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Analysis of Polarization in Neutron-Proton Scattering*

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An analysis of polarization in neutron-proton scattering experiments is described, and phase shifts of S and D waves for 310 Mev are presented. The calculation to obtain the S and D phase shifts has been arranged, assuming charge independence, to make use of experimental p-p polarization information in order not to depend on specific phase shift analyses of these data. The results indicate ranges of values of the phase shifts in which charge-independent fits to n-p and p-p data may be expected to exist.

I. INTRODUCTION

In this note an account is given of a phase shift analysis of the data on polarization in neutron-proton scattering at 310 Mev, and some preliminary results of the analysis are presented and discussed. It makes use of an Argand diagram treatment[1] of polarization data and an equivalent analytical computation. Resultant phase shifts are limited in size by the total scattering cross section.

If it is assumed that states for $L>3$ are not important at 310 Mev, then the analysis of polarization is in terms of ten triplet phase shifts: $\delta_1{}^S$, $\delta_0{}^P$, $\delta_1{}^P$, $\delta_2{}^P$, $\delta_1{}^D$, $\delta_2{}^D$, $\delta_3{}^D$, $\delta_2{}^F$, $\delta_3{}^F$, $\delta_4{}^F$. The possible couplings between ${}^3S_1$ and ${}^3D_1$ as well as ${}^3P_2$ and ${}^3F_2$ states, such as might arise as a result of tensor forces, add two additional parameters. These couplings are neglected in the present note in order to obtain a simplified, though incomplete survey of the possibilities.

In Sec. II, the formula for polarization in neutron-proton scattering taking into account partial waves with $L\leqq 3$ is presented. Section III is devoted to the analysis of experiments on n-p polarization in terms of Legendre polynomials. In Sec. IV a phase shift analysis for D and S waves is made, and the effects of experimental uncertainty are considered. In this analysis the theoretical expressions for the coefficients of a Legendre polynomial expansion of n-p polarization are separated into a part containing P and F phase shifts in the same combination as they appear for p-p polarization, and a part containing S and D phase shifts only. The experimental values of the coefficients of a Legendre polynomial analysis of p-p polarization were used, assuming charge independence, for the former part in order to make the present analysis independent of assumptions about the number or values of phase shifts in p-p scattering states.

In Sec. V selected sets of D and S phase shifts are used together with sets of P and F waves from analyses of p-p data to calculate theoretical values of the n-p polarization coefficients. Since only the coefficients of odd polynomials can be separated in the manner described, specific values of S, P, D, F phase shifts must be assumed in order to compare the theoretical fit with the experimental data. In Sec. VI the effect on the fits of introducing small ${}^3G$ phase shifts is looked into.

The spirit of this investigation has been more that of providing knowledge of regions of phase shift space where over-all fits are to be sought, rather than that of finding final solutions, which probably can be obtained more effectively by means of high speed computing machines once the region in which the phase shift search is to be made is sufficiently limited. Starting points for such searches, intended to lead to charge-independent fits for n-p and p-p data, will be found in results given below.

II. FORMULA FOR POLARIZATION IN NEUTRON-PROTON SCATTERING FOR $L\leq 3$

The general formula for polarization in neutron-proton scattering was given by Breit, Ehrman, and Hull.[2] An expression for the calculation of polarization taking into account wave for $L\leq 3$ is shown below. Coupling between ${}^3S_1$ and ${}^3D_1$ and between ${}^3P_2$ and ${}^3F_2$ states is neglected, but otherwise the most general condition described by a set of phase shifts for states of definite orbital as well as total angular momentum is considered. The result is written in a form convenient for numerical work.

If P is the polarization, defined as twice the expectation value of the y component of the spin, then

$$k^2(P\sigma)_{n-p}/\sin\theta=\alpha P_0(\cos\theta)+\beta P_1(\cos\theta)+\gamma P_2(\cos\theta)+\delta P_3(\cos\theta)+\epsilon P_4(\cos\theta)+\zeta P_5(\cos\theta), \quad (1)$$

where σ is the differential cross section for single scattering, k the wave number of the incident nucleons. Explicit expressions for the six coefficients are as follows:

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[1] Hull, Ehrman, Hatcher, and Durand, Phys. Rev. **103**, 1047 (1956).
[2] Breit, Ehrman, and Hull, Phys. Rev. **97**, 1051 (1955).

