Limitations of the Microwave Cavity Method of Measuring Electron Densities in a Plasma

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The limitations of the conventional microwave cavity method of measuring the electron density are derived. The conventional method permits the electron density to be measured over a range of approximately two decades. The upper limit of the measurement of the electron density, roughly 5×10^9 cm^{-3}, is caused by plasma resonance due to the macroscopic polarization of the plasma and by the overlapping from higher order modes. The lower limit of the measurement of the electron density, roughly 5×10^7 cm^{-3}, is determined by how accurately the resonant frequency can be measured. The macroscopic electric polarization can be eliminated and the overlapping modes suppressed by designing the cavity so that the probing microwave field and the plasma have rotational symmetry around the same axis. The electric polarization limit is then replaced by a magnetic polarization limit and the available range is increased approximately one additional decade at 3000 Mc/sec. By decreasing the frequency from 3000 Mc/sec to 1 Mc/sec and by measuring the Q or the losses of the plasma in a properly designed solenoid instead of a cavity the magnetic polarization limit can be raised even more. At 1 Mc/sec and at a pressure of 1 mm Hg the electron density corresponding to the magnetic polarization limit is 10^{15} to 10^{16} cm^{-3}. The lower limit for the measurable electron density or the conductivity is determined by the sensitivity of the detecting arrangement, and the noise originating in the electron-ion plasma and is probably 10^4 to 10^5 times less than the maximum measurable electron density.

INTRODUCTION

THE microwave method of measuring various aspects of the behavior of the electron-ion plasma has become a rather important tool during the last ten years. In spite of its popularity no integrated effort has been made to find its limitations. It now seems rather necessary to determine the present limitations and possible extensions, especially since some of the conclusions derived from measurements with the method are disputed.

The presence of an electron-ion plasma inside a microwave cavity causes its resonance frequency to shift. In a first order approximation the frequency shift is directly proportional to the average electron density. The microwave cavity method has primarily been used to measure the electron density during the afterglow period of the microwave gas discharge. The frequency shift as function of time has in those cases been interpreted as directly proportional to the average electron density without regard to the limitations that necessarily follow with an approximation. An interpretation of this kind cannot be accepted until it has been shown, either experimentally or theoretically, that the approximation is applicable within the range of the measured electron density.

It is the purpose of this paper to show that the frequency shift is not always directly proportional to the average electron density within the range of electron density as measured by the conventional microwave cavity method. The limits of the simple theory, which applies when there is a linear relationship between the average electron density and the frequency shift, will be derived and it will be shown that some of the limits can be removed by a proper design of the cavity and choice of mode. The resonant frequency of a microwave cavity containing an electron-ion plasma depends on the macroscopic polarization of the plasma, the losses of the plasma, and the presence of excited higher order modes. To find the limitations of the simple theory, in which these phenomena can be neglected, it is necessary to derive a theory for resonance where these phenomena are included to a first order approximation.

GENERAL THEORY

The power loss in the walls of the microwave cavity can in general be neglected when compared with the power loss in the plasma. The plasma is then for all practical purposes contained in a cavity with perfectly reflecting walls. The necessary resonance criterion for the cavity containing the plasma can be obtained from the complex form of Poynting's theorem. The equation describing the energy balance of the cavity as a whole is obtained by integrating Poynting's equation over the volume of the cavity. The energy balance equation for the cavity therefore becomes

$$\langle \mathbf{E}\times\mathbf{H}^*\rangle_S = -j\omega\{\mu_0\langle\mathbf{H}\cdot\mathbf{H}^*\rangle_V - \epsilon_0\langle\mathbf{E}\cdot\mathbf{E}^*\rangle_V\} - \langle\mathbf{E}\cdot\mathbf{J}^*\rangle_V, \quad (1)$$

where $\mathbf{E}$ is the electric field ($\mathbf{E}^*$ is the conjugate complex field), $\mathbf{H}$ is the magnetic field, and $\mathbf{J}$ is the current density in the plasma. The inside surface and the volume of the cavity are denoted S and V respectively. The average signs, used in the expression above, are defined as

$$\langle\mathbf{E}\times\mathbf{H}^*\rangle_S \equiv \int_S (\mathbf{E}\times\mathbf{H}^*)\cdot d\mathbf{S}$$

and

$$\langle\mathbf{H}\cdot\mathbf{H}^*\rangle_V \equiv \int_V \mathbf{H}\cdot\mathbf{H}^* dV. \quad (2)$$

The resonance of the cavity is observed through a coaxial line or wave guide which is connected to the cavity through a small loop, probe or iris. The area of this opening is S' and the inside surface of the cavity is totally reflecting except for this small area S'. The surface integral $\langle \mathbf{E}\times\mathbf{H}^*\rangle_S$ is therefore equal to the integral $\langle \mathbf{E}\times\mathbf{H}^*\rangle_{S'}$.

