PHYSICAL REVIEW VOLUME 106, NUMBER 2 APRIL 15, 1957

Remarks on Possible Noninvariance under Time Reversal and Charge Conjugation*

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(Received January 7, 1957)

Interrelations between the nonconservation properties of parity, time reversal, and charge conjugation are discussed. The results are stated in two theorems. The experimental implications for the K-$\bar{K}$ complex are discussed in the last section.

IN a recent paper[1] the question has been raised as to whether the weak interactions are invariant under a space inversion. It was also pointed out there that similar to the situation for space inversion there exists at present no experimental proof that weak interactions are invariant under charge conjugation. Consequently the absolute invariance under charge conjugation is also an open question.

The present note is devoted to a study of questions concerning the invariance under charge conjugation C, and under time reversal T (which is defined to be the Wigner time[2] reversal. It *does not* switch a particle into its antiparticle; nor does it change the sign of the spatial coordinates).

1. *CPT* THEOREM

For the discussion of the experimental consequences of possible nonconservation of P, C, and/or T, a theorem[3] which we shall call the CPT theorem proves very important.

To understand the meaning of the theorem one recalls first that the operations P and C in any many-particle system (with possibilities of creation and annihilation) are represented by unitary operators that operate on the state vectors. The operation T, on the other hand, is represented[2] by the *operator* of complex conjugation *multiplied* by a unitary operator. In the Schrödinger representation the transformation of a second quantized spin 0 field described by $\varphi(r)$ and $\pi(r)$ under these operations[4] can be brought into the following form:

$$\begin{aligned} P\phi(r)P^{-1}&=\eta_P\phi(-r), & P\pi(r)P^{-1}&=\eta_P^*\pi(-r),\\ C\phi(r)C^{-1}&=\eta_C\phi^\dagger(r), & C\pi(r)C^{-1}&=\eta_C^*\pi^\dagger(r),\\ T\phi(r)T^{-1}&=\eta_T\phi(r), & T\pi(r)T^{-1}&=-\eta_T^*\pi(r),\end{aligned}\tag{1}$$

where † means Hermitian conjugate and the phases η_P, η_C, and η_T have absolute values equal to 1. For the spin $\frac{1}{2}$ field $\psi(r)$, the transformations are

$$\begin{aligned} P\psi(r)P^{-1}&=\eta_P\gamma_4\psi(-r),\\ C\psi(r)C^{-1}&=\eta_C\psi^\dagger(r),\\ T\psi(r)T^{-1}&=\eta_T\gamma_1\gamma_2\gamma_3\psi(r),\end{aligned}\tag{2}$$

where the γ matrices are so chosen that γ_1, γ_2, and γ_3 are real and γ_4 is pure imaginary. The phases η_P, η_C, and η_T also have absolute values equal to unity. The transformation properties of fields of higher spin are similar.

From the CPT theorem one concludes that for any local Hermitian Hamiltonian H which is invariant under proper Lorentz transformations (i.e., Lorentz transformations that involve neither space nor time inversions), there always exists a choice of the phases η_C, η_P, and η_T for the various fields (usually in more than one way) with the following properties: (a) H commutes with the product of the operators P, C, and T taken in any order; and (b) if this choice of phases does not make H commute with P, then no other choice does, and the theory is not invariant under space inversion. (Of course, if this choice of phases makes H commute with P, then the theory is invariant under space inversion.) The same holds for C and T.

We shall illustrate this theorem by an example where H is invariant under proper Lorentz transformations. Let ψ_p, ψ_n, ψ_e, and ψ_ν be the fields describing the proton, the neutron, the electron, and the neutrino. The neutrino is assumed to be a non-Majorana particle with a nonvanishing mass. Consider

$$\begin{aligned} H=H_{\text{free}}+\int\{&g_1(\psi_p^\dagger\gamma_4\psi_n)(\psi_e^\dagger\gamma_4\psi_\nu)\\ &+g_2(\psi_p^\dagger\gamma_4\psi_n)(\psi_e^\dagger\gamma_4\gamma_5\psi_\nu)\\ &+g_3(\psi_p^\dagger\gamma_4\gamma_5\psi_n)(\psi_e^\dagger\gamma_4\gamma_5\psi_\nu)\\ &+g_4(\psi_p^\dagger\gamma_4\gamma_5\psi_n)(\psi_e^\dagger\gamma_4\psi_\nu)\\ &+\text{Hermitian conjugate}\}d^3(r),\end{aligned}\tag{3}$$

* *Note added in proof.*—This paper was written in December, 1956, before parity nonconservation was experimentally established.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[2] E. P. Wigner, Gött. Nachr., Math. Naturw. Kl. (1932), p. 546.

