152×10^{10}	0.623×10^7	0.456×10^9	0.463×10^9	0.668×10^7
$\Xi_s\to\Lambda_{\frac{1}{2}}^0+\nu+\nu$	0.287×10^8	0.289×10^8	0.861×10^8	0.863×10^8	0.910×10^5	0.273×10^8	0.274×10^8	0.102×10^6
$\Xi_s\to\Sigma_{\frac{1}{2}}^++\nu+\nu$	0.327×10^7	0.330×10^7	0.983×10^7	0.985×10^7	0.773×10^4	0.317×10^7	0.318×10^7	0.685×10^4

TABLE II. Calculated values of reciprocal lifetimes for some Fermi modes of decay of hyperons for the interactions predicted by the hypothesis of a universal Fermi interaction with spin assignments of $\frac{1}{2}$ or $\frac{3}{2}$.

Spin	1/2			3/2
Interaction	$S-T+P$	$V-A$	$S-A-P$	$S-V-P$
$\Lambda_s^0\to\mathfrak{N}+\nu+\nu$	0.540×10^8	0.540×10^8	0.539×10^8	0.256×10^8
$\Sigma_s^+\to\mathfrak{N}+\nu+\nu$	0.288×10^9	0.288×10^9	0.287×10^9	0.135×10^9
$\Sigma_s^+\to\Lambda_{\frac{1}{2}}^0+\nu+\nu$	0.868×10^6	0.866×10^6	0.852×10^6	0.423×10^6
$\Xi_s\to\mathfrak{N}+\nu+\nu$	0.203×10^{10}	0.203×10^{10}	0.202×10^{10}	0.926×10^9
$\Xi_s\to\Lambda_{\frac{1}{2}}^0+\nu+\nu$	0.115×10^9	0.115×10^9	0.115×10^9	0.547×10^8
$\Xi_s\to\Sigma_{\frac{1}{2}}^++\nu+\nu$	0.131×10^8	0.131×10^8	0.131×10^8	0.635×10^7

interaction. The lifetime was found to be 5 times longer than for the decay $\Lambda_{\frac{1}{2}}\to\mathfrak{N}+\nu+\nu$ with the same interaction. This may be expected to be typical, because of the smaller amount of energy liberated in the former process. The lifetimes for processes involving two particles with higher spin, such as $\Xi_{\frac{3}{2}}\to\Lambda_{\frac{3}{2}}+\nu+\nu$ may be calculated in a straightforward way from Eq. (17) or the similar one that holds when $s_1=s_2$. Decays in which 3 or even 4 particles have higher spins may also be considered, but at present the calculations would be quite tedious.

We shall now discuss some of the branching ratios (i.e., the ratio between observed lifetimes for normal decay, and the calculated lifetimes for Fermi decay) for some of the processes listed above. In general, for decays involving the same emitted particles, the branching ratio increases with an increase of the initial hyperon mass, and decreases for any one hyperon as the mass of the final particles increases. As an explicit example we consider the branching ratios for the universal Fermi interaction. The results may easily be obtained for any other combination of coupling constants. The following experimental lifetimes for the normal modes of decay have been used:

$$\Lambda\to\mathfrak{N}+\pi,\quad \tau=2.8\times10^{-10}\ \text{sec},^{24}$$

$$\Sigma\to\mathfrak{N}+\pi,\quad \tau\sim3\times10^{-10}\ \text{sec},^{25}$$

$$\Xi\to\Lambda+\pi,\quad \tau\sim10^{-10}\ \text{sec}.^{25}$$

$\Lambda_s\to\mathfrak{N}+\nu+\nu$.—For a Λ spin assignment of $\frac{1}{2}$, the branching ratio $B\sim1/66$; for $\frac{3}{2}$, $B\sim1/140$; for greater than $\frac{3}{2}$, $B=0$.

$\Sigma_s\to\mathfrak{N}+\nu+\nu$.—For a Σ spin assignment of $\frac{1}{2}$, the branching ratio $B\sim\frac{1}{12}$; for $\frac{3}{2}$, $B\sim1/25$; for greater than $\frac{3}{2}$, $B=0$.

$\Sigma_s\to\Lambda_{\frac{1}{2}}+\nu+\nu$.—For Λ and Σ spin assignment of $\frac{1}{2}$, the branching ratio $B\sim1/4000$.

$\Xi_s\to\mathfrak{N}+\nu+\nu$.—For Ξ spin assignment of $\frac{1}{2}$, the branching ratio $B\sim\frac{1}{5}$; for $\frac{3}{2}$, $B\sim1/11$; and for spin greater than $\frac{3}{2}$, $B=0$.

These branching ratios indicate that the Fermi decays for all the hyperons are within the range of experimental detection. If more massive hyperons exist, say $m>3000\,m_e$, which have lifetimes for the emission of a heavy particle and a pion of the same order as the presently known hyperons, and they have spins of $\frac{1}{2}$ or $\frac{3}{2}$, the dominant mode of decay will be the Fermi mode. Even for the known hyperon, the Ξ, the Fermi mode would become an appreciable fraction of the normal mode.