$$\alpha=-\frac{13}{4}(F_4,D_3)-\frac{5}{4}(F_4,D_2)-\frac{9}{8}(F_4,D_1)+\frac{7}{4}(F_3,D_3)-\frac{7}{24}(F_3,D_1)+\frac{3}{2}(F_2,D_3)+\frac{5}{4}(F_2,D_2)+\frac{17}{12}(F_2,D_1)$$
$$-\frac{9}{8}(F_4,S_1)+\frac{7}{24}(F_3,S_1)+\frac{5}{6}(F_2,S_1)-\frac{9}{4}(D_3,P_2)-\frac{3}{4}(D_3,P_1)-\frac{1}{2}(D_3,P_0)+\frac{5}{4}(D_2,P_2)$$
$$+(D_1,P_2)+\frac{3}{4}(D_1,P_1)+\frac{1}{2}(D_1,P_0)-\frac{5}{4}(P_2,S_1)+\frac{3}{4}(P_1,S_1)+\frac{1}{2}(P_0,S_1), \quad (1.1)$$

$$\beta=-\frac{21}{4}(F_4,F_3)-\frac{39}{8}(F_4,F_2)-\frac{21}{8}(F_3,F_2)-\frac{51}{8}(F_4,P_2)-\frac{9}{4}(F_4,P_1)-\frac{3}{2}(F_4,P_0)+\frac{21}{8}(F_3,P_2)$$
$$+\frac{15}{4}(F_2,P_2)+\frac{9}{4}(F_2,P_1)+\frac{3}{2}(F_2,P_0)-\frac{15}{4}(D_3,D_2)-\frac{13}{4}(D_3,D_1)-\frac{5}{4}(D_2,D_1)-\frac{7}{2}(D_3,S_1)$$
$$+\frac{5}{4}(D_2,S_1)+\frac{9}{4}(D_1,S_1)-\frac{9}{4}(P_2,P_1)-\frac{3}{2}(P_2,P_0), \quad (1.2)$$

$$\gamma=-\frac{25}{14}(F_4,D_3)-\frac{25}{4}(F_4,D_2)-\frac{45}{8}(F_4,D_1)+5(F_3,D_3)-\frac{35}{24}(F_3,D_1)+\frac{51}{7}(F_2,D_3)+\frac{5}{2}(F_2,D_2)+\frac{1}{3}(F_2,D_1)$$
$$-\frac{45}{8}(F_4,S_1)+\frac{35}{24}(F_3,S_1)+\frac{25}{6}(F_2,S_1)-\frac{3}{4}(D_3,P_2)-\frac{15}{4}(D_3,P_1)-\frac{5}{2}(D_3,P_0)+\frac{5}{2}(D_2,P_2)+\frac{9}{2}(D_1,P_2), \quad (1.3)$$

$$\delta=-\frac{133}{18}(F_4,F_3)-\frac{791}{72}(F_4,F_2)-\frac{21}{8}(F_3,F_2)-\frac{11}{8}(F_4,P_2)-\frac{21}{4}(F_4,P_1)-\frac{7}{2}(F_4,P_0)$$
$$+\frac{21}{8}(F_3,P_2)+\frac{15}{2}(F_2,P_2)-\frac{15}{4}(D_3,D_2)-\frac{27}{4}(D_3,D_1), \quad (1.4)$$

$$\epsilon=-\frac{13}{28}(F_4,D_3)-5(F_4,D_2)-9(F_4,D_1)+\frac{15}{4}(F_3,D_3)+\frac{75}{7}(F_2,D_3), \quad (1.5)$$

$$\zeta=-\frac{175}{36}(F_4,F_3)-\frac{125}{9}(F_4,F_2), \quad (1.6)$$

where

$$(L_J,L'_{J'})=\sin\delta_J{}^L\,\sin\delta_{J'}{}^{L'}\,\sin(\delta_J{}^L-\delta_{J'}{}^{L'}).$$

