

expected to be found among the products of binary fission. At this time, the only sixth-group possibility appears to be an isotope of bromine ( $A \sim 93$ ), or an isotope of selenium or arsenic with mass  $A \sim 90$ .

\* This Letter summarizes briefly the predictions given in a review paper on delayed neutrons, presented at the Washington meeting of the American Physical Society, April 1957. A more detailed account of this work is in preparation.

<sup>1</sup> G. R. Keepin, *Bull. Am. Phys. Soc. Ser. II*, **2**, 194 (1957); based on work of Keepin, Wimett, and Zeigler, *Phys. Rev.* (to be published). The six nuclides are  $U^{235}$ ,  $U^{233}$ ,  $U^{238}$ ,  $Pu^{239}$ ,  $Pu^{240}$ , and  $Th^{232}$ .

<sup>2</sup> L. E. Glendenin and E. P. Steinberg, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Paper No. 614 (8B).

<sup>3</sup> Calculations of neutron-to- $\beta$  branching ratios (also called "delayed-neutron emission probability,"  $P_n$ ) have previously been carried out and the results compared with experimental branching ratios for  $U^{235}$  fission (see references 4 and 5).

<sup>4</sup> A. C. Pappas, Massachusetts Institute of Technology Laboratory for Nuclear Science Report No. 63, 1953 (unpublished).

<sup>5</sup> G. R. Keepin, in *Progress in Nuclear Energy Series I, Physics and Mathematics* (Pergamon Press, London, 1956), Vol. 1, p. 191.

<sup>6</sup> N. Sugarman, *J. Chem. Phys.* **15**, 544 (1947).

<sup>7</sup> N. Sugarman, *J. Chem. Phys.* **17**, 11 (1949).

<sup>8</sup> P. J. Bendt and F. R. Scott, *Phys. Rev.* **97**, 744 (1955).

## Longitudinal Polarization of Bremsstrahlung and Pair Production at Relativistic Energies\*

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SINCE the discovery of longitudinal polarization in  $\beta$  decay,<sup>1</sup> it has become important to calculate the cross sections for bremsstrahlung and pair production with specified longitudinal polarization for the incident and outgoing particles. We have found a simple argument which determines these cross sections when the energies of all the particles concerned are highly relativistic.

In the case of bremsstrahlung, we consider an electron of energy  $E_0$  radiating a photon of energy  $k$  and going out with energy  $E = E_0 - k$ . In the case of pair production, we consider a photon of energy  $k$  producing a positron and an electron with energies  $E_+$ ,  $E_-$ . Let the  $z$  axis be along the direction of the photon in both cases. In the bremsstrahlung process, we take the component of momentum transverse to the  $z$  axis to be given by the vector  $(u, v, 0)$  for the incident electron and by  $(s, t, 0)$  for the outgoing electron. In the pair production, we take the transverse momenta to be  $(-u, -v, 0)$  for the positron and  $(s, t, 0)$  for the electron. In the relativistic range the differential cross sections are

$$d\sigma_B = C(dk/kE_0^2)dudvdsdt(q^2)^{-2}|M|^2 \quad (1)$$

for bremsstrahlung, and

$$d\sigma_p = C(dE_+/k^3)dudvdsdt(q^2)^{-2}|M|^2 \quad (2)$$

for pair production. Here  $C$  is a constant,  $q$  is the momentum transferred to the nucleus, and  $M$  is the element of a particular Dirac matrix between the electron or positron spin-states. It is simple to calculate  $M$  by using the explicit form of the spinors for longitudinally polarized particles.<sup>2</sup> In this way the complicated apparatus of projection operators and spin-sums is completely avoided.

For calculating cross sections between states of purely longitudinal polarization, only the absolute values of  $M$  are required. Since we are also interested in transverse polarizations, we have taken some trouble to calculate the phase of each  $M$  according to a consistent set of definitions. We define any particle to be "forward" or "backward" according as its spin and its velocity constitute a right-handed or left-handed screw. (Thus a forward photon is left circularly polarized according to the old optical convention.) The Dirac spinors for forward and backward electrons and positrons have their phases fixed by the convention that the numerically largest component is in each case real and positive.

