expressions in neutrino operators nor a two-component neutrino theory of gravitation using quadrilinear expression in neutrino operators is possible.

VI. CONCLUSION

The situation can be summarized as follows:

(1) Using a four-component theory of half-integral spin fields, we can form three essentially independent fields of zero spin using bilinear expressions.

(2) The restriction to a two-component theory reduces these to one spin-zero field. This field is a parity mixture with equal amounts of scalar and pseudoscalar.

(3) Expressions of mth order in the operators of a two-component theory of spin s cannot produce operators for spin ms (m even).

This situation may be compared with that starting with an integral spin-s field. Bilinear operators of either a two- or four-component theory cannot be constructed which will describe either spin 0 or spin $2s$.

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Parity Mixing Effects in τ-θ Decay*

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The recent results on nonconservation of parity in decay processes involving neutrinos do not provide an unambiguous solution of the τ-θ puzzle. In fact the 2π and 3π decay modes of θ and τ involve no neutrinos, whereas the Lee-Yang two-component neutrino theory attributes the nonconservation of parity to special properties of the neutrino. However, even if the τ and θ are different particles with opposite parities, the neutrino decay modes (μ,ν), (e,π,ν), and (μ,π,ν), which presumably violate parity conservation, will cause mixing of the two particles τ and θ—in the sense that the states with definite lifetimes will be certain linear combinations, K_1 and K_2, of τ and θ. Both K_1 and K_2 will then decay into both 2π and 3π as well as the neutrino modes, but with different lifetimes. An explanation of the apparent equality of lifetimes of τ and θ may be that under present experimental conditions only the long-lived component K_2 is observed. If this is the case, interference effects between the K_1 and K_2 components should be found in experiments performed at shorter times. Conversely, absence of such effects would constitute strong evidence that τ and θ are identical.

Phenomenological expressions are derived by the Wigner-Weisskopf method for the decay rates, including interference effects; and various experimental possibilities are discussed.

I. INTRODUCTION

THE recent spectacular developments concerning parity nonconservation in β decay and π- and μ-meson decay[1–5] have not as yet led to a clear understanding of the familiar problem that motivated them, namely, the τ-θ puzzle. It is of course now possible to suppose that the puzzle has vanished: that τ and θ are one and the same particle, which may decay, with violation of parity conservation, into both the 2π and 3π modes observed in K-meson disintegrations. But in the processes where parity nonconservation has been established experimentally, neutrinos are always involved among the decay products; and it appears that the parity conservation law is violated here in a very special way "attributable" to special properties of the neutrino.[3] No equally natural and compelling picture for parity nonconservation in decay processes not involving neutrinos has as yet been put forward. It is thus still conceivable that the τ and θ mesons are different particles. If this is so, one is still faced with the familiar problem of understanding, among other things, the apparent equality in their lifetimes.[6]

It was in fact this problem that represented one of the major difficulties with the parity-doublet scheme introduced by Lee and Yang.[7] We may now, however, re-examine this scheme in the light of the apparent fact that parity conservation is always violated in decay processes involving neutrinos.[4,5] Suppose, as did Lee and Yang, that τ and θ are members of a parity doublet, and that between them they account for all the decay modes observed among K mesons; for the charged K mesons: $K_{2\pi}$, $K_{3\pi}$, $K_{\mu\nu}$, $K_{e\pi\nu}$, $K_{\mu\pi\nu}$. In the original scheme parity conservation was of course assumed. Let us now abandon this conservation law, *but only for those processes involving neutrinos*. Even in the original scheme, it was always conceivable that τ and θ could both contribute to neutrino decay modes, but they could do so only to states of opposite parity. Now, however, it appears that both τ and θ could contribute to neutrino processes involving the same states. If this were the situation, it would give rise to a "parity"

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1 T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
2 Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).
3 T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
4 Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
5 Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).
6 See, e.g., *Report of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York), Sec. V, p. 2.
7 T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956).

mixing of τ and θ very similar to the "charge conjugation" mixing of θ^0 and $\bar{\theta}^0$ introduced long ago by Gell-Mann and Pais.[8]

In fact this idea of parity mixing was already discussed, before the experiments on parity conservation had been carried out, by Arnowitt and Teutsch.[9] The remarks which will be made here are similar in spirit to those contained in the above paper. But because of our new knowledge concerning parity nonconservation, it will be possible to adopt a somewhat more definite model, and our detailed discussion will lead to some qualitatively new phenomena (namely, interference effects).