If charge independence for nucleon-nucleon interactions is assumed, then some parts of expressions β, δ, and ζ, namely,

$$\frac{1}{4}\beta'=-\frac{21}{4}(F_4,F_3)-\frac{39}{8}(F_4,F_2)-\frac{21}{8}(F_3,F_2)-\frac{51}{8}(F_4,P_2)-\frac{9}{4}(F_4,P_1)-\frac{3}{2}(F_4,P_0)$$
$$+\frac{21}{8}(F_3,P_2)+\frac{15}{4}(F_2,P_2)+\frac{9}{4}(F_2,P_1)+\frac{3}{2}(F_2,P_0)-\frac{9}{4}(P_2,P_1)-\frac{3}{2}(P_2,P_0), \quad (1.2')$$

$$\frac{1}{4}\delta'=-\frac{133}{18}(F_4,F_3)-\frac{791}{72}(F_4,F_2)-\frac{21}{8}(F_3,F_2)-\frac{11}{8}(F_4,P_2)-\frac{21}{4}(F_4,P_1)-\frac{7}{2}(F_4,P_0)+\frac{21}{8}(F_3,P_2)+\frac{15}{2}(F_2,P_2), \quad (1.4')$$

$$\frac{1}{4}\zeta'=-\frac{175}{36}(F_4,F_3)-\frac{125}{9}(F_4,F_2), \quad (1.6')$$

can be replaced by the numerical values of β', δ', and ζ', which are the coefficients for polarization in proton-proton scattering:

$$k^2(P\sigma)_{p-p}/\sin\theta=\beta' P_1(\cos\theta)+\delta' P_3(\cos\theta)+\zeta' P_5(\cos\theta). \quad (1')$$

Coulomb interference is omitted in Eq. (1′).

III. ANALYSIS OF EXPERIMENTS ON POLARIZATION IN TERMS OF LEGENDRE POLYNOMIALS

The data of Chamberlain, Donaldson, Segrè, Tripp, Wiegand, and Ypsilantis[3] (Fig. 1) are analyzed in terms of Legendre polynomials. The coefficients in the Legendre polynomial expansion were obtained by numerical quadrature from curves through the data, employing the orthogonality of the polynomials. Five curves were drawn, representing various interpretations of the data, and the experimental values of the coefficients are averages of the five values of each coefficient obtained in this way, while the errors measure the spread in the values.

The results of the analysis are given below and are shown graphically in Fig. 1. The subscript e on the coefficients signifies that the experimental value is meant.

$$\begin{aligned}\alpha_e&=0.101\pm0.033, &\beta_e&=0.827\pm0.081,\\ \gamma_e&=0.581\pm0.079, &\delta_e&=0.459\pm0.065,\\ \epsilon_e&=0.433\pm0.059, &\zeta_e&=-0.084\pm0.053.\end{aligned} \quad (2)$$

Moreover, the results of a least squares Legendre polynomial analysis of proton-proton polarization data[1] are added, giving the experimental values of the coefficients in Eq. (1′),

$$\beta_e'=1.015\pm0.057,\quad \delta_e'=0.316\pm0.069,\quad \zeta_e'=0.099\pm0.073. \quad (2')$$

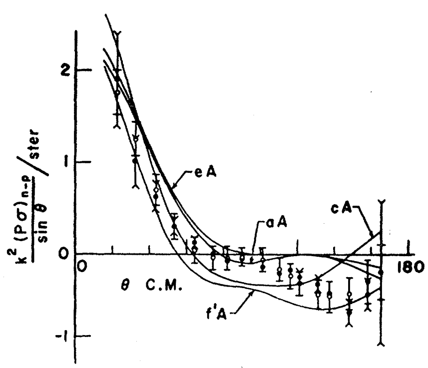

FIG. 1. The experimental data[3] used in this analysis are represented by solid circles, while open circles designate the points calculated from the experimental coefficients of Eq. (2). Experimental uncertainties are represented by vertical lines through the points, the extent of the uncertainty being indicated by v-shaped ends. The possible spread in the value calculated from the experimental coefficients is indicated by straight ends. Four sample theoretical fits are shown for comparison. Theoretical coefficients were taken from Table IV, and the curves carry the same designation which labels the table entries.

[3] Chamberlain, Donaldson, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. 95, 850 (1954).

It will be noted that $\frac{1}{4}\zeta_e'\neq\zeta_e$, contrary to what would happen if only states for $L\leq3$ are necessary and charge independence is obeyed. However, the uncertainties in determining the coefficients from the data are numerous enough so that a concrete conclusion is hard to draw from this disparity. Taking the disagreement literally, one might expect to find phase shifts for $L>3$ entering in a significant manner if charge independence is maintained.