[3] See W. Pauli's article in *Niels Bohr and the Development of Physics* (Pergamon Press, London, 1955). G. Lüders, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 5 (1954). J. Schwinger, Phys. Rev. **91**, 720, 723 (1953); **94**, 1366, formula (54) and p. 1576, discussions after formula (208). We are indebted to Professor Pauli for informing us of the work of Schwinger.

[4] We discuss here only the usual "type" of fields. The possibility of the existence of unusual "types" has been pointed out by Wigner. [An account of these unusual types has been given by L. Michel and A. S. Wightman, Princeton University lecture notes (unpublished).] An examination of these unusual "types" would be an important task if space-time conservation laws should indeed be found to break down for the weak interactions.

where $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$. This example is a special case of an example considered by Pauli.[3]

Writing H as

$$H=H(g_1,g_2,g_3,g_4),$$

one easily proves that

$$PHP^{-1}=H(g_1\eta_P,\ -g_2\eta_P,\ g_3\eta_P,\ -g_4\eta_P), \tag{4}$$

$$CHC^{-1}=H(g_1^*\eta_C^*,\ -g_2^*\eta_C^*,\ g_3^*\eta_C^*,\ -g_4^*\eta_C^*), \tag{5}$$

$$THT^{-1}=H(g_1^*\eta_T,\ g_2^*\eta_T,\ g_3^*\eta_T,\ g_4^*\eta_T). \tag{6}$$

In deriving these formulas, use has been made of the fact that $TgT^{-1}=g^*$ and $T\gamma_iT^{-1}=\gamma_i^*$. The phases η_P, η_C, and η_T are products of the respective phases of the four interacting fields. They are given by

$$\begin{aligned}\eta_P&=\eta_P^*(p)\eta_P(n)\eta_P^*(e)\eta_P(\nu),\\ \eta_C&=\eta_C^*(p)\eta_C(n)\eta_C^*(e)\eta_C(\nu),\\ \eta_T&=\eta_T^*(p)\eta_T(n)\eta_T^*(e)\eta_T(\nu).\end{aligned} \tag{7}$$

Using (4), (5), and (6), one can calculate the commutation relation between H and the six operators TCP, TPC, $\cdots$, PCT. It is found that with suitable choices of the phases η, the Hamiltonian H commutes with all of the six, as required by the CPT theorem. In fact the conditions on the phases η are simply

$$\eta_P=\eta_C\eta_T=\pm 1. \tag{8}$$

It follows from the CPT theorem that, if one of the three operators P, C, and T is not conserved, at least one other must also be not conserved. It is of course also possible that all three are separately not conserved. In the example above, by assigning suitable values to the coupling constants g, one can construct examples for all the five possibilities of conservation or nonconservation of P, C, and T. These examples are displayed in Table I.

2. LIFETIME OF CHARGE CONJUGATE PARTICLES AGAINST WEAK DECAY

Consider now a Hamiltonian

$$H=H_{\text{strong}}+H_{\text{weak}},$$

where both terms are invariant under a proper Lorentz transformation. In all subsequent discussions we shall assume that H_{strong} is invariant under C, P, and T. The phases η of the fields are defined (up to, possibly, some arbitrary factors) by this invariance; i.e., by the requirements that

$$CH_{\text{strong}}C^{-1}=H_{\text{strong}},\ \text{etc.}$$

On the other hand, the weak interactions may violate the invariance of C, P, and T. One can prove the following theorem.

Theorem 1.—If a particle A decays through the interaction H_{weak}, and if the particle and its antiparticle $\bar{A}$ do not decay into the same final products (as e.g. when A is charged), then to the lowest order of H_{weak} the lifetimes of A and $\bar{A}$ are the same, even if H_{weak} is *not* invariant under charge conjugation.