The situation may be summarized in the following way. Suppose that τ and θ are different particles, as in the Lee-Yang parity-doublet scheme. Then if we make the assumption that both τ and θ contribute to neutrino decay modes, the τ and θ will be mixed in their decay processes and neither will have a definite lifetime, even if parity conservation is violated only for decay processes involving neutrinos. The two states of definite lifetime will be certain linear combinations of the τ and θ states. The decay curves for the various K-meson decay modes would then have a more or less complicated time dependence, as would, correspondingly, the branching ratios into the various modes. We are considering here charged K mesons. For the neutral K mesons one would be dealing with an even more complicated situation in which there are four particles of definite lifetime, described by states which are linear combinations of θ^0, $\bar{\theta}^0$, τ^0, $\bar{\tau}^0$.

All of these phenomena can be subjected to experimental test. One may take the view that if τ and θ are indeed different particles, then—on perhaps any model—the phenomena discussed here would very likely occur, assuming of course that the two particles have the same spin but opposite parity. If it should turn out that the charged K mesons in fact show a pure-exponential decay curve, this would strongly imply that τ and θ are identical. On the other hand, it is still possible that the apparent equality in τ and θ lifetimes is illusory; i.e., that under current experimental conditions one is observing only the long-lived component of a more complicated phenomenon.

II. DECAY FUNCTIONS

We suppose that in the absence of weak interactions τ and θ are particles of the same mass and opposite parity. For simplicity suppose that both have spin zero. Then the τ meson can decay into three pions, the θ into two pions—but not vice versa (in accordance with our assumption of parity conservation for processes not involving neutrinos). Both τ and θ may couple, however, to the same states involving neutrinos; e.g., τ and θ may couple to both parity states of $\mu+\nu$.

[8] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).
[9] R. Arnowitt and W. B. Teutsch, Phys. Rev. **105**, 285 (1957).

Let the symbols τ and θ, in appropriate context, stand for the state vectors of the τ and θ mesons. We want to find the linear combinations of τ and θ which describe states which decay exponentially in time. The time-dependent amplitude for such a state may be written

$$\psi(t)=(C_\theta\theta+C_\tau\tau)e^{-\frac{1}{2}\gamma t},$$

where the real part of γ is the reciprocal lifetime, and the imaginary part describes the mass shift associated with the decay. The decay constant γ and the coefficients C_θ, C_τ can be obtained by the methods of Wigner and Weisskopf; in fact, as discussed by Lee, Oehme, and Yang,[2] γ and the coefficients C_θ, C_τ are, respectively, the eigenvalue and eigenfunction of a matrix $\Gamma+iM$:

$$(\Gamma+iM)\begin{pmatrix}C_\theta\\C_\tau\end{pmatrix}=\gamma\begin{pmatrix}C_\theta\\C_\tau\end{pmatrix}.$$

The 2×2 Hermitian matrix Γ has diagonal elements which are equal to the respective transition rates for θ and τ into all their final states. The off-diagonal elements describe the coupling of θ and τ via parity nonconserving processes. The elements of the Hermitian matrix M describe the corresponding mass shifts associated with these processes. It may be noted that, unlike the case discussed by Lee, Oehme, and Yang,[2] the two diagonal elements of Γ are here not equal, nor are the two diagonal elements of M.

Without a detailed dynamical model we of course cannot make numerical predictions. We shall, however, give the form of the solutions of the above eigenvalue problem and discuss the physical consequences in a qualitative way. Even here, however, matters are simplified considerably if we adopt the assumption of invariance under time reversal, a symmetry principle which has not as yet been ruled out experimentally. This principle leads to the result that the off-diagonal elements of Γ are real and therefore equal; and likewise for the matrix M. This restriction does not remove any of the qualitative physical effects which we want to consider.