IV. PHASE SHIFT ANALYSIS FOR D AND S WAVES

Trial values of $\delta_1{}^D$ and $\delta_2{}^D$ were chosen, and the coefficients of the Legendre polynomial analysis were used to determine $\delta_3{}^D$ and $\delta_1{}^S$. Since the expression for δ,

$$\delta=\tfrac{1}{4}\delta'+\tfrac{3}{4}[9(D_1,D_3)+5(D_2,D_3)], \quad (3.1)$$

depends only on the ^3D phase shifts under the assumptions made, selection of $\delta_1{}^D$ and $\delta_2{}^D$ determines $\delta_3{}^D$ from the experimental values of δ and δ'. Similarly, the expression for β,

$$\begin{aligned}\beta=\tfrac{1}{4}\beta'+\tfrac{1}{4}[5(D_1,D_2)+15(D_2,D_3)-13(D_3,D_1)]\\ +\tfrac{1}{4}[9(D_1,S_1)+5(D_2,S_1)-14(D_3,S_1)],\end{aligned} \quad (3.2)$$

depends only on the ^3D and ^3S phase shifts. Selection of $\delta_1{}^D$ and $\delta_2{}^D$, and the corresponding $\delta_3{}^D$, therefore determines $\delta_1{}^S$ from the values of β and β'.

It is to be emphasized that the *experimental values* of β' and δ' were used in the following calculation. Thus the analysis is independent of the phase shift analysis of proton-proton polarization at this stage.

From Eq. (4.1)

$$\tfrac{4}{3}(\delta_e-\tfrac{1}{4}\delta_e')=\mathrm{Im}[(9Q_1{}^D+5Q_2{}^D)Q_3{}^{D*}], \quad (4.1)$$

where

$$Q_J{}^L=[\exp(2i\delta_J{}^L)-1]/2i.$$

On an Argand diagram, the points representing $(9Q_1{}^D+5Q_2{}^D)Q_3{}^{D*}$ for given $\delta_1{}^D$, $\delta_2{}^D$ lie on a circle passing through the origin with radius $\frac{1}{2}|9Q_1{}^D+5Q_2{}^D|$, and with a diameter which passes through the origin at an angle $\arg(9Q_1{}^D+5Q_2{}^D)$ measured counterclockwise from the negative imaginary axis. The intersections of the line parallel to the real axis giving the experimental value of $\frac{4}{3}(\delta_e-\frac{1}{4}\delta_e')=0.5$ with the circle give the values of $\delta_3{}^D$ which are desired: they are the angles measured clockwise from the line through the origin at angle $\arg(9Q_1{}^D+5Q_2{}^D)$ with respect to the positive real axis (namely, the tangent to the circle at origin) to lines joining the origin with the points of intersection.

Equivalent analytical computation is as follows: from Eq. (4.1),

$$\rho\sin\delta_3{}^D\sin(\theta-\delta_3{}^D)=S=0.5,$$

TABLE I. ^3S and ^3D phase shifts resulting from the analysis. Table entries are as follows: the location of a number in the table corresponds to a choice of $\delta_1{}^D$ (column) and $\delta_2{}^D$ (row). At each location will be found values of $\delta_3{}^D$ (not in parentheses) and $\delta_1{}^S$ (in parentheses). Values of all phase shifts are given in degrees. Only solutions for $\delta_1{}^S>0$ included.