Proof.—Consider the case that particle A has spin zero. [The proof for the general case follows along the same lines.] Then the final states B and $\bar{B}$ in the decays

$$A\rightarrow B,\quad \bar{A}\rightarrow\bar{B}$$

also have spin zero. Using the identity

$$\langle\psi_1|\psi_2\rangle^*=\langle T\psi_1|T\psi_2\rangle, \tag{9}$$

one obtains

$$\begin{aligned}\langle B|H_{\text{weak}}|A\rangle^*&=\langle TB|TH_{\text{weak}}T^{-1}|TA\rangle\\ &=\langle TB|C^{-1}P^{-1}H_{\text{weak}}PC|TA\rangle,\end{aligned}$$

by the CPT theorem. Consider first the case that H_{weak} commutes (or anticommutes) with P. Then

$$\langle B|H_{\text{weak}}|A\rangle^*=\pm\langle TB|C^{-1}H_{\text{weak}}C|TA\rangle.$$

For a spinless system,

$$|TA\rangle=|A\rangle,\quad |TB\rangle=|B\rangle. \tag{10}$$

Hence

$$\begin{aligned}\langle B|H_{\text{weak}}|A\rangle^*&=\pm\langle B|C^{-1}H_{\text{weak}}C|A\rangle\\ &=\pm\langle CB|H_{\text{weak}}|CA\rangle=\pm\langle\bar{B}|H_{\text{weak}}|\bar{A}\rangle.\end{aligned} \tag{11}$$

This shows that the lifetimes of A and $\bar{A}$ are the same. If H_{weak} does not commute with P, we write

$$H_{\text{weak}}=H_1+H_2, \tag{12}$$

where

$$\begin{aligned}H_1&=\tfrac{1}{2}[H_{\text{weak}}+PH_{\text{weak}}P^{-1}],\\ H_2&=\tfrac{1}{2}[H_{\text{weak}}-PH_{\text{weak}}P^{-1}].\end{aligned} \tag{13}$$

Then

$$PH_1P^{-1}=H_1, \tag{14}$$

$$PH_2P^{-1}=-H_2. \tag{15}$$

The decays of A through H_1 and through H_2 lead to states B_1 and B_2 with opposite parities. They are orthogonal to the order considered, and hence they contribute independently without interference to the decay rate of A. The lifetimes of A and $\bar{A}$ are therefore again the same.

A consequence of this theorem has already been mentioned in a previous paper[1]: The identity of the experimental lifetimes of $\pi^\pm$ and of $\mu^\pm$ does not con-

TABLE I. Examples of theories with various possible nonconservation properties.

Value of coupling constants	Conserved operators	Nonconserved operators
g_1=real, g_3=real, $g_2=g_4=0$	P, C, T	$\cdots$
g_1=real, g_3=complex, $g_2=g_4=0$	P, CT, TC	C, T
g_1=real, g_2=imaginary, $g_3=g_4=0$	C, PT, TP	P, T
g_1=real, g_2=real, $g_3=g_4=0$	T, CP, PC	C, P
g_1=real, g_2=complex, $g_3=g_4=0$	PCT, *and* permutations	P, C, T

stitute a proof that charge conjugation invariance holds for the weak interactions.

For a discussion of a case where A and $\bar{A}$ may decay into the same final channels, see Sec. 4.

3. DEPENDENCE OF INTERFERENCE EFFECTS ON CONSERVATION OF C AND T

One would like to ask what are the experimentally detectable manifestations of a weak nonconservation of P, C, or T? For the nonconservation of parity, the answer is clearly to be sought in experiments to differentiate the right-handed screw from the left-handed. Some such experiments have been discussed before.[1]

If parity is indeed not strictly conserved, some of these experiments could *also reveal whether C and/or T are or are not conserved*. To illustrate this let us consider the experiment of the angular distribution of β decay from oriented nuclei. The degree of asymmetry was given in the appendix of reference 1 as proportional to

$$\mathrm{Re}\left[C_TC_T'^*-C_AC_A'^*+i\frac{Ze^2}{\hbar cp}(C_AC_T'^*+C_A'C_T^*)\right]. \quad (16)$$

Applying the arguments of Sec. 1 we recognize that the two terms in (16) are present or absent depending on whether C or T are not conserved. To be more specific: The first term vanishes if C is strictly conserved, the second term vanishes if T is strictly conserved. If this experiment shows any asymmetry, the p dependence and the Z dependence of the asymmetry could therefore reveal whether C and/or T are nonconserved. (The existence of any asymmetry rules out the possibility that both C and T are conserved, a conclusion we already drew on general grounds in Sec. 1.)

