

## Composite Particles of Zero Mass

K. M. CASE\*

*The Institute for Advanced Study, Princeton, New Jersey*

(Received March 8, 1957)

A neutrino theory of spin-zero particles of zero mass is constructed in terms of bilinear neutrino operators. The restrictions of a two-component neutrino theory are shown to reduce the possible fields from three to one. Further, by using a two-component neutrino theory it is shown that it is impossible by using bilinear expressions to form a theory of light and that using quadrilinear expressions cannot produce a theory of gravitation.

### I. INTRODUCTION

IN an article in 1938 Pryce<sup>1</sup> showed that the neutrino theories of light formulated up to that time were subject to a grave fault. They were not invariant under spatial rotations. The proposal of Lee and Yang<sup>2</sup> that the neutrino is to be described by a two-component equation suggests that the question be reinvestigated. This is done below.

It should be remarked that since the original argument uses only invariance properties with respect to the proper inhomogeneous Lorentz group and since the two-component theory merely describes an invariant decomposition (with respect to this group) of the four-component theory, the same conclusions must follow. There are, however, two impressions prevalent which we shall show to be false.

The first impression is that a composite neutrino theory is tied to one dimension. After some formal preliminaries in Sec. II, this is shown to be false in Sec. III by an explicit construction of a spinless, massless field in three dimensions using a two-component neutrino theory. To see the reflection properties of the spinless field constructed, we consider the same problem using a four-component neutrino theory in Sec. IV. From the results obtained there, it follows that the field constructed with the two-component neutrino describes an equal "mixture" of scalar and pseudoscalar particles.

The second impression is that the difficulty with the "neutrino theory of light" arises because light is transversely and neutrinos are longitudinally polarized. However, the two-component neutrino has the same type polarization as light. On the basis of the theorem provided in Sec. V, it can be concluded that the "two-component neutrino theory of light" still does not work. Another consequence of this theorem is that one cannot construct a "neutrino theory of gravitation" by using four two-component neutrinos.

### II. PRELIMINARIES

As Wigner has shown,<sup>3</sup> invariance and irreducibility under the inhomogeneous proper Lorentz group requires

\* Permanent address: Physics Department, University of Michigan, Ann Arbor, Michigan.

<sup>1</sup> M. H. L. Pryce, Proc. Roy. Soc. (London) **A165**, 247 (1938).

<sup>2</sup> T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

<sup>3</sup> E. P. Wigner, Ann. Math. **40**, 149-204 (1939).

that for particles of mass zero and spin  $s$  the spin must be either parallel or antiparallel to the momentum. If invariance under spatial reflection is also required, both possibilities must occur. We shall refer to the case where only one or the other are present as a "two-component theory" while the presence of both will be a "four-component theory."

It is assumed that we are given a field describing massless particles of half-integral spin  $s$  quantized according to Fermi-Dirac statistics. The question we wish to discuss is the following: Can we construct operators from the given field operators which will describe an integral-spin field quantized according to Einstein-Bose statistics?

First let us consider two-component theories. Corresponding to a momentum of magnitude  $P_1$  and direction  $\mathbf{n}_1$ , we have absorption and emission operators for the particles which will be denoted by  $a(P_1\mathbf{n}_1)$ ,  $a^*(P_1\mathbf{n}_1)$  respectively. There will be similar operators for the antiparticle which will be written as  $b(P_1\mathbf{n}_1)$  and  $b^*(P_1\mathbf{n}_1)$ . The commutation relations are

$$[a(P_1\mathbf{n}_1), a^*(P_2\mathbf{n}_2)]_+ = [b(P_1\mathbf{n}_1), b^*(P_2\mathbf{n}_2)]_+ = \delta(P_1\mathbf{n}_1, P_2\mathbf{n}_2), \quad (1)$$

while all other anticommutators vanish. Under translations in space and time,  $a(P_1\mathbf{n}_1)$  and  $b(P_1\mathbf{n}_1)$  will transform as  $\exp[iP_1(\mathbf{n}_1 \cdot \mathbf{r} - t)]$  while  $a^*$ ,  $b^*$  transform as  $\exp[-iP_1(\mathbf{n}_1 \cdot \mathbf{r} - t)]$ . We choose our convention so that under rotations around the direction  $\mathbf{n}_1$  the transformation of  $a$  is:

$$a(P_1\mathbf{n}_1) \rightarrow a'(P_1\mathbf{n}_1) = e^{-is\theta} a(P_1\mathbf{n}_1). \quad (2a)$$