$\delta_2{}^D$ \ $\delta_1{}^D$	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0	−5	−10	−15	−20	−25	−30	−35	−40	−45	−50	−55	−60	−65	−70
45																													
40																		11(10)	9(4)	8(−1)									
35																		13(16)	10(10)	8(5)	7(0)	6(−3)							
30																		16(23)	12(15)	10(8)	8(4)	6(2)	6(−2)						
25					5(9)	6(14)												19(29)	14(20)	11(14)	8(10)	7(6)	6(3)	5(1)	5(−2)				
20				5(6)	6(10)	7(16)												22(34)	15(25)	11(19)	9(14)	7(10)	6(7)	5(5)	5(2)	4(2)			
15			4(7)	5(9)	6(12)													23(36)	16(28)	12(22)	10(17)	8(13)	6(11)	6(7)	5(6)	4(5)	4(3)		
10		4(7)	4(10)	5(12)	6(16)													23(36)	17(29)	13(24)	10(20)	8(17)	7(13)	6(11)	5(9)	4(8)	4(7)	4(5)	
5	4(9)	4(11)	4(13)	5(16)	6(22)													23(35)	17(30)	13(25)	10(22)	8(19)	7(16)	6(14)	5(12)	5(10)	4(10)	4(8)	4(7)
0		4(14)	4(17)	5(21)														22(33)	16(30)	12(26)	10(22)	8(20)	7(17)	6(15)	5(14)	5(12)	4(12)	4(11)	4(10)
−5			4(22)	5(26)													26(36)	19(31)	15(28)	12(25)	10(22)	8(20)	7(18)	6(17)	5(15)	4(15)	4(14)	4(13)	
−10																	22(32)	17(29)	14(26)	11(24)	9(22)	8(20)	6(19)	6(17)	5(16)	4(16)	4(15)		
−15																25(30)	20(27)	16(25)	13(23)	10(22)	9(20)	7(19)	6(18)	5(18)	5(16)	4(17)			
−20															25(28)	20(26)	17(24)	14(22)	11(22)	9(21)	8(19)	7(18)	6(17)	5(17)	5(16)				
−25														22(26)	20(24)	17(22)	14(21)	12(20)	10(19)	9(17)	7(17)	6(17)	5(16)	5(15)					
−30												16(31)	18(25)	17(24)	16(21)	14(19)	13(17)	11(16)	9(16)	8(15)	7(14)	6(14)	5(14)						
−35													14(24)	14(21)	13(19)	12(16)	11(14)	10(13)	8(13)	7(13)	6(13)								
−40														12(19)	11(17)	11(13)	10(11)	9(10)	8(9)	7(9)									
−45																													

where

$$\rho=[(9\sin\delta_1{}^D\cos\delta_1{}^D+5\sin\delta_2{}^D\cos\delta_2{}^D)^2+(9\sin^2\delta_1{}^D+5\sin^2\delta_2{}^D)^2]^{\frac{1}{2}},$$
$$\theta=\arctan\left(\frac{9\sin^2\delta_1{}^D+5\sin^2\delta_2{}^D}{9\sin\delta_1{}^D\cos\delta_1{}^D+5\sin\delta_2{}^D\cos\delta_2{}^D}\right). \quad (5.1)$$

The sum of the two values of $\delta_3{}^D$ which are solutions of this equation must equal $\arg(9Q_1{}^D+5Q_2{}^D)$. If one uses

$$(\delta_3{}^D)_1+(\delta_3{}^D)_2=0$$

together with Eq. (5.1), the two values of $\delta_3{}^D$ can be checked or one of them can be computed.

In order to obtain $\delta_1{}^S$, the same method can be used for the following equation:

$$4\beta_e-\beta_e'-[5(D_1,D_2)+15(D_2,D_3)-13(D_3,D_1)]=\mathrm{Im}[(9Q_1{}^D+5Q_2{}^D-14Q_3{}^D)Q_1{}^{S*}]. \quad (4.2)$$

Moreover, only sets of triplet phase shifts satisfying the condition

$$\frac{\pi}{k^2}\sum_{L=\text{even}}[(2L+3)\sin^2\delta_{L+1}{}^L+(2L+1)\sin^2\delta_L{}^L+(2L-1)\sin^2\delta_{L-1}{}^L]\leq\sigma_{\text{tot}}{}^{n-p}-\tfrac{1}{4}\sigma_{\text{tot}}{}^{p-p} \quad (6)$$

were accepted. The total neutron-proton scattering cross section at 310 Mev is 35 mb,[4] and the total proton-proton nuclear scattering cross section at 310 Mev is about $4\pi\times3.75$ mb.[5] Therefore, the condition of Eq. (6) implies

$$3\sin^2\delta_1{}^S+3\sin^2\delta_1{}^D+5\sin^2\delta_2{}^D+7\sin^2\delta_3{}^D\leq2.79. \quad (6.1)$$

Results of this phase shift analysis are given in Tables I and II. Table I contains values of $\delta_1{}^D$, $\delta_2{}^D$, $\delta_3{}^D$, and $\delta_1{}^S$ in degrees which fit the experimentally determined coefficients β_e, δ_e in the expansion of $P\sigma$, as limited by the total cross section, for $\delta_1{}^S>0$. Table II is the same for $\delta_1{}^S<0$. Entries in the table are as follows: the position of squares in the table corresponds to a choice of $\delta_1{}^D$ (column) and $\delta_2{}^D$ (row). In the square will be found values of $\delta_3{}^D$ (no parentheses) and $\delta_1{}^S$ (in parentheses). The same values of $\delta_1{}^D$ and $\delta_2{}^D$ were investigated for $\delta_1{}^S<0$ as for $\delta_1{}^S>0$ but, as Table II indicates, a more limited range was found for allowed values.