We notice that if C is strictly conserved, the asymmetry discussed above vanishes in the absence of the Coulomb distortion of the electron wave function. In fact, when C is conserved the asymmetry is directly dependent on the existence of a difference of the Coulomb phase shifts for opposite parities. It turns out that this is a consequence of a general theorem which we state and prove below:

Theorem 2.—If, in addition to the assumptions stated in Sec. 2 concerning H_{strong} and H_{weak}, we assume that H is strictly invariant under charge conjugation, (i.e., $[H,C]=0$) and if the decay products in the final state B are *free* particles, then to the lowest order of H_{weak} there is no interference between the parity-conserving and the parity-nonconserving parts of H in the decay of A, provided the interference is sought for in experiments measuring a term of the form $\boldsymbol{\sigma}\cdot\mathbf{p}^1$.

Proof.—We again illustrate the proof by considering the case that A is spinless. The general proof follows along the same lines. We perform the decomposition of H_{weak} as in Eqs. (12)–(15). The final state B consists of two states B_1 and B_2 of opposite parities reached from A through H_1 and H_2, respectively. Clearly H_1 commutes with C, and also, by the CPT theorem, commutes with CPT. Hence using identity (9) and Eq. (10) one obtains

$$\begin{aligned}\langle B_1|H_1|A\rangle^* &= \langle TB_1|TH_1T^{-1}|TA\rangle \\ &= \langle B_1|TH_1T^{-1}|A\rangle \\ &= \langle B_1|P^{-1}C^{-1}H_1CP|A\rangle \\ &= \langle B_1|P^{-1}H_1P|A\rangle = \langle B_1|H_1|A\rangle.\end{aligned}$$

Thus $\langle B_1|H_1|A\rangle$ is real. Similarly one easily proves that $\langle B_2|H_2|A\rangle$ is pure imaginary.

In the above the states B_1 and B_2 are taken as stationary states of H_{strong} consisting of standing waves. [Otherwise Eq. (10) does not hold.] Transition amplitudes into them have a relative phase factor which is, according to the above, pure imaginary. The observed final states are equal to these amplitudes multiplied by the *outgoing* part of the stationary states B_1 and B_2. Such outgoing parts always have real relative amplitudes if the stationary states B_1 and B_2 represent free particles. The theorem now follows immediately.

Using this theorem, one concludes that if any left-right asymmetry of the form $\boldsymbol{\sigma}\cdot\mathbf{p}$ is found, the part of this asymmetry that is independent of the distortion of the final-state wave functions can arise only if charge conjugation symmetry breaks down for the weak interactions. In particular, in decays where there is no strong final-state interactions, as, e.g., in $\pi\rightarrow\mu+\nu$ and $\mu\rightarrow e+\nu+\nu$ decays, the detection[1] of parity nonconservation through the observation of $\boldsymbol{\sigma}\cdot\mathbf{p}$ becomes impossible if C is strictly conserved.

4. K^0, $\bar{K}^0$ DECAY MODES

The existence of the particle $\bar{K}^0$ and some properties of its decay were predicted[5] and discussed under the assumption that charge conjugation is strictly conserved. We wish to discuss in this section the decay of K^0 and $\bar{K}^0$ under the assumption that C, P, and T are conserved for the strong interactions, but are not necessarily conserved in the weak decay interactions.

In the first place, the conservation of strangeness with respect to the strong interactions still requires that two particles K^0 and $\bar{K}^0$ with opposite strangeness exist. To understand their decay processes it is interesting to consider the charge conjugation symmetrical and antisymmetrical combinations introduced in reference 5 (compare, however, footnote 11):

$$K_1=\frac{1}{\sqrt{2}}(K^0+\bar{K}^0), \quad K_2=\frac{1}{\sqrt{2}}(K^0-\bar{K}^0). \quad (17)$$

Unlike the situation in reference 5, if C is now not invariant in the decay process, K_1 and K_2 can decay into the same final states:

$$\begin{aligned}&K_1\rightarrow\pi^++\pi^-,\\ &K_2\rightarrow\pi^++\pi^-,\\ &K_1\rightarrow\pi^\pm+e^\mp+\nu,\\ &K_2\rightarrow\pi^\pm+e^\mp+\nu,\\ &K_1\rightarrow\pi^++\pi^-+\pi^0,\\ &K_2\rightarrow\pi^++\pi^-+\pi^0.\end{aligned}$$

[5] M. Gell-Mann and A. Pais, Phys. Rev. 97, 1387 (1955).