Then the other operators will transform so that

$$a^*(P_1\mathbf{n}_1) \rightarrow a'^*(P_1\mathbf{n}_1) = e^{is\theta} a^*(P_1\mathbf{n}_1), \quad (2b)$$

$$b(P_1\mathbf{n}_1) \rightarrow b'(P_1\mathbf{n}_1) = e^{is\theta} b(P_1\mathbf{n}_1), \quad (2c)$$

$$b^*(P_1\mathbf{n}_1) \rightarrow b'^*(P_1\mathbf{n}_1) = e^{-is\theta} b^*(P_1\mathbf{n}_1). \quad (2d)$$

Suppose we attempt to construct operators for a new field which are to be bilinear combinations of the above. If  $\xi(\mathbf{k}\mathbf{n})$  is to be the absorption operator for a particle of momentum  $\mathbf{k}\mathbf{n}$ , energy  $k$ , it must transform as  $\exp[ik(\mathbf{n} \cdot \mathbf{r} - t)]$  under space-time translations. There are three types of terms that can occur in the expression

for  $\xi(k\mathbf{n})$ . These are

$$C^*(P_1\mathbf{n}_1)C'(P_2\mathbf{n}_2), \quad (3a)$$

$$C(P_1\mathbf{n}_1)C'(P_2\mathbf{n}_2), \quad (3b)$$

$$C^*(P_1\mathbf{n}_1)C^*(P_2\mathbf{n}_2). \quad (3c)$$

(Here  $C, C'$  can be either  $a$ 's or  $b$ 's.) What restrictions are implied by the translation properties? If terms of the form (3a) appear, we must have

$$k\mathbf{n} = P_2\mathbf{n}_2 - P_1\mathbf{n}_1, \quad (4a)$$

$$k = P_2 - P_1. \quad (4b)$$

Squaring Eqs. (4) and subtracting gives

$$0 = 2P_1P_2(1 - \mathbf{n}_1 \cdot \mathbf{n}_2). \quad (5)$$

Hence  $\mathbf{n}_1 = \mathbf{n}_2$ . From Eq. (4a) we see this common direction must be  $\mathbf{n}$ . The only expressions of the form (3a) that can occur are then

$$C^*(P_1)C'(P_2)\delta(P_2 - P_1, k), \quad (6a)$$

where we have suppressed the common direction  $\mathbf{n}$ . Similarly the only expressions of the form (3b) must be

$$C(P_1)C'(P_2)\delta(P_2 + P_1, k), \quad (6b)$$

while there can be no terms of the form (3c). Introducing particle and antiparticle operators explicitly, we thus have the eight possibilities

$$\alpha_1 = a^*(P_1)a(P_2)\delta(P_2 - P_1, k),$$

$$\alpha_2 = b^*(P_1)b(P_2)\delta(P_2 - P_1, k),$$

$$\alpha_3 = a^*(P_1)b(P_2)\delta(P_2 - P_1, k),$$

$$\alpha_4 = b^*(P_1)a(P_2)\delta(P_2 - P_1, k),$$

$$\beta_1 = a(P_1)b(P_2)\delta(P_2 + P_1, k),$$

$$\beta_2 = b(P_1)a(P_2)\delta(P_2 + P_1, k),$$

$$\beta_3 = b(P_1)b(P_2)\delta(P_2 + P_1, k),$$

$$\beta_4 = a(P_1)a(P_2)\delta(P_2 + P_1, k).$$

Here, of course, all  $P$ 's are restricted to be positive. A notational simplification is obtained by defining  $a$ 's for negative  $P$  by

$$a(-|P|) = b^*(|P|), \quad a^*(-|P|) = b(|P|). \quad (7)$$

The anticommutation relations are simply

$$[a(P), a^*(P')]_{\pm} = \delta(P, P'), \quad -\infty < P, P' < \infty, \quad (8a)$$

and

$$[a(P), a(P')]_{\pm} = 0, \quad -\infty < P, P' < \infty. \quad (8b)$$

We note that these definitions are consistent with the rotational properties expressed by Eqs. (2). Thus, under rotations around  $\mathbf{n}$ , all  $a$ 's transform as  $e^{-is\theta}$  while all  $a^*$ 's transform as  $e^{+is\theta}$ . The eight possibilities listed above can now be lumped in three groups. All