Let us denote by γ_1 and γ_2 the two eigenvalues of our matrix equation and by K_1 and K_2 the corresponding eigenstates. Because of the assumption of time-reversal invariance, we have that $(C_\theta/C_\tau)_1=-(C_\tau/C_\theta)_2$. Setting $(C_\theta)_1=\alpha$ and $(C_\tau)_1=\beta e^{-i\phi}$, we can write for the eigenstates

$$K_1=\alpha\theta+\beta e^{-i\phi}\tau,\quad K_2=\beta e^{-i\phi}\theta-\alpha\tau, \tag{1}$$

where, with no loss of generality, we may choose α and β real, the relative phase of the coefficients being contained in the factor $e^{-i\phi}$. Normalization of the state vectors gives

$$\alpha^2+\beta^2=1. \tag{2}$$

The state vectors θ and τ can be expressed in terms of

K_1 and K_2 as follows:

$$\theta=d^{-1}(\alpha K_1+\beta e^{-i\phi}K_2),$$
$$\tau=d^{-1}(\beta e^{-i\phi}K_1-\alpha K_2), \quad (3)$$
$$d=\alpha^2+\beta^2 e^{-2i\phi}.$$

Let us now consider the situation where at time $t=0$, a θ meson is produced. The wave function of the system at some later time, t, is then given by

$$\psi(t)=d^{-1}\{\alpha K_1 e^{-\frac{1}{2}\gamma_1 t}+\beta e^{-i\phi}K_2 e^{-\frac{1}{2}\gamma_2 t}\}. \quad (4)$$

With no loss of generality in all physical applications, we can take one of the decay constants, say γ_1, to be real. So we set

$$\gamma_1=\lambda_1, \quad \gamma_2=\lambda_2+2i\Delta,$$

where λ_1 and λ_2 are real and Δ represents the mass separation of the particles K_1 and K_2. Decomposing $\psi(t)$ into the states θ and τ, we have from Eq. (1)

$$\psi(t)=d^{-1}\{[\alpha^2 e^{-\frac{1}{2}\lambda_1 t}+\beta^2 e^{-\frac{1}{2}\lambda_2 t-i(\Delta t+2\phi)}]\theta + \alpha\beta e^{-i\phi}[e^{-\frac{1}{2}\lambda_1 t}-e^{-\frac{1}{2}\lambda_2 t-i\Delta t}]\tau\}. \quad (5)$$

To find the rate of decay into the 2π mode (which receives contributions only from the θ particle) we multiply the absolute square of the coefficient of θ in Eq. (5) by the transition rate $r_{2\pi}$ for the process $\theta\rightarrow 2\pi$. We denote by $R_\theta(2\pi)$ the rate of decay into the 2π mode, where the subscript θ denotes that we are considering the case where the particle initially produced is a θ meson. Thus

$$R_\theta(2\pi)=r_{2\pi}\frac{1}{|d|^2}\{\alpha^4 e^{-\lambda_1 t}+\beta^4 e^{-\lambda_2 t} + 2\alpha^2\beta^2 e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\cos(\Delta t+2\phi)\}. \quad (6)$$

In the same way we can calculate the rate $R_\theta(3\pi)$ of decays into the 3π mode (again, the subscript θ denotes that the particle initially produced is a θ meson). The result is

$$R_\theta(3\pi)=r_{3\pi}\frac{1}{|d|^2}\alpha^2\beta^2\{e^{-\lambda_1 t}+e^{-\lambda_2 t}-2e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\cos\Delta t\}, \quad (7)$$

where $r_{3\pi}$ is the transition rate for the process $\tau\rightarrow 3\pi$.

Finally, we want to obtain the rate of decay via modes involving neutrinos. In this case both the θ and τ particles contribute. Consider, for example, the $\mu+\nu$ mode. Let $f_{\mu\nu}$ be the transition *amplitude* for the process $\theta\rightarrow\mu+\nu$, and $g_{\mu\nu}$ the transition amplitude for $\tau\rightarrow\mu+\nu$. For simplicity we shall assume that f and g differ at most by a minus sign. In the present model this appears to be the most natural assumption; but even if the situation were otherwise the effects discussed here would not be altered in a qualitative way. The total transition rates for the processes $\theta\rightarrow\mu+\nu$ and $\tau\rightarrow\mu+\nu$ are now identical; we denote this rate by $r_{\mu\nu}$. The rate of decays into the $\mu+\nu$ mode is then given by