Interference effects would therefore set in in these decay processes. The questions that one would like to ask are then: what the the lifetimes of the particles K^0 and $\bar{K}^0$? What are the branching ratios into various decay modes?

These questions can be answered by using a Weisskopf-Wigner type of treatment[6] of the time-dependent Schrödinger equation. We write the time-dependent amplitudes of the particles[7] K^0 and $\bar{K}^0$ as $a(t)$ and $b(t)$. The various channels of decay are denoted by j. $F_j(\omega)e^{-i\omega t}$ represents the amplitude of the decay product in the channel j with the energy ω.[8] [We choose units such that $\hbar=1$.] The zero of energy is taken to be the rest energy of K. The Schrödinger equations are then

$$i\frac{da}{dt}=\sum_{j,\omega} H_{aj}(\omega)F_j(\omega)e^{-i\omega t}, \tag{18}$$

$$i\frac{db}{dt}=\sum_{j,\omega} H_{bj}(\omega)F_j(\omega)e^{-i\omega t}, \tag{19}$$

$$i\frac{\partial F_j(\omega)}{\partial t}=e^{i\omega t}[H_{ja}(\omega)a+H_{jb}(\omega)b], \tag{20}$$

where $H_{ja}=H_{aj}^*$ are the matrix elements. The Weisskopf-Wigner treatment consists of first assuming an exponential time dependence for a and b, and then in sums over ω neglecting the variation of the matrix elements with ω in the interval $|\omega|\lesssim$uncertainty of energy of the original state. Using this treatment, one obtains

$$\begin{pmatrix} a \\ b \end{pmatrix}=\psi e^{-\frac{1}{2}\lambda t}. \tag{21}$$

The amplitude ψ and the decay constant λ are given by the eigenequation

$$\Gamma\psi=\lambda\psi. \tag{22}$$

Γ is a 2×2 Hermitian matrix with matrix elements given by

$$\begin{aligned}\Gamma_{11}=\Gamma_{22}&=\sum_j \Gamma_{aj}=\sum_j \Gamma_{bj},\\ \Gamma_{12}&=\sum_j (\Gamma_{aj}\Gamma_{bj})^{\frac{1}{2}}e^{i\delta_j}=\Gamma_{21}^*,\end{aligned} \tag{23}$$

where

$$\begin{aligned}\Gamma_{aj}&=2\pi|H_{aj}|^2 \text{ (density of states per unit } d\omega)_{\omega=0},\\ \Gamma_{bj}&=2\pi|H_{bj}|^2 \text{ (density of states per unit } d\omega)_{\omega=0},\end{aligned} \tag{24}$$

and

$$e^{i\delta_j}=(\text{phase of } H_{aj}H_{bj}^{-1})_{\omega=0}. \tag{25}$$

In the foregoing derivation, use has been made of Eq. (11) which leads to

$$H_{aj}^*=\pm H_{bj'},$$

where j' is the charge conjugate channel of j (which may or may not be the same as j). It is important to notice that this equation is a consequence of the CPT theorem.

The two eigenvalues λ_+, λ_- of (22) correspond to the two decay lifetimes. The general solution is a linear superposition of two solutions $\psi_\pm$ of the form (21), each of which is characterized by a pure exponential decay. Since the 2×2 matrix Γ is Hermitian, the two solutions represent linear orthogonal combinations of the states K and $\bar{K}$.

