1.8×10^{-9}-sec component in liquid helium is close to that for aluminum absorbers and is distinctly less than that to be expected if this component were due to some form of triplet positronium. The result is consistent with that to be expected for free positrons in liquid helium.

A short-lived component ($\tau\sim2\times10^{-10}$ sec) is also observed in these experiments which could be due in part to singlet positronium. However, an intensity correction for positrons which annihilate in the source materials and cryostat walls cannot be made accurately enough to prove conclusively that singlet positronium is formed in liquid helium.

These experiments have been described[7] at the New York meeting of the American Physical Society, January 30 to February 2, 1957.

ACKNOWLEDGMENTS

We would like to thank Dr. D. G. Henshaw for his assistance and for the loan of some his equipment. We are also indebted to Dr. L. Yaffe of McGill University who prepared one of the Na^{22} sources.

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[7] D. A. L. Paul and R. L. Graham, Bull. Am. Phys. Soc. Ser. II, 2, 38 (1957) and R. L. Graham and D. A. L. Paul, Bull. Am. Phys. Soc. Ser. II, 2, 38 (1957).

PHYSICAL REVIEW VOLUME 106, NUMBER 1 APRIL 1, 1957

Annihilation of Positrons in Liquid Helium*

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(Received December 26, 1956)

The annihilation scheme of positrons in liquid helium at 4.2°K was studied. Three components in the decay were observed with mean lives: $\tau_1<5\times10^{-10}$ sec, $\tau_2=(2.6\pm0.2)\times10^{-9}$ sec, and $\tau_3=(1.2\pm0.2)\times10^{-7}$ sec. The abundance of the τ_3 component is $(16\pm2)\%$ of all positrons which annihilate in the liquid. Three-quantum annihilation occurs in about 15% of all positron annihilations. This leads to the conclusion that the three components are due to parapositronium, free positron annihilation, and orthopositronium.

INTRODUCTION

WHEN positrons annihilate in amorphous materials the decay curve is not always a simple exponential, but commonly shows two exponential components. The longer mean life is associated with the presence of orthopositronium in the material, the shorter mean life with an unresolved mixture of parapositronium annihilation and the annihilation of positrons which do not form positronium.[1]

Recently two mean lifetimes have been reported in the annihilation of positrons in liquid helium.[2] It has been suggested that these two components are similar to the components in other insulators; i.e., the component of longer mean life is associated with the presence of orthopositronium, the one of shorter life with the annihilation of parapositronium and of positrons not forming positronium.[3]

This assignment is not satisfactory in view of the relative abundances of the two components. More than $\frac{2}{3}$ of all positrons are in the longer-lived component, while on the basis of energy considerations of the positron-helium interaction in which positronium is formed, only between 20% and 35% of all positrons should form positronium.[4]

The difficulty is very easily resolved: there are present in the annihilation of positrons in liquid helium not two mean-lifetime components but three. The longest mean life is associated with the formation of orthopositronium, the intermediate one with the annihilation of free positrons, and the short one with the annihilation of parapositronium.

Experiments on which this assignment is based and the mean lives and abundances of the three components are reported here.

EXPERIMENTAL RESULTS

Positrons from the decay of Na^{22} were used in these experiments. $Na^{22}Cl$ was deposited from solution on an aluminum foil $\sim$4 mg/cm² and covered with a foil $\sim$1 mg/cm². The entire source was submerged in liquid helium at 4.2°K for measurements described here. All results are corrected for the effect of those positrons which annihilate in the aluminum of the source.

The mean lifetimes of the three components were measured by the delayed-coincidence method, using a circuit similar to that of Bell and Graham.[5]

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* Part of a thesis submitted by Jerry Wackerle in partial fulfillment of the requirements for the degree of Doctor of Philosophy in physics at the University of Kansas.

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[1] Telegdi, Sens, Yovanovitch, and Warshaw, Phys. Rev. **104**, 867 (1955).

[2] Graham, Paul, and Henshaw, Bull. Am. Phys. Soc. Ser. II, 1, 68 (1956).

[3] R. A. Ferrell, Revs. Modern Phys. **28**, 308 (1956).

[4] A. Ore, Univ. i Bergen Arbok, Naturvitenskap. Rekke, 1949, No. 9 (1949).

[5] R. E. Bell, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955).