The expression (1) is rather inconvenient for a demonstration of the influence of the electron-ion plasma on the resonance of the cavity. For this purpose it is more suitable to introduce the applied fields $\mathbf{E}_a$ and $\mathbf{H}_a$, the scalar potential ϕ_p, and the vector potential $\mathbf{A}_p$ defined as follows:

$$\nabla\times\mathbf{E}_a=-j\omega\mu_0\mathbf{H}_a, \qquad \nabla\cdot\mathbf{E}_a=0, \qquad \nabla\times\mathbf{H}_a=j\omega\epsilon_0 E_a, \qquad \nabla\cdot\mathbf{H}_a=0, \tag{3}$$

$$\nabla^2\phi_p+(\omega/c)^2\phi_p=-\rho/\epsilon_0, \qquad \nabla\cdot\mathbf{j}+j\omega\rho=0, \qquad \nabla^2\mathbf{A}_p+(\omega/c)^2\mathbf{A}_p=-\mathbf{j}/\epsilon_0, \qquad \nabla\cdot\mathbf{A}_p+j\omega\phi_p=0. \tag{4}$$

The total fields $\mathbf{E}$ and $\mathbf{H}$ are then

$$\mathbf{E}=\mathbf{E}_a-\nabla\phi_p-(j\omega/c^2)\mathbf{A}_p, \qquad \mathbf{H}=\mathbf{H}_a+\epsilon_0\nabla\times\mathbf{A}_p. \tag{5}$$

The ions in the plasma are for all practical purposes stationary when compared with the electrons, and since the magnetic field $\mathbf{H}$ can be neglected in the momentum balance equation for the electrons, the relation between the current density $\mathbf{J}$ and the electric field $\mathbf{E}$ becomes

$$\mathbf{j}=e^2n/[m(\nu_m+j\omega)], \tag{6}$$

where n is the electron density, ν_m the momentum transfer collision frequency of the electrons, and ω the radian frequency of the applied field. The area S' or the hole in the cavity can be made arbitrarily small as only the resonance of the cavity is observed. Provided the Q of the cavity containing the plasma is sufficiently high, the presence of the hole can be entirely neglected and the boundary conditions are then

$$\mathbf{E}_a\times d\mathbf{S}=0, \quad \mathbf{H}_a\cdot d\mathbf{S}=0, \quad \mathbf{A}_p=0, \quad \phi_p=0. \tag{7}$$

One more condition must be imposed on the fields. The scalar potential ϕ_p and the vector potential $\mathbf{A}_p$ must both be identically equal to zero when the space charge ρ and the current density $\mathbf{J}$ are zero.

With the restrictions and definitions mentioned above it is now possible to transform the energy balance Eq. (1) of the cavity into the following more descriptive form

$$\langle\mathbf{E}\times\mathbf{H}^*\rangle_{S'}=-j\omega\left\{\mu_0\langle\mathbf{H}_a\cdot\mathbf{H}_a^*\rangle_V-\epsilon_0\langle\mathbf{E}_a\cdot\mathbf{E}_a^*\rangle_V+2T_e+\frac{1}{c^2}\langle\mathbf{j}^*\cdot\mathbf{A}_p\rangle_V-\langle\rho^*\phi_p\rangle_V\right\}-2\nu_m T_e, \tag{8}$$

where T_e, the total peak kinetic energy of the ordered motion of the electrons, is

$$T_e=\frac{e^2}{2m(\omega^2+\nu_m^2)}\langle n\mathbf{E}\cdot\mathbf{E}^*\rangle_V. \tag{9}$$