The effects of the experimental uncertainty in the values of the coefficients was investigated by solving Eq. (3.1) and (3.2) using not only the central values of δ and β given in Eq. (2), but also the extreme values $\delta+\Delta\delta$, $\beta+\Delta\beta$, $\delta-\Delta\delta$, and $\beta-\Delta\beta$. It was found by this

[4] Kelly, Leith, Segrè, and Wiegand, Phys. Rev. **79**, 96 (1950); Fox, Leith, Wouters, and MacKenzie, Phys. Rev. **80**, 23 (1950); J. DeJuren and B. J. Mayer, Phys. Rev. **81**, 919 (1951).

[5] Chamberlain, Pettengill, Segrè, and Wiegand, Phys. Rev. **83**, 923 (1951); **93**, 1424 (1954); **95**, 1348 (1954).

TABLE II. ${}^3S$ and ${}^3D$ phase shifts resulting from the analysis. Table entries are labeled as in Table I. Only solutions for $\delta_1{}^S<0$ are included. The same range of choices for $\delta_1{}^D$ and $\delta_2{}^D$ was investigated as for Table I.

$\delta_2{}^D \backslash \delta_1{}^D$	20	15	10	5	0	−5	−10	−15	−20	−25	−30	−35	−40
30													
25								19(−29)	14(−35)				
20								22(−26)	15(−34)	11(−40)			
15								23(−25)	16(−31)	12(−37)	10(−40)		
10								23(−24)	17(−30)	13(−35)	10(−40)	8(−45)	
5							35(−16)	23(−23)	17(−29)	13(−34)	10(−40)	8(−45)	
0							30(−17)	22(−22)	16(−28)	12(−35)	10(−38)	8(−44)	
−5						37(−12)	26(−18)	19(−24)	15(−29)	12(−34)	10(−38)	8(−43)	
−10						30(−13)	22(−20)	17(−25)	14(−29)	11(−34)	9(−39)	8(−42)	
−15					32(−9)	25(−14)	20(−19)	16(−24)	13(−28)	10(−34)	9(−37)	7(−42)	
−20				30(−9)	25(−13)	20(−18)	17(−21)	14(−25)	11(−31)	9(−35)	8(−38)		
−25		24(−21)	24(−16)	22(−16)	20(−17)	17(−20)	14(−24)	12(−27)	10(−31)	9(−33)			
−30			18(−27)	17(−25)	16(−23)	14(−24)	13(−24)	11(−27)	9(−31)				
−35					13(−29)	12(−27)	11(−26)	10(−28)					
−40													

means that the boundaries of the region of allowed values of $\delta_1{}^D$ and $\delta_2{}^D$ showed in Tables I and II was changed by at most $\pm5°$ in any case. The solutions $\delta_3{}^D$ and $\delta_1{}^S$ also changed relatively little: of the order of $\pm2°$ in the first case and $\pm4°$ in the second.

The regions of fit and ranges of phase shifts shown in Tables I and II, therefore, are relatively stable against small changes in the coefficients, and provide a reasonably reliable indication of the limits within which a charge-independent fit is to be found.

V. REPRODUCTION OF EXPERIMENTAL DATA

In order to compare the theoretical fits with the experimental data, some sets of S and D phase shifts were selected, and P and F phase shifts available from analyses of proton-proton data[6] were used. The selected sets of phase shifts are shown in Table III, and designations for convenient reference are indicated.

Results are given in Table IV and Fig. 1. Table IV

TABLE III. Selected sets of S and D, and P and F phase shifts.