In writing down Eqs. (18) and (19) we did not include a slight difference of mass in the form of a mass operation M for the states of the K particle. This restriction can be easily removed by adding to the right-hand sides of (18) and (19) the terms $\frac{1}{2}(M_{11}a+M_{12}b)$ and $\frac{1}{2}(M_{21}a+M_{22}b)$, respectively. The mathematical treatment is very similar to the above simple case except that we have now

$$-d\psi/dt=(\Gamma+iM)\psi, \tag{26}$$

where Γ is the same Hermitian matrix given by Eq. (23). (iM) is an anti-Hermitian matrix representing the effects of the mass shifts. By using Eq. (11) one can show that, similar to Eq. (23), M is a Hermitian matrix with

$$M_{11}=M_{22}. \tag{27}$$

Equation (26) can now be readily solved. Its eigenstates, defined by

$$(\Gamma+iM)\psi_\pm=\lambda_\pm\psi_\pm,$$

are

$$\psi_\pm=\begin{pmatrix} p \\ \pm q \end{pmatrix}(|p|^2+|q|^2)^{-\frac{1}{2}}, \tag{28}$$

with the corresponding time constants

$$\lambda_\pm=\Gamma_{11}+iM_{11}\pm(pq); \tag{29}$$

where p and q are two complex numbers given by

$$p^2=\Gamma_{12}+iM_{12},\quad q^2=\Gamma_{21}+iM_{21}=\Gamma_{12}^*+iM_{12}^*. \tag{30}$$

If at $t=0$ a K particle is produced, then at a later time the state function ψ can be expressed in terms of these two eigenstates $\psi_\pm$ as

$$\psi(t)=\left(\frac{1}{2p}\right)(|p|^2+|q|^2)^{\frac{1}{2}}[\psi_+e^{-\frac{1}{2}\lambda_+t}+\psi_-e^{-\frac{1}{2}\lambda_-t}]. \tag{31}$$

It is convenient to separate the real and imaginary parts of $\lambda_\pm$. Without loss of generality, we may write

$$\lambda_+=\gamma_+,\quad \lambda_-=\gamma_-+2i\Delta, \tag{32}$$

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[6] V. F. Weisskopf and E. P. Wigner, Z. Physik **63**, 54 (1930); **65**, 18 (1930).

[7] In this paper we assume that the K^0 particle (strangeness $=+1$) is a single state.

[8] Each channel j represents a possible decay state that is an eigenstate of H_{strong}. Thus it has a definite spin, charge, and parity.

where γ_+, γ_- are two real numbers representing the reciprocal lifetimes of the short-lived ones and the long-lived ones respectively and Δ is the mass difference between these two eigenstates. One notices that *these two eigenstates* ψ_+ *and* ψ_- *do not in general represent the states* K_1 *and* K_2 introduced in (17). *In fact they even may not be orthogonal to each other* (see footnote 11).

A general discussion of the decay processes is rather involved. We shall make only the following remarks:

(*A*) The fractional number of K mesons that decay at time t after its production is given by

$$N(t)dt=-d[\psi^\dagger\psi]. \tag{33}$$

Using (26), one easily shows that

$$-\frac{d}{dt}[\psi^\dagger\psi]=\psi^\dagger\Gamma\psi.$$

By using (28)–(31), Eq. (33) becomes

$$N(t)=\tfrac{1}{2}(1+\alpha)^{-1}\{\gamma_+e^{-\gamma_+t}+\gamma_-e^{-\gamma_-t}+\alpha e^{-\frac{1}{2}(\gamma_++\gamma_-)t} \times[(\gamma_++\gamma_-)\cos\Delta t-2\Delta\sin\Delta t]\}, \tag{34}$$

where

$$\alpha=\psi_+{}^\dagger\psi_-=[|p|^2-|q|^2][|p|^2+|q|^2]^{-1} \tag{35}$$

is a real number representing the nonorthogonality of these two eigenstates. The four real numbers γ_+, γ_-, Δ, and α characterize the decay of the K particle. They satisfy the inequalities

$$\gamma_\pm\geqq 0,\quad \alpha^2\leqq\frac{4\gamma_+\gamma_-}{(\gamma_++\gamma_-)^2+4\Delta^2}, \tag{36}$$

which follow from the fact that Γ is a positive Hermitian matrix. These conditions also insure that $N(t)\geqq 0$ for all t.

Experimentally $N(t)$ is measurable. From $N(t)$ one can in principle determine all four constants γ_+, γ_-, Δ, and α. Indications from presently existing experiments[9] show that probably $\gamma_+/\gamma_->100$. Equation (36) then shows that $\alpha^2<4\gamma_-/\gamma_+<0.04$.