It can be shown that all individual terms in Eq. (8) are real. The first two terms inside the curly brackets on the right-hand side constitutes the Lagrangian for the applied field while the three remaining terms inside the same brackets is the Lagrangian for the plasma. The cavity containing the plasma therefore acts like two coupled oscillators, one represented by the applied fields $\mathbf{E}_a$ and $\mathbf{H}_a$, the other represented by the electron ion plasma. Resonance is obtained when the integral $\langle\mathbf{E}\times\mathbf{H}^*\rangle_{S'}$ is real. The general criterion for resonance can now be written as follows

$$2(W_H-W_E)+2T_e+\frac{1}{c^2}\langle\mathbf{j}^*\cdot\mathbf{A}_p\rangle_V-\langle\rho^*\phi_p\rangle_V=0, \tag{10}$$

where W_H is the peak stored energy in the applied magnetic field and W_E is the peak stored energy in the applied electric field. The electromagnetic oscillator represented by E_a and H_a in reality consist of an infinite number of oscillators represented by the various electromagnetic modes that can be excited by either the coupling mechanism (loop or iris) or by the nonuniform electron ion plasma. Before the resonance criterion can be evaluated in terms of the frequency of the applied field, it is necessary to find the relative distribution of the stored energies of the various modes. This can easily be done if the orthonormal set of eigenvectors (E_i and H_i) devised by Slater[1] are used. If W_{Ei} and W_{Hi} are respectively the peak magnetic energy and the peak electric energy of the ith mode and if the cross coupling between modes as caused by the nonuniform plasma is neglected, it can be shown that

$$W_{Hi}=\frac{\mu_0(\omega_i/c)^2\langle\mathbf{E}_i\times\mathbf{H}^*\rangle_{S'}}{(\omega_i^2-\omega^2)^2+(\omega\omega_i/Q_i)^2}=\left(\frac{\omega_i}{\omega}\right)^2W_{Ei}, \tag{11}$$

where ω_i is the radian resonant frequency and Q_i is the Q of the ith mode. Ordinarily the lowest mode ($i=1$) is used for measuring the average electron density with the microwave method. The source term $\langle\mathbf{E}_i\times\mathbf{H}^*\rangle_{S'}$ is constant, that is, independent of the mode number as long as the wavelength is very large compared with the dimensions of the coupling mechanism. The expression W_H-W_E can then easily be expressed in terms of the peak stored energy W_{E1} and Q_1 of the first mode, the resonant frequencies f_i, and the frequency f of the probing signal. In a first-order approximation where f is very close to f_1, one finds that

$$\frac{W_H-W_E}{W_{E1}}\cong\left(\frac{f_1}{f}\right)^2-1+\frac{K}{Q_1^2}, \tag{12}$$

where

$$\frac{1}{Q_1}=\frac{\nu_m}{\omega}\frac{T_e}{W_{E1}}, \quad K=\sum\left(1+\frac{f_1}{f_i}\right)^{-2}\left(\frac{f_i}{f_1}-1\right)^{-1}. \tag{13}$$

[1] J. C. Slater, *Microwave Electronics* (D. Van Nostrand Company, Inc., New York, 1950), pp. 59–63.

The first-order approximation of the resonance criterion can now be written as follows:

$$\left(\frac{f_1}{f}\right)^2-1+\frac{T_e}{W_{E1}}\left[1+K\left(\frac{\nu_m}{\omega}\right)^2\frac{T_e}{W_{E1}}+\frac{1}{c^2}\frac{\langle \mathbf{j}^*\cdot\mathbf{A}_p\rangle_V}{2T_e}-\frac{\langle\rho^*\varphi_p\rangle}{2T_e}\right]=0. \quad (14)$$

With the definitions and the restrictions mentioned above, it also can be shown that

$$j\omega\left(\frac{1}{c^2}\langle\mathbf{j}^*\cdot\mathbf{A}_p\rangle_V-\langle\rho^*\varphi_p\rangle_V\right)+\langle\mathbf{j}^*\cdot\mathbf{E}\rangle_V=\langle\mathbf{j}^*\cdot\mathbf{E}_a\rangle_V. \quad (15)$$

Introducing this result into Eq. (10), one obtains the same resonance criterion as can be obtained from Slater's formula for the impedance of a microwave cavity.[2] However the expression (14) above illustrates in a much more convenient way the influence of the electron-ion plasma on the frequency shift of the cavity.

LIMITS OF THE SIMPLE THEORY

The resonance criterion (14) is in general very difficult to evaluate in terms of the average electron density. In order to find the relative importance of the various terms in the resonance criterion, it is necessary to make some simplifying assumptions. We assume first that the cavity has some symmetry around the center. Its shape may be cubical, cylindrical, or spherical. The mathematics becomes particularly simple if we assume that the plasma is uniform in the density and is confined within a spheroidal surface determined by the following equation:

$$a_xx^2+a_yy^2+a_zz^2=R^2, \quad (16)$$

where a_x, a_y, and a_z are positive real numbers independent of the coordinates and where R is a sort of average radius of the spheroid. This spheroid of plasma is placed in the center of the cavity with one of the principal axes coinciding with one of the principal axes of the cavity. The average radius R of the spheroid is made sufficiently smaller than the wavelength λ of the applied field so that in a first order approximation the terms $(\omega/c)^2\varphi_p$ and $(\omega/c)^2\mathbf{A}_p$ of Eqs. (4) can be neglected.