Designation of set	$\delta_1{}^S$	$\delta_1{}^D$	$\delta_2{}^D$	$\delta_3{}^D$
a	−22	−15	0	22
b	33	−15	0	22
c	17	−40	0	7
d	12	−55	0	5
e	21	0	−30	16
f	21	55	0	5
f'	16	45	20	7
f''	12	50	15	6
f'''	12	55	10	5
f^{iv}	14	65	0	4
(f)[a]	24.5	45	15	7

Designation of set	$\delta_0{}^P$	$\delta_1{}^P$	$\delta_2{}^P$	$\delta_2{}^F$	$\delta_3{}^F$	$\delta_4{}^F$
A	−19.8	−19.8	20	10	− 5	4.8
A'	−23	− 8.9	21.5	10	−10	3.7
B	−38.5	− 3.4	20	10	−10	2.8
C	−26.5	−19.7	19	10	−10	2.8
E	−33.6	− 4.1	21	10	− 5	4.8
S[b]	−43.0	− 2.00	15.6	−1.53	− 4.05	1.50

[a] This is an exceptional case which is obtained using the central value of δ and a little smaller value of β than the central one.
[b] From Saperstein.[6] Other P, F sets from Hull, Ehrman, Hatcher, and Durand,[1] with designations as in their Table I except for A', which is one of their unpublished fits.

[6] Hull, Ehrman, Hatcher, and Durand, reference 1; A. M. Saperstein, Dissertation, Yale University, 1956 (unpublished).

TABLE IV. Theoretical values of coefficients of Legendre polynomials obtained from the sets of phase shifts shown in Table III.

Designation of set ${}^3S,{}^3D$	${}^3P,{}^3F$	α	β	γ	δ	ϵ	ζ
a	A	0.276	0.818	0.727	0.477	−0.020	0.0243
a	A'	0.291	0.800	0.667	0.454	0.036	0.0300
a	B	0.279	0.815	0.786	0.473	0.022	0.0239
a	C	0.293	0.821	0.848	0.476	0.022	0.0239
a	E	0.270	0.812	0.686	0.474	−0.020	0.0243
a	S	0.254	0.874	0.901	0.476	0.1052	0.00138
b	A	0.411	0.821	0.787	0.477	−0.020	0.0243
b	A'	0.363	0.803	0.768	0.454	0.036	0.0300
b	B	0.447	0.818	0.881	0.473	0.022	0.0239
b	C	0.484	0.824	0.943	0.476	0.022	0.0239
b	E	0.390	0.815	0.746	0.474	−0.020	0.0243
b	S	0.477	0.877	0.963	0.476	0.1052	0.00138
c	A	0.132	0.822	1.158	0.469	0.361	0.0243
c	A'	0.129	0.804	1.142	0.446	0.293	0.0300
c	B	0.197	0.819	1.085	0.465	0.227	0.0239
c	C	0.130	0.825	1.037	0.468	0.227	0.0239
c	E	0.190	0.816	1.023	0.466	0.361	0.0243
c	S	0.310	0.878	0.883	0.468	0.1113	0.00138
d	A	0.010	0.820	1.55	0.499	0.553	0.0243
d	A'	0.037	0.802	1.48	0.476	0.435	0.0300
d	B	0.086	0.817	1.37	0.495	0.333	0.0239
d	C	−0.029	0.823	1.32	0.498	0.333	0.0239
d	E	0.104	0.814	1.61	0.496	0.553	0.0243
d	S	0.256	0.876	1.112	0.498	0.168	0.00138
e	A	0.334	0.834	0.716	0.454	0.100	0.0243
e	A'	0.319	0.816	0.688	0.431	0.117	0.0300
e	B	0.350	0.831	0.748	0.450	0.0928	0.0239
e	C	0.360	0.837	0.763	0.453	0.0928	0.0239
e	E	0.331	0.828	0.712	0.451	0.100	0.0243
e	S	0.376	0.890	0.816	0.453	0.0840	0.00138
f	A	−0.134	0.835	1.016	0.451	0.493	0.0243
f	A'	−0.096	0.817	0.882	0.428	0.400	0.0300
f	B	−0.112	0.832	0.842	0.447	0.313	0.0239
f	C	−0.196	0.838	0.823	0.450	0.313	0.0239
f	E	−0.071	0.829	1.030	0.448	0.493	0.0243
f	S	0.045	0.891	0.832	0.450	0.163	0.00138
f'	A	−0.129	0.834	0.763	0.475	0.400	0.0239
f''	A	−0.186	0.832	0.869	0.474	0.448	0.0239
f'''	A	−0.215	0.835	0.995	0.455	0.498	0.0239
	A'	−0.170	0.817	0.834	0.430	0.404	0.0296
f^{iv}	A	−0.214	0.832	1.299	0.454	0.608	0.0239
(f)	A	−0.060	0.826	0.737	0.457	0.382	0.0239

TABLE V. Choices of ^3G phase shifts.