$$R_\theta(\mu\nu)=r_{\mu\nu}\frac{1}{|d|^2}\{\alpha^2(1\pm 2\alpha\beta\cos\phi)e^{-\lambda_1 t} + \beta^2(1\mp 2\alpha\beta\cos\phi)e^{-\lambda_2 t}-2\alpha\beta[2\alpha\beta\sin\phi\sin(\Delta t+\phi) \pm(\alpha^2-\beta^2)\cos(\Delta t+\phi)]e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\}, \quad (8)$$

where the $(\pm)$ sign corresponds to $f/g=\pm 1$.

For the decay rates $R_\theta(e\pi\nu)$ and $R_\theta(\mu\pi\nu)$ into the other modes involving neutrinos one obtains expressions identical to that in Eq. (8), except that the factor $r_{\mu\nu}$ in Eq. (8) is replaced by the appropriate transition rate $r_{e\pi\nu}$ or $r_{\mu\pi\nu}$.

So far we have considered the case where at $t=0$ it is the θ meson which is produced. If the initial particle is a τ meson we find, as above, the expressions

$$R_\tau(2\pi)=r_{2\pi}\frac{1}{|d|^2}\alpha^2\beta^2\{e^{-\lambda_1 t}+e^{-\lambda_2 t}-2e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\cos\Delta t\}, \quad (9)$$

$$R_\tau(3\pi)=r_{3\pi}\frac{1}{|d|^2}\{\beta^4 e^{-\lambda_1 t}+\alpha^4 e^{-\lambda_2 t} + 2\alpha^2\beta^2 e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\cos(\Delta t-2\phi)\}, \quad (10)$$

$$R_\tau(\mu\nu)=r_{\mu\nu}\frac{1}{|d|^2}\{\beta^2(1\pm 2\alpha\beta\cos)e^{-\lambda_1 t} + \alpha^2(1\mp 2\alpha\beta\cos\phi)e^{-\lambda_2 t} + 2\alpha\beta[2\alpha\beta\sin\phi\sin(\Delta t-\phi) \pm(\alpha^2-\beta^2)\cos(\Delta t-\phi)]e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\}. \quad (11)$$

In the model under discussion the actual decay rates which one would observe depend on the relative numbers of θ and τ mesons produced at the source. In the parity-doublet scheme of Lee and Yang, θ and τ mesons are always produced in equal numbers in collisions involving ordinary particles. If we maintain this feature of the parity-doublet scheme, then the observed decay rates would be given by the average: $R=\frac{1}{2}(R_\theta+R_\tau)$. We find then the following expressions, which are the ones to be compared with experiment.

$$R(2\pi)=r_{2\pi}\frac{1}{2|d|^2}\{\alpha^2 e^{-\lambda_1 t}+\beta^2 e^{-\lambda_2 t} - 4\alpha^2\beta^2\sin\phi\, e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\sin(\Delta t+\phi)\}, \quad (12)$$

$$R(3\pi)=r_{3\pi}\frac{1}{2|d|^2}\{\beta^2 e^{-\lambda_1 t}+\alpha^2 e^{-\lambda_2 t} + 4\alpha^2\beta^2\sin\phi\, e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\sin(\Delta t-\phi)\}, \quad (13)$$

$$R(\mu\nu)=r_{\mu\nu}\frac{1}{2|d|^2}\{1\pm 2\alpha\beta\cos\phi)e^{-\lambda_1 t} + (1\mp 2\alpha\beta\cos\phi)e^{-\lambda_2 t} - 4\alpha\beta e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\sin\phi[2\alpha\beta\sin\phi\cos\Delta t \mp(\alpha^2-\beta^2)\sin\Delta t]\}. \quad (14)$$

For the other modes involving neutrinos, the rates $R(e\pi\nu)$ and $R(\mu\pi\nu)$ are given by expressions identical to that in Eq. (14) except that the factor $r_{\mu\nu}$ is replaced by the appropriate transition rate $r_{e\pi\nu}$ or $r_{\mu\pi\nu}$. It is to be recalled that we have simplified matters by assuming that the transition amplitudes for $\theta\rightarrow\mu+\nu$ and $\tau\rightarrow\mu+\nu$ are either equal or opposite in sign. The $(\pm)$ signs in Eq. (14) correspond to these two possibilities. Even with this limitation, however, the relative signs need not be the same for all three K-meson decay modes involving neutrinos.