The transverse momenta  $(u, v, s, t)$  are of the order of magnitude of  $m$  when the energies are highly relativistic. Keeping terms only of the leading order in  $(m/E_0)$  and  $(m/k)$ , we find for the bremsstrahlung matrix elements

$$M_{FFF} = E_0 A^*, \quad M_{BBB} = E_0 A, \quad (3)$$

$$M_{FFB} = mkB, \quad M_{BBF} = -mkB, \quad (4)$$

$$M_{FBF} = EA, \quad M_{BFB} = EA^*, \quad (5)$$

$$M_{FBB} = 0, \quad M_{BFF} = 0. \quad (6)$$

Here the three suffixes ( $F$ =forward and  $B$ =backward) refer to the spin-states of the incident electron, the photon, and the outgoing electron, reading from left to right. The pair production matrix elements are

$$M_{FFF} = -mkB, \quad M_{BBB} = mkB, \quad (7)$$

$$M_{FFB} = E_+ A, \quad M_{BBF} = E_+ A^*, \quad (8)$$

$$M_{FBF} = -E_- A, \quad M_{BFB} = -E_- A^*, \quad (9)$$

$$M_{FBB} = 0, \quad M_{BFF} = 0, \quad (10)$$

with the three suffixes referring respectively to the photon, the positron, and the electron. The quantities  $A$  and  $B$  are given in both cases by

$$A = (u + iv)(m^2 + u^2 + v^2)^{-1} - (s + it)(m^2 + s^2 + t^2)^{-1}, \quad (11)$$

$$B = (m^2 + u^2 + v^2)^{-1} - (m^2 + s^2 + t^2)^{-1}. \quad (12)$$

To a sufficient accuracy in the relativistic range, we have

$$q^2 = Q^2 + (u - s)^2 + (v - t)^2, \quad (13)$$

where  $Q$  is a constant determined by the size of the atom

in case screening is important. If there is no screening, then  $Q = (m^2 k / 2E_0 E)$  for bremsstrahlung and  $Q = (m^2 k / 2E_+ E_-)$  for pair production.

Now consider the problem of integrating  $d\sigma_B$  given by Eq. (1) over the angular variables  $u$ ,  $v$ ,  $s$ , and  $t$ . For purely dimensional reasons the result of the integration can be a function only of  $(Q/m)$  multiplied by  $E_0^2$ ,  $k^2$ , or  $E^2$  according as  $M$  is given by Eqs. (3)–(5). But from the Bethe-Heitler formula,<sup>3</sup> the cross section summed over final spin-states is

$$(2E_0^2 + 2E^2 + k^2)(dk/3kE_0^2 R), \quad (14)$$

where  $R$  is a function of  $(Q/m)$  which is given the name of "radiation length." From this it necessarily follows that the angular integrals of Eq. (1) must have the values  $(\frac{2}{3}R^{-1}, \frac{1}{3}R^{-1}, \frac{2}{3}R^{-1})$  for the three separate final spin combinations. We have also checked these values by a direct integration. We therefore conclude that the integrated cross sections for bremsstrahlung with assigned polarizations are given by

$$[d\sigma_{FFF}, d\sigma_{FFB}, d\sigma_{FBF}, d\sigma_{FBB}] \\ = [2E_0^2, k^2, 2E^2, 0](dk/3kE_0^2 R), \quad (15)$$

where the suffixes refer respectively to the incident electron (energy  $E_0$ ), the photon (energy  $k$ ), and the outgoing electron (energy  $E$ ).

A precisely similar argument applied to the pair-production process gives the integrated cross sections

$$[d\sigma_{FFF}, d\sigma_{FFB}, d\sigma_{FBF}, d\sigma_{FBB}] \\ = [k^2, 2E_+^2, 2E_-^2, 0](dE_+/3k^3 R), \quad (16)$$

where the suffixes refer to the polarization of photon, positron, and electron, respectively.