One can now distinguish between two fundamentally different cases. The first case is obtained when the electric field $\mathbf{E}_1$, of the mode used for measuring the electron density, is essentially uniform within the plasma and parallel with one of the principal axes of the spheroid. The second case is obtained when the magnetic field $\mathbf{H}_1$ is essentially uniform within the plasma and parallel to one of the principal axes of the spheroid. The first order approximation of the resonance criterion in the first case can be written as follows:

$$\left(\frac{f_1}{f}\right)^2-1+\frac{\alpha\eta}{1+\gamma^2}\times\left[1+K\frac{\alpha\gamma^2}{1+\gamma^2}\eta+\left(\frac{R}{\lambda}\right)^2\pi\delta-\beta_i\eta\right]=0, \quad (17)$$

where

$$\delta=\int_{V_p}\int_{V_p}\frac{\mathbf{j}(\mathbf{r}_0)\cdot\mathbf{j}(\mathbf{r})}{|\mathbf{r}-\mathbf{r}_0|}d^3rd^3r_0\Big/\int_{V_p}|\mathbf{j}|^2d^3r,$$
$$\gamma=\frac{\nu_m}{\omega},\quad \eta=\frac{n}{n_p},\quad n_p=\frac{\omega^2m\epsilon_0}{e^2},\quad \alpha=\frac{V_p|E_1|^2}{\langle|E_1|^2\rangle_V}. \quad (18)$$

The density n_p is the electron density for which plasma resonance would occur at the radian frequency ω of the applied field in the plane parallel case. The factor α is essentially the ratio between the volume of the plasma V_p and the volume of the cavity modified by the nonuniform electric field. The factor β_1 is the polarization factor for the principal axis i, which is parallel with the field E_1 ($\beta_i=\frac{1}{3}$ for the sphere). The factor δ depends on the shape of the plasma and the current distribution. In the first case, when E_a is essentially uniform within the plasma and the plasma is confined within a spherical surface, δ can very easily be calculated and is $8\pi/5$.

The frequency shift resulting from the presence of the plasma is proportional to the average electron density, represented by the normalized electron density η, only if the plasma terms within the bracket of the expression (17) can be neglected. The first plasma term inside the bracket is caused by the presence of excited higher modes. The closer the excited modes are in terms of the frequency the more important is this term. The factor K has been derived assuming that the higher modes are excited only by the coupling mechanism (iris or loop). In practice, however, higher modes may be excited by the nonuniform or asymmetric plasma. The K evaluated from (13) must therefore be considered as a minimum value. It is difficult to find the exact theoretical value for K and it should therefore be considered as an experimental parameter characteristic for each experimental setup. To the author's knowledge this correction term, which is caused by the overlapping from higher modes, has not been considered in any previous publication. Judging from the designs of the commonly used cavities it seems that a reasonable range for K in previously reported works should be from 1 to 100 with a probable value higher than 10. With an allowed error of five %, the maximum measurable electron density corresponding to the overlapping mode limit (O.M.L.) is determined by the following expression:

$$\alpha\eta\leqq\frac{1}{20K}\left(\frac{1+\gamma^2}{\gamma^2}\right). \quad (19)$$

[2] J. C. Slater, *Microwave Electronics* (D. Van Nostrand Company, Inc., New York, 1950), Vol. 78.

The next term that causes a deviation from the linear relation between the average electron density and the frequency shift is due to the macroscopic magnetic polarization of the plasma while the third term is caused by the electric polarization of the plasma. These two terms are roughly equal when the electron distribution is described by the fundamental diffusion mode. The term caused by the magnetic polarization can be neglected when the effective radius of the plasma is small compared with the wavelength λ. The maximum measurable electron density corresponding to these two terms is obtained by neglecting the magnetic polarization term and counting only the electric polarization term. This limit, which properly is called the electric polarization limit (E.P.L.) is therefore determined by the following expression:

$$\eta \leqq 1/20\beta_i. \tag{20}$$

The macroscopic electric polarization is causing the ordinary plasma resonance. When $R \ll \lambda$ and thus the magnetic polarization term can be neglected, resonance for the uniform plasma, confined within a spheroidal surface, is obtained when $2T_e = \langle \rho^* \varphi_p \rangle_V$.