Designation of set	$\delta_3{}^G$	$\delta_4{}^G$	$\delta_5{}^G$
I	3	1	0
II	3	0	1
III	0	1	3
IV	1	0	3
I′	5	2	0
II′	5	0	2
III′	0	2	5

gives values of 44 theoretical sets of coefficients of Legendre polynomials obtained from these sets of phase shifts. Four of them are also drawn in Fig. 1. A comparison of the results in Table IV with the experimental values of the coefficients given in Eq. (2) shows that none of the sets of calculated coefficients is entirely satisfactory. For example, set cA gives coefficients reasonably close to the experimental values except for γ and ζ. The discrepancy in ζ has already been discussed, and is general for all fits. The error in γ is small enough to allow the order of magnitude of the experimental n-p polarization to be reproduced, but doubling the $P_2(\cos\theta)$ contribution compared to the experimentally determined amount prevents the fit to angular distribution from being anything but qualitative. Set $j'A$ gives a value of γ only a little larger than allowed by the errors on the experimental value, but α is of the wrong sign. Set $(f)A$ is a fit of similar characteristics, giving coefficients even closer to the experimental ones. From the standpoint of using the present results as a basis for further work, this is the most promising type of fit, since the angular distribution is fairly well reproduced and the whole curve needs only to be shifted up 0.2 mb/sterad or so. Such an adjustment is more easily accomplished than other types.

Since no special attempt has yet been made to arrive at final fits by further adjustment of phase shifts, the present results are considered to be hopeful indications that charge-independent fits to n-p and p-p data can be obtained.

VI. EFFECTS OF G WAVES

Effects of higher angular momentum states were investigated by introducing small ^3G phase shifts. If it is assumed that states for $L\leq 4$ must be taken into account and G phase shifts are comparatively small, the general formula for polarization by Breit, Ehrman and Hull [Eq. (18) of reference 2] can be expressed as a sum of two terms such that the first term is just Eq. (1), and the second includes the effects of G waves to first order. To estimate the second term, for simplicity, numerical values were used. For S, P, D, F phase shifts two sets, cA and $(f)A$, were employed. Choices of G phase shifts are shown in Table V. Sets I, II, III, IV were used with cA and sets I′, II′, III′ with $(f)A$, respectively. Results are plotted in Fig. 2, where additions to the polarization due to the G waves are shown on the same scale as the theoretical curves cA and $(f)A$.

The addition of the ^3G waves has little effect on the angular distribution above $\theta=60°$, even for 5° phase shifts. At smaller angles, the effects become appreciable,

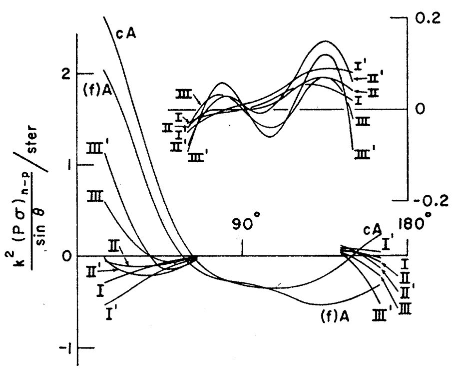

FIG. 2. The curves labeled cA and $(f)A$ were obtained from coefficients with the same labels in Table IV. The curves labeled I, II, III are additions to curve cA resulting from the inclusion of sets of ^3G waves from Table V used with the S, P, D, F phase shifts of set cA. Curves labeled I′, II′, III′ are to be added in the same way to curve $(f)A$. The additions between $\theta=60°$ and 140° are similar for all cases and near zero, so they are plotted with an expanded vertical scale in the inset.

especially for the larger phase shifts, but the data are less certain here also. The calculations indicate that the addition of ^3G phase shifts of the order of 5° or less, when used with the present fits, is not excluded by the data, but also that such an addition does not improve present fits, so that immediate improvement is probably not to be sought in the adding of higher L states.

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