The total decay rate into all modes may be obtained by summing the above individual rates. It can also be calculated by taking the negative time-derivative of $|\psi(t)|^2$. In the latter way one finds

$$R_{\text{total}}=\frac{1}{2|d|^2}\{\lambda_1 e^{-\lambda_1 t}+\lambda_2 e^{-\lambda_2 t}-4\alpha^2\beta^2\sin^2\phi e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\times[(\lambda_1+\lambda_2)\cos\Delta t+2\Delta\sin\Delta t]\}. \quad (15)$$

That this expression does not contain the factors $r_{2\pi}$, $r_{3\pi}$, etc., merely reflects the fact that these factors and the parameters λ_1, λ_2, α/β, ϕ, and Δ are not independent. In fact, from the physical requirement that the negative time-derivative of $|\psi(t)|^2$ be positive definite—for an arbitrary initial state—one easily derives the important inequality

$$\alpha^2\beta^2\sin^2\phi<\frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_2)^2+4\Delta^2}. \quad (16)$$

III. DISCUSSION

The essential idea of the present discussion may now be restated as follows. If τ and θ are different particles, of the same spin but opposite parity, then the two particles could be interconverted through weak parity-nonconserving processes involving neutrinos, e.g., $\theta\rightarrow\mu+\nu\rightarrow\tau$. The particles τ and θ would then not have definite lifetimes. Instead, for charged K mesons the two particles of definite lifetime, K_1 and K_2, would be certain linear combinations of τ and θ. Given a θ or a τ meson produced at the initial time, the subsequent decay function would display a complicated pattern, characterized by the lifetimes λ_1^{-1} and λ_2^{-1} of the particles K_1 and K_2 and by their interference effects. The qualitative behavior is indicated by the formulas obtained above.

It is clear now how the present argument might bear on the $\tau-\theta$ problem. Quantitative experiments on charged K-meson lifetimes have so far been carried out at distances from the K-meson source corresponding to transit times of order 10^{-8} sec.[6,10] Suppose that $\lambda_1^{-1}\ll\lambda_2^{-1}$ and that $\lambda_1^{-1}\ll 10^{-8}$ sec. Then in the above experimental circumstances one would always detect essentially the pure long-lived particle K_2 and, of course, all K-meson decay modes would appear to have the same lifetime, λ_2^{-1}. If further, we assume that τ and θ mesons are always produced in the same proportions, as in the parity-doublet scheme, then the K-meson branching ratios would be the same in all experiments carried out at distances corresponding to transit times large compared to λ_1^{-1}.

On the other hand, the ideas discussed here could be tested by studying the K-meson decay functions and branching ratios[11] over a time interval which includes times short compared to 10^{-8} sec. If it indeed turns out that the decay functions are pure exponentials, we would argue that this constitutes strong evidence against the idea that τ and θ are different particles, of the same spin and opposite parity.

Parity mixing could lead to observable effects in absorption experiments similar to those proposed by Pais and Piccioni[12] for neutral K mesons. Suppose, for example, that an absorber is placed in a pure K_2 beam (i.e., it is placed far from the source). If the τ and θ interact differently with nuclear matter, one would expect regeneration of the K_1 component in the transmitted beam. In the parity-doublet scheme of Lee and Yang, however, the absorption and scattering cross sections in nuclear matter are identical for τ and θ, so that in fact the transmitted beam would still be pure K_2. On the other hand, the *scattered* beam could have a different composition. This would come about if "parity exchange" scattering occurs, i.e.,

$$\theta+\text{nucleus}\leftrightarrows\tau+\text{nucleus}.$$

[10] R. Motley and V. Fitch, Phys. Rev. **105**, 265 (1957); and references therein.

[11] In experiments where K particles are stopped, say in emulsions, after a transit time, t, the number of observed decays in a particular mode would be given by $\int_t^\infty R dt$.

[12] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).