The plasma resonance mechanism or the macroscopic electric polarization can be avoided under certain circumstances. When the divergence of the plasma current is zero the oscillating space charge is also zero and the term $\langle \rho^* \varphi_p \rangle_V$ is then eliminated. This corresponds to the case where $\mathbf{H}_1$ is essentially uniform inside the plasma. In order to avoid the oscillating space charge completely, it is necessary that $\mathbf{H}_1$ and the plasma have rotational symmetry around the same axis. If $\mathbf{H}_1$ and the plasma have rotational symmetry around the y axis ($a_x = a_y$), then the first-order approximation of the resonance criterion can be written in the following way:

$$\left(\frac{f_1}{f}\right)^2 - 1 + \frac{\alpha\eta}{1+\gamma^2}\left[1 + K\frac{\alpha\gamma^2}{1+\gamma^2}\eta + \left(\frac{R}{\lambda}\right)^2 \pi\delta\eta\right] = 0. \tag{21}$$

This formula has the same form as in the previous case. The term due to the magnetic polarization has changed somewhat but is of the same order of magnitude as in the previous case. The electric polarization term has disappeared and one must here consider the influence of the magnetic polarization. The corresponding limit which should be called the magnetic polarization limit (M.P.L.) is given by the following expression:

$$\eta \leqq \frac{1}{20}\left(\frac{\lambda}{R}\right)^2 \frac{1}{\pi\delta}. \tag{22}$$

In this case $\mathbf{H}_a$ is essentially constant within the plasma and the current density can then in the first approximation be written as $\mathbf{\Omega} \times \mathbf{r}$, where $\mathbf{\Omega}$ is a vector independent of the coordinates. As in previous cases δ can easily be evaluated when the uniform plasma is confined within a spherical surface and is then $8\pi/21$. It is important to notice that the magnetic polarization limit is independent of γ or the pressure.

Two more limitations of the microwave cavity method should be mentioned. As the frequency of a resonance phenomenon is measured, it is necessary that the Q is sufficiently high so that the resonance can be recognized. The corresponding limit, properly called the "low-Q limit" (L.Q.L.) is determined more or less arbitrarily by setting the lowest admissible value of Q equal to 10. The low-Q limit then becomes

$$\eta \leqq \frac{1}{10}\left(\frac{1+\gamma^2}{\alpha\gamma}\right). \tag{23}$$

Secondly, there is a lower limit for η, determined by how accurately a frequency shift can be measured. With the methods used until now the error in the frequency measurement is approximately 1 part in 10^5 parts. Allowing for a relative error of five %, this lower limit for η is determined by

$$\eta \geqq \frac{4(1+\gamma^2)}{\alpha} \times 10^{-4}. \tag{24}$$

DISCUSSION AND SUGGESTIONS

The five limits derived in the foregoing are functions of the normalized electron density η, the normalized momentum transfer frequency γ, the polarization factor β, and the volume ratio α. The limits are illustrated in Fig. 1 where they are plotted as function of η and γ with $\alpha = 1$, $\beta = \frac{1}{3}$, $\delta = 8\pi/21$, and $\lambda/R \cong 6$. The shaded area gives the allowed range of η and γ values for the conventional microwave cavity method. The most commonly used frequency is 3000 Mc/sec. The electron density n_p corresponding to this frequency is 1.12×10^{11} cm^{-3}. The momentum transfer frequency ν_m is proportional to the pressure p and the proportionality constant is of the order of 10^8 to 10^9 sec^{-1} (mm Hg)$^{-1}$ for ordinary gases. For γ less than 10^{-1}, the upper limit of η is caused by the electric polarization limit (E.P.L.) and is approximately 5×10^{-2} corresponding to an electron density of 6×10^9 cm^{-3}. The lower limit of η is determined by the relative error in the frequency measurement (F.M.L.) and corresponds to an electron density of 5×10^7 cm^{-3}. For γ values around unity, the upper

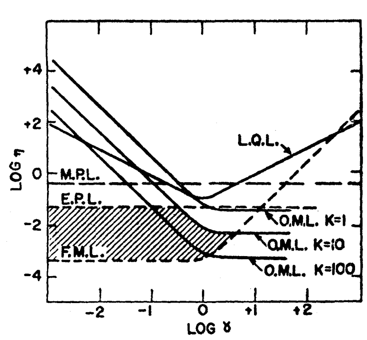

FIG. 1. The limit of the microwave method $\alpha = 1$, $\beta = \frac{1}{3}$.

limit of η is the overlapping mode limit (O.M.L.) provided K is larger than unity. It can be seen from the resonance criterion (17) that the frequency shift in this range includes a term proportional to the square of the electron density. This term could possibly explain some of the unexplained electron-ion recombination coefficients that have been observed with the microwave cavity method.[3–6] Unfortunately the constant K, that must be measured on the cavity containing the electron-ion plasma, is not available in the papers that have been published so far.