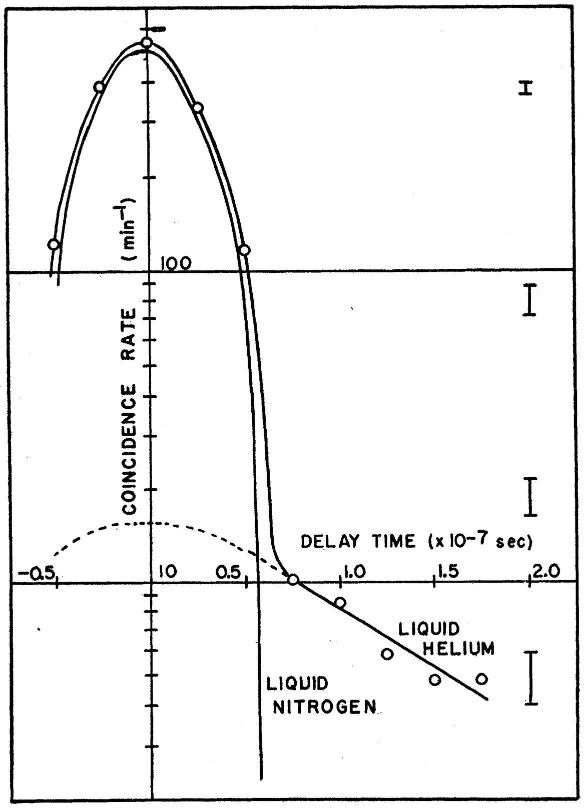

FIG. 1. Delay curve for liquid helium at 4.2°K obtained with slow coincidence circuit.

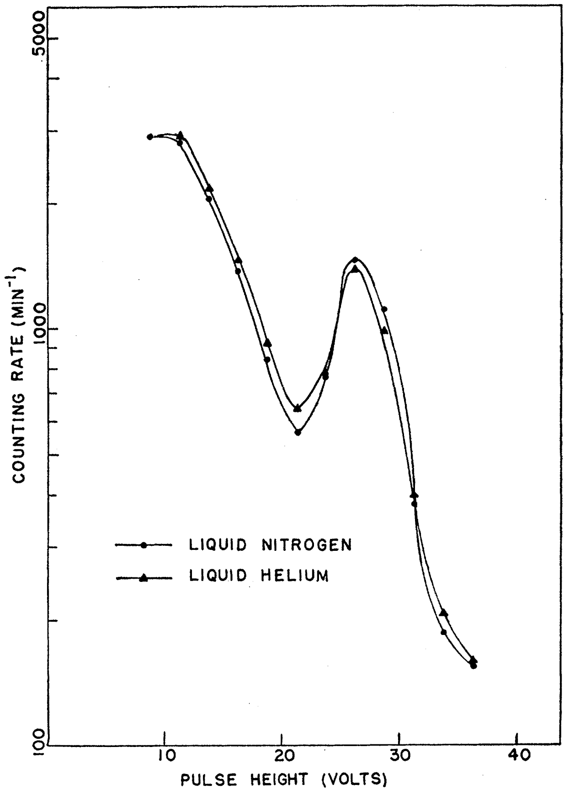

FIG. 2. Comparison of annihilation gamma-ray spectra for positrons in liquid helium at 4.2°K and liquid nitrogen at 77.3°K.

The width of the prompt curve, about 5×10^{-9} sec, was too long to permit accurate measurement of the component with short mean life, but was adequate for the one with intermediate mean life. To measure the mean life of the long-lifetime component, the width of the prompt curve was increased to about 9×10^{-8} sec.

The mean lives observed are:

$$\tau_1 < 5\times10^{-10}\text{ sec},$$
$$\tau_2 = (2.6\pm0.25)\times10^{-9}\text{ sec},$$
$$\tau_3 = (1.2\pm0.2)\times10^{-7}\text{ sec}.$$

The values of τ_1 and τ_2 are in good agreement with previously reported measurements.[2]‡ A typical delayed-coincidence curve showing the long-lifetime component is shown in Fig. 1.

The abundance of the τ_1 component is less than the number of positrons annihilating in the aluminum backing of the source, and so cannot be accurately measured. The abundance of the τ_3 component is $(15\pm2)\%$ of all positron decays in liquid helium. In calculating the abundance, the counter efficiency correction was based on the assumption that the τ_3 component is due to orthopositronium. If this assumption is wrong, the abundance would be increased somewhat.

If, as assumed, the τ_3 component is due to orthopositronium annihilation, then one would expect that almost 15% of all positrons in liquid helium would undergo three-quantum annihilation. On the other hand, if the τ_2 component is due to orthopositronium, and the τ_3 component is not, then fewer than 2% of the positrons would undergo three-quantum annihilation. Thus, the component associated with orthopositronium can be identified by determining the amount of three-quantum annihilation.