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APPENDIX

In the rest system the conditions defining Θ are

(a) $$\Theta^2=\Theta=\Theta^\dagger,$$

(b) $$\Theta^{\cdots}_{\cdots\alpha_i\cdots\alpha_j\cdots}=\Theta^{\cdots}_{\cdots\alpha_j\cdots\alpha_i\cdots},$$

(c) $$\sum_{i=1}^{3}\Theta^{\cdots}_{ii\cdots}=0,$$

(d) $$\Theta^{\cdots}_{\cdots4\cdots}=0.$$

[24] Blumenfeld, Booth, Lederman, and Chinowsky, Phys. Rev. **102**, 1194 (1956).

[25] H. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, 1955), Vol. II, p. 374.

The adjoint here means the transposed. Because of (a) and (d), Θ reduces to a three-dimensional matrix:

$$\Theta^{\beta_1\cdots\beta_s}_{\alpha_1\cdots\alpha_s},\quad \alpha_i,\beta_i=1,2,3.$$

The most general matrix of this type, that satisfies conditions (a) and (b), is the expression (8). The constants $a_r^{(s)}$ are easily determined by imposing condition (c). Setting $i=j$ and summing, we obtain

$$(s-2r)(s-2r-1)a_r^{(s)}+[3\cdot 2(r+1)+2(r+1)2r+2\cdot 2(r+1)(s-2r-2)]a_{r+1}^{(s)}=0,$$

which gives (9) immediately.

In order to prove Eq. (10), giving Θ for half-odd-integer spin in terms of those for integer spin, we use the theorem proved in the paper referred to above that the conditions (2) uniquely determine Θ. It is then sufficient to prove that the right-hand side of (10) satisfies (2). But this is quite obvious except for condition (2d). The latter yields the factor $(s+\frac{1}{2})/(2s+2)$ as follows. Consider

$$\text{(e)}\quad \Theta^{\beta_1\beta_2\cdots}_{\alpha_1\alpha_2\cdots}(n-\tfrac{1}{2})\Theta^{\gamma_1\gamma_2\cdots}_{\beta_1\beta_2\cdots}(n-\tfrac{1}{2}) =c^2\gamma^\alpha\gamma_\beta\gamma^{\beta'}\gamma_\gamma\Theta^{\beta\beta_1\cdots}_{\alpha\alpha_1\cdots}(n)\Theta^{\gamma\gamma_1\cdots}_{\beta'\beta_1\cdots}(n).$$

Since the traces of the Θ are all zero, the only contributing part of the $\Theta^{\beta\cdots}_{\alpha\cdots}(n)$ are the first two terms of the expansion (8). Of this the second also drops out, because

$$\gamma_\alpha\gamma^{\alpha'}\gamma_\gamma g^{\alpha\alpha_1}\Theta^{\gamma\gamma_1\cdots}_{\alpha'\alpha_1\cdots}(n)=\gamma^{\alpha_1}\gamma^{\alpha'}\gamma_\gamma\Theta^{\gamma\gamma_1\cdots}_{\alpha'\alpha_1\cdots}(n)=0.$$

The only remaining part of the right-hand side of (e) is

$$c^2\frac{1}{n!}\sum_{P(\alpha)}(g_\beta{}^\alpha g_{\beta_1}{}^{\alpha_1}\cdots)\gamma^\beta\gamma_\alpha\gamma^{\alpha'}\gamma_\gamma\Theta^{\gamma\gamma_1\cdots}_{\alpha'\alpha_1\cdots}(n)$$
$$=c\left[\frac{3}{n}+2\left(1-\frac{1}{n}\right)\right]\Theta^{\gamma_1\gamma_2\cdots}_{\beta_1\beta_2\cdots}(n-\tfrac{1}{2}),$$

where $n=s+\frac{1}{2}$. This fixes the constant in Eq. (10).

In a very similar way, we may prove that

$$\text{(f)}\qquad g_{\beta_1}{}^{\alpha_1}\Theta^{\beta_1\cdots}_{\alpha_1\cdots}(s)=\frac{2s+1}{2s-1}\Theta^{\beta_2\cdots}_{\alpha_2\cdots}(s-1),$$

while Eq. (8) gives

$$\text{(g)}\quad \frac{1}{p^{2s}}p_{\beta_1}\cdots p_{\beta_s}p^{\alpha_1}\cdots p^{\alpha_s}\Theta^{\beta_1\cdots\beta_s}_{\alpha_1\cdots\alpha_s}(p') =(-)^s\sinh^{2s}\theta\sum_{r=0}a_r^{(s)},$$

where $\cosh^2\theta=(p\cdot p')^2/p^2p'^2$ and $a_0^{(s)}\equiv 1$. Using (f) and (g), we find

$$\text{(h)}\quad \Theta^{\alpha_1\cdots\alpha_s}_{\beta_1\cdots\beta_s}(p)\Theta^{\beta_1\cdots\beta_s}_{\alpha_1\cdots\alpha_s}(p')=\sum_{r=0}a_r^{(s)}\sum_{n=0}^{s-2r}\binom{s-2r}{n} \times\frac{2s+1}{(2s-2n+1)}(\sinh\theta)^{2(s-n)}\sum_{t=0}a_t^{(n-s)}.$$

These various formulas are useful for the evaluation of transition probabilities.