These cross sections are of interest for two reasons. First, they show more clearly than the unpolarized cross sections the symmetry between bremsstrahlung and pair production, and they explain the origin of the unsymmetrical factors  $(2E_0^2 + 2E^2 + k^2)$  and  $(k^2 + 2E_+^2 + 2E_-^2)$  which appear in the unpolarized cross sections. Second, they clearly indicate the possibility of a large-scale persistence of longitudinal polarization in an electromagnetic cascade originated by a single polarized electron of high energy.<sup>4</sup> The latter effect will be the subject of a separate communication.

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<sup>1</sup> T. D. Lee and C. N. Yang, *Phys. Rev.* **104**, 254 (1956); Frauenfelder, Bobone, von Goeler, Levine, Lewis, Peacock, Rossi, and De Pasquali, *Phys. Rev.* **106**, 386 (1957); Goldhaber, Grodzins, and Sunyar, *Phys. Rev.* **106**, 826 (1957).

<sup>2</sup> Kirk W. McVoy, *Phys. Rev.* **106**, 828 (1957).

<sup>3</sup> W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1954), third edition, p. 248, Eq. (21). In the case of complete screening the cross section is given by Heitler's Eq. (26), and the extra (2/9) in this formula makes a slight change in the coefficients (2, 2, 1) in our Eq. (14). We have neglected the (2/9) term.

<sup>4</sup> This possibility was suggested by M. Goldhaber (private communication).

## Further Experiments on $\beta$ Decay of Polarized Nuclei\*

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IN a previous communication<sup>1</sup> we reported that we observed a large asymmetry in the angular distribution of electrons from polarized  $\text{Co}^{60}$  nuclei. It was concluded that unequivocal proof was thereby established of the nonconservation of parity as well as of noninvariance under charge conjugation in beta decay. It was also pointed out that according to Lee, Oehme, and Yang,<sup>2</sup> invariance under time reversal could also be investigated by studying the momentum dependence of the asymmetry parameter  $\beta$ . Since then we have made further measurements and checks. In particular we have carried out similar experiments<sup>3</sup> with  $\text{Co}^{58}$  and observed an asymmetry in the positron emission with a coefficient opposite in sign and roughly one third of that from  $\text{Co}^{60}$ . Through more detailed measurements on  $\text{Co}^{60}$  we have obtained the general behavior of the momentum dependence of  $\beta$ . The linear dependence of  $\beta$  on  $v/c$  in the range from 0.4 to 0.75 is good.

In order to put upper limits on possible spurious effects in our experimental method, we have performed a similar experiment with  $\text{Bi}^{210}$  incorporated in the crystal. Since the bismuth ion in cerium magnesium nitrate is diamagnetic, there can be no significant nuclear polarization set up and therefore no beta asymmetry should be expected. In fact no effect was observed to an accuracy of better than  $\frac{1}{2}\%$ .

Although no changes were made in the apparatus, a simpler and more effective method was found for preparing samples; a high-specific-activity cobalt nitrate solution was spread on the surface of a crystal so that a small part of it was dissolved. When this was allowed to dry, the solution again crystallized with apparently the same crystallographic orientation as the parent crystal and also formed a very thin source.

The experiment with  $\text{Co}^{58}$  is very similar to that of  $\text{Co}^{60}$ .  $\text{Co}^{58}$  decays by positron emission or electron capture to the first excited state of  $\text{Fe}^{58}$  and then to the ground state with the emission of a  $\gamma$  ray of energy 0.805 Mev. The anisotropy of the 0.805-Mev  $\gamma$  ray was used to determine the degree of nuclear polarization. The sign of the coefficient  $\alpha$  is positive; i.e., more positrons are emitted in the same direction as the spin of the  $\text{Co}^{58}$  nuclei. The reversal of the sign of the coefficient  $\alpha$  in the case of the positron emission as compared with electron emission can best be understood from the two-component theory of the neutrino and pure G-T interaction. The negative coefficient found in our  $\text{Co}^{60}$  experiment can be interpreted by supposing that electron emission is associated with a left-handed