(*B*) The above discussion also leads easily to a determination of the branching ratio of the long-lived component (and the short-lived component) into the various decay modes. If charge conjugation is conserved, the long-lived component is an eigenstate of charge conjugation.[5] Consequently its decay into charge conjugate channels such as $\pi^+e^-\nu$ and $\pi^-e^+\bar{\nu}$ must be equally probable, as is well known. If charge conjugation is not strictly conserved, decays into $\pi^+e^-\nu$ and $\pi^-e^+\nu$ may have different probabilities for the long-lived component.

A more complete discussion of the charge asymmetry of the decay of the long-lived K^0 will be given in the appendix. We mention here only that Lederman[10] has kindly informed us that experimental work in this direction is in progress. It is important to notice that if the experiments should yield a large asymmetry, and a small α (as mentioned above), Eq. (A7) would impose very strict conditions on the relative magnitudes of the amplitudes f_1, g_1, f_2 and g_2. (To see this roughly we need only examine the limiting case $\alpha=0$ discussed below.)

(*C*) If $\alpha=0$, the two eigenstates are orthogonal.[11] Also $|p|=|q|$. [See (35).] This is the case if the mass matrix is negligible. In this case $\psi_\pm$ are both 1:1 superpositions of the particle K^0 and $\bar{K}^0$. The fraction of particles decaying in dt, namely $N(t)dt$, becomes the sum of two pure exponentials by (34). Furthermore (A7) shows that the decays of the long-lived component into charge conjugate channels such as $\pi^++e^-+\nu$ and $\pi^-+e^++\bar{\nu}$ are equally probable.

One of us (Reinhard Oehme) would like to express his gratitude to Professor Robert Oppenheimer for his kind hospitality at the Institute for Advanced Study.

APPENDIX

In this appendix we shall show the interrelationship between the parameters p, q and the branching ratio for the decay of, say, the long-lived component of K particle into various charge conjugate states.

Consider first the following decay channel of the K particle

$$K\rightarrow e^-+\pi^++\nu. \tag{A1}$$

The final product may be in states with either parity $=+1$ or parity $=-1$. Let us denote the matrix elements for the decay process into these two types of states by f_1 and f_2. Similarly, we denote the matrix elements for

$$K\rightarrow e^++\pi^-+\bar{\nu}, \tag{A2}$$

with the final state having parity $=+1$ and parity $=-1$, by g_1 and g_2.

By using the *CPT* theorem and Eq. (11), the corresponding matrix elements for the decay of $\bar{K}$,

$$\bar{K}\rightarrow e^++\pi^-+\bar{\nu}, \tag{A3}$$

are related to that of (A1). These elements are f_1^* and $-f_2^*$. Similarly the matrix elements for

$$\bar{K}\rightarrow e^-+\pi^++\nu \tag{A4}$$

are g_1^* and $-g_2^*$. Let ψ_+ represent the long-lived component K_+ of the K particle. The matrix elements for

[9] K. Lande *et al.*, Phys. Rev. **103**, 1901 (1956).

[10] L. Lederman (private communication).

[11] We recall that since the strangeness S is conserved in the strong interaction, the phase η_c of a K particle ($S=+1$) under charge conjugation is not fixed by the strong interactions. If the weak interaction is not invariant under charge conjugation, the phase η_c is defined only up to a factor $e^{iS\theta}$. If ψ_+ is orthogonal to ψ_-, there exists however a most convenient choice which makes $\psi_\pm$ identical with the K_1, K_2 defined in (17).

the decay of K_+,

$$K_+ \to e^- + \pi^+ + \nu, \qquad (A5)$$

into the two different final parity states are proportional to $pf_1 + qg_1^*$ and $pf_2 - qg_2^*$, respectively, while the corresponding elements for

$$K_+ \to e^+ + \pi^- + \bar{\nu} \qquad (A6)$$

are proportional to $pg_1 + qf_1^*$ and $pg_2 - qf_2^*$. The branching ratio r for the decay of K_+ into $e^- + \pi^+ + \nu$ and $e^+ + \pi^- + \bar{\nu}$ is, therefore,

$$r = \frac{|pf_1 + qg_1^*|^2 + |pf_2 - qg_2^*|^2}{|pg_1 + qf_1^*|^2 + |pg_2 - qf_2^*|^2}. \qquad (A7)$$

PHYSICAL REVIEW VOLUME 106, NUMBER 2 APRIL 15, 1957

Fermi Decay of Higher Spin Particles*

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(Received November 26, 1956)

The explicit form for the spin projection operators introduced by Fronsdal is calculated for arbitrary spin and applied to first-order processes involving four fermions. The matrix element for the most general nonderivative interaction is found for the special case in which two of the particles have spin $\frac{1}{2}$. The method of relating matrix elements written in different orders is extended to this case.