The conventional microwave method measures the electron density at the most over a range of somewhat less than three decades. If the electric polarization is removed through a proper choice of mode, the electric polarization limit (E.P.L.) is replaced by the magnetic polarization limit (M.P.L.) The available range is then extended approximately one more decade for $\gamma<1$. The available range is moved towards higher electron densities if α is decreased. This is, however, not too desirable as some of the more interesting properties of the plasma can be found only by using a large volume of the plasma.

Some of the difficulties with the limits can, however, be resolved by abandoning the measurements of the resonant frequency and replacing it with a measurement of the Q or the losses of the plasma. This in general means that we no longer measure the electron density directly but rather the conductivity of the plasma. Knowledge of the collision frequency ν_m then gives us the electron density. The sacrifice is not too large, however, as direct measurements of the electron density can be done only for low pressures, when $\gamma<1$. The electric polarization of the plasma is avoided if the applied field and the plasma have rotational symmetry around the same axis. This can be obtained in a cylindrical cavity or in a solenoid. It is simplest to consider a long solenoid and a long cylindrical plasma. The end effects can then be neglected and the following first-order approximation of the change in the impedance Z of the solenoid is obtained owing to the presence of the plasma:

$$\frac{\Delta Z}{L}=-\frac{\alpha}{2}\left(\frac{1+j\gamma}{1+\gamma^2}\right)\frac{n}{n_0}, \tag{25}$$

where

$$n_0=\left(\frac{2c}{R}\right)^2\left(\frac{m\epsilon_0}{e^2}\right),$$

where c is the velocity of light, R is the radius of the plasma, L is the inductance of the solenoid, and α is the ratio between the cross-sectional area of the plasma and the cross-sectional area of the solenoid. The foregoing approximation is good only as long as the following inequalities are satisfied:

$$n<n_0\gamma \quad \text{for} \quad \gamma>1, \tag{26}$$

$$n<n_0 \quad \text{for} \quad \gamma<1, \tag{27}$$

and

$$\lambda/\pi R<1. \tag{28}$$

The last inequality insures that the undisturbed magnetic field within the volume corresponding to the plasma is uniform. The first inequality applies when the plasma conductivity is ohmic while the second inequality applies when the plasma currents are primarily inductive. It is obvious from the above formulas that the maximum measurable electron density is determined by the inequalities (26) and (27). If the plasma has a radius of one centimeter, the corresponding electron density n_0 is 10^{12} cm^{-3}. The change in the impedance ΔZ is inductive and directly proportional to the electron density provided $\gamma<1$. The corresponding maximum measurable electron density is equal to n_0. When $\gamma>1$ the change in the impedance is resistive and the corresponding maximum measurable electron density is $n_0\gamma$. Since $\gamma=\nu_m/\omega$, it is obvious that the maximum measurable electron density will increase if the pressure is increased and if the frequency of the applied field is decreased. The minimum measurable electron density depends on how small a change, ΔZ, that can be measured. If a bridge arrangement is used, it is not unreasonable to assume that the impedance can be measured to 1 part in 10^5 parts. The corresponding minimum measurable electron density is then determined by the expression

$$n\geqq 10^{-5}n_0\times 2(1+\gamma^2)^{\frac{1}{2}}/\alpha. \tag{29}$$

The discussion above shows that the so-called microwave cavity method has rather limited use. To be able to investigate the high-density plasmas it is necessary to use other methods. One way of avoiding the limitations of the conventional microwave method is to measure the losses instead of the frequency shift. In order to avoid the effect of macroscopic electric polarization it is necessary to use solenoidal applied electric fields and plasmas with rotational symmetry. With the bridge method, suggested above, including at least one solenoid, the available range in the electron density is, at 1 Mc/sec and a pressure of 1 mm Hg, approximately 10^{15} cm^{-3} to 10^{10} cm^{-3}.

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