Two methods were used to observe the three-quantum annihilation: the spectrum of the gamma rays in a NaI scintillation spectrometer and the triple-coincidence rate. Figure 2 shows a typical comparison between the annihilation gamma-ray spectra from positrons in liquid helium and in liquid nitrogen, in which no three-quantum annihilation occurs.[6] The lowering of the peak due to the 511-kev gamma ray and the filling of the valley at lower energy are due to a decrease in the number of two-quantum annihilations, and a corresponding increase in the number of low-energy gamma rays due to the three-quantum annihilation. The effect is partially masked by the fact that in

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[‡] *Note added in proof.*—As pointed out by Graham and Paul [Bull. Am. Phys. Soc. Ser. II, 2, 38 (1957)] this value should be corrected for the presence of the long-lived component. The corrected value is $\tau_2 = (2.3\pm0.25)\times10^{-9}$ sec.

[6] S. de Benedetti and R. Siegel, Phys. Rev. 94, 995 (1954).

liquid nitrogen, Compton scattering causes degradation in the energy of some of the 511-kev gamma rays. This is probably the reason that no three-quantum radiation was observed in an earlier experiment.[7]

From the change in the ratio of peak to valley counting rates, and the resolution curve of the spectrometer, the abundance of three-quantum events may be calculated. The abundance of three-quantum annihilation in liquid helium is $(16\pm3)\%$. This amount of three-quantum annihilation radiation clearly indicates that the τ_3 component is due to orthopositronium.

Triple-coincidence measurements were made to verify the existence of orthopositronium. Three NaI scintillation counters were placed coplanar with the source every 120° around the source. The triple-coincidence rate was compared with the rate in liquid nitrogen. The coincidence rate in helium was greater than that in nitrogen by (0.7 ± 0.2) count per minute. From this coincidence rate, the abundance of three-quantum annihilation is $(13\pm4)\%$. This is in satisfactory agreement with the abundance calculated from the gamma-ray spectrum analysis.

DISCUSSION

The three mean lives, τ_1, τ_2, and τ_3 are associated with parapositronium, free positron annihilation, and orthopositronium annihilation, respectively. The orthopositronium component has an abundance of $16\pm2\%$, based on the three abundances listed above. Any error in the mean life τ_3 would affect this value. If τ_3 is smaller than 1.2×10^{-7} sec, the abundance would be increased. If the long-lived component is due to orthopositronium, the very short-lived one must be due to parapositronium, and will have an abundance of about 5%. The remaining 79% of the positrons do not form positronium.

The mean life of the short-lived component, $\tau_1<5\times10^{-10}$ sec, indicates that positrons rapidly lose energy in liquid helium, so that in a time less than 5×10^{-10} sec the positron has either formed positronium or has lost so much energy that positronium formation is no longer possible. The intermediate mean life, $\tau_2=2.6\times10^{-9}$ sec, is longer than that generally found for free positrons in condensed materials. The component of long mean life, $\tau_3=1.2\times10^{-7}$ sec, indicates that very little pickoff annihilation occurs. From this it may be concluded that the exchange repulsion between the positronium atom and the helium atom is much stronger than any polarization effects which would tend to raise the electron density at the positron.

The behavior of positrons in liquid helium is just about what would be expected in helium gas at the same density, i.e., about 700 atmospheres at 0°C. Computation of the annihilation probability for free positrons using Ore's correction for Coulomb effects[4] for helium gas at 700 atmospheres gives a mean life which is in exact agreement with the value measured. The fraction of the positrons forming positronium is consistent with the predictions of Ore[4] and with the triple-coincidence studies of helium gas by de Benedetti and Siegel.[6]

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[7] F. L. Hereford, Phys. Rev. 95, 1097 (1954).

PHYSICAL REVIEW VOLUME 106, NUMBER 1 APRIL 1, 1957

Ground-State Energy of a Hard-Sphere Gas

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(Received December 17, 1956)

Rigorous lower and upper bounds are established for the energy of the ground state of a Bose gas with hard-sphere interaction between particles.

1. STATEMENT OF RESULTS

We consider a Bose gas consisting of N identical nonrelativistic particles in a cubical box of volume V. Between each pair of particles there is a hard-sphere repulsion of range a, and no other interaction. Let m be the mass of each particle, and $\rho=N/V$, $\rho_1=(N-1)/V$. We suppose $N\geq2$.

Let E be the ground-state energy of the gas. Calculations by Lee and Yang[1] have shown that

$$E\sim[2\pi\hbar^2N\rho a/m], \tag{1}$$

as $N\rightarrow\infty$ and $a\rightarrow0$. The meaning of Eq. (1) is that as $a\rightarrow0$ the error is of higher than first order in a. An asymptotic formula of this kind has the disadvantage that there seems to be no way to convert it into a precise inequality. The error introduced by breaking off the asymptotic expansion in powers of a at any term is not controllable; it is unlikely that the power series converges in the strict sense for any value of a.

The purpose of this paper is to supplement Eq. (1) with precise inequalities.

Theorem 1.— $$E>[\tfrac{1}{10}\sqrt{2}\pi\hbar^2N\rho a/m]. \tag{2}$$

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[1] T. D. Lee and C. N. Yang (private communication).