The theory is applied to the decay of the mu meson, extending the work of Caianiello. It is found that the experimental decay spectrum can be equally well fitted by an assignment of spin $\frac{1}{2}$ or $\frac{3}{2}$. The method is then applied to the Fermi decay of hyperons. Lifetimes are calculated for decays in which the initial particle has a spin of $\frac{1}{2}$ or $\frac{3}{2}$, and the final particles all have spin $\frac{1}{2}$. All the lifetimes are less than 2 orders of magnitude longer than the corresponding observed lifetimes for the normal mode of decay.

The hypothesis of a universal Fermi interaction is extended to include fermions of arbitary spin. Under this hypothesis, the experimental muon spectrum is most closely reproduced with spin $\frac{3}{2}$. The results also indicate that the muon has the same particle-antiparticle character as an electron of the same charge.

INTRODUCTION

RECENT experimental evidence has indicated the possibility that the "strange" particles may have spins larger than unity. Ruderman and Karplus[1] have found, by an analysis of mesonic and nonmesonic decay of hyperfragments, that the spin of the Λ^0 is either $\frac{1}{2}$ or $\frac{3}{2}$. Walker and Shephard[2] analyzed the angular correlations between the planes of production and decay of the Σ and the Λ^0 and found the spins to be $\frac{3}{2}$, $\frac{5}{2}$, or $\frac{7}{2}$. In addition to the strange particles, the long-known mu meson may conceivably have spin higher than $\frac{1}{2}$.

When considering the possibility that some of the hyperons might be fermions with spin higher than $\frac{1}{2}$, we meet a difficulty in that some of them are charged, and so interact with the electromagnetic field. A gauge-invariant way of describing this interaction has been given by Fierz and Pauli.[3] Only very few calculations have been carried out on the electromagnetic properties of particles described by the Fierz-Pauli equation, and the only result of interest to us is that of Mathews,[4] who calculated the Compton scattering cross section and the bremsstrahlung in the case of spin $\frac{3}{2}$. His result definitely rules out the possibility that the muon is such a particle, while the conclusions that can be made with regard to hyperons are less definite.

In the present paper we have calculated the lifetimes and spectra of Fermi decays of higher spin particles to first order, i.e., using the field-free wave functions. This calculation has been applied to the hyperons, the heavy mesons,[5] and the muon. We have included the muon on the basis that the electromagnetic properties of higher spin particles might be different than those predicted by Fierz and Pauli and calculated by Mathews.

FREE FIELDS

The wave function appropriate for describing a free particle of integral spin s is a tensor of rank s, and satisfies the wave equation

$$(p^2 + m^2)\Phi_{\alpha_1 \cdots \alpha_s} = 0, \qquad (1a)$$

and the subsidiary conditions

$$\Phi_{\cdots \alpha_i \cdots \alpha_j \cdots} = \Phi_{\cdots \alpha_j \cdots \alpha_i \cdots}, \qquad (1b)$$

$$p^{\alpha_1}\Phi_{\alpha_1 \cdots \alpha_s} = 0, \qquad (1c)$$

$$g^{\alpha_1\alpha_2}\Phi_{\alpha_1\alpha_2 \cdots \alpha_s} = 0, \qquad (1d)$$

where $g^{\mu\nu}$ is the metric tensor.

* This work was supported in part by the National Science Foundation.
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[2] W. Walker and W. Shephard, Phys. Rev. **101**, 1810 (1956).
[3] M. Fierz and W. Pauli, Proc. Roy. Soc. (London) **A173**, 211 (1939).
[4] J. Mathews, Phys. Rev. **102**, 270 (1956).
[5] Results for the heavy mesons will be given in a separate publication.