terms of the form  $\alpha_1, \alpha_2, \beta_1, \beta_2$  can be written as

$$a^*(P_1)a(P_2)\delta(P_2 - P_1, k), \quad -\infty < P_1, P_2 < \infty, \quad (9a)$$

while the terms of the form  $\alpha_3, \beta_3$  are just

$$a^*(P_1)a^*(P_2)\delta(-P_2 - P_1, k), \quad -\infty < P_1, P_2 < \infty, \quad (9b)$$

and the terms  $\alpha_4, \beta_4$  are

$$a(P_1)a(P_2)\delta(P_2 + P_1, k), \quad -\infty < P_1, P_2 < \infty. \quad (9c)$$

From operators of the form (9) we can clearly only form absorption operators for particles of spin 0 and spin  $2s$ . For spin 0 only the form (9a) can occur, while for particles of spin  $2s$  only (9c) or (9b) can occur depending on whether we are describing particles which behave as  $e^{-i2s\theta}$  or  $e^{+i2s\theta}$  under rotation.

With a four-component theory of our particle of spin  $s$ , the situation is quite analogous except for a certain increase in freedom. Instead of a single absorption operator for a particle of momentum  $P\mathbf{n}$ , there are two, which we denote by  $a_1$  and  $a_2$ . Under translations these have the same behavior, but they have opposite behavior under rotation. Thus if we take as a convention that  $a_1$  behaves as  $e^{-is\theta}$ , then  $a_2$  will behave as  $e^{+is\theta}$ . The possibilities expressed by (9) are increased since each  $a$  can either be an  $a_1$  or  $a_2$ . For example, for absorption operators for a particle of spin zero, we can have instead of (9a) the four possibilities

$$a_1^*(P_1)a_1(P_2)\delta(P_2 - P_1, k), \quad (10a)$$

$$a_2^*(P_1)a_2(P_2)\delta(P_2 - P_1, k), \quad (10b)$$

$$a_1^*(P_1)a_2^*(P_2)\delta(-P_2 - P_1, k), \quad (10c)$$

$$a_1(P_1)a_2(P_2)\delta(P_2 + P_1, k). \quad (10d)$$

### III. CONSTRUCTION OF A SPIN-ZERO FIELD

We wish here to construct the absorption and emission operators for a massless, spinless field in terms of bilinear combinations of a two-component spin- $s$  field. The considerations of the previous section shows that the absorption operator  $\xi(k)$  for momentum  $k\mathbf{n}$  must be of the form

$$\xi(k) = \sum_P a^*(P)a(P+k)f(P, k). \quad (11)$$

It is fortunate that only spin- $s$  operators corresponding to the direction  $\mathbf{n}$  appear. The conditions

$$[\xi(k\mathbf{n}), \xi(k'\mathbf{n}')]_{-} = [\xi(k\mathbf{n}), \xi^*(k'\mathbf{n}')]_{-} = 0 \quad \text{for } \mathbf{n} \neq \mathbf{n}'$$

are then automatically satisfied. Here the  $c$  numbers  $f(P, k)$  are to be determined so that

$$[\xi(k), \xi(k')]_{-} = 0, \quad (12)$$

and

$$[\xi(k), \xi^*(k')]_{-} = \delta(k, k'). \quad (13)$$

For simplicity a box normalization has been assumed so that integrals over momenta can be reduced to sums over integers. Following Pryce,<sup>1</sup>  $\sum$  is taken to mean  $\sum_{-R}^R$ , where  $R$  is some large fixed number. It is

assumed that all particle and antiparticle states are empty for sufficiently high momenta, i.e.,

$$a(|P|\Psi) = a^*(-|P|\Psi) = 0, \quad |P| > Q, \quad \text{where } Q < R. \quad (14)$$

Using Eqs. (11) and (9) we obtain

$$[\xi(k), \xi(k')]_- = \sum_P \{ f(P, k) f(P+k, k') - f(P+k', k) f(P, k') \} \{ a^*(P) a(P+k+k') \}. \quad (15)$$

The condition of Eq. (12) is obviously satisfied if  $f(P, k)$  is independent of  $k$ , i.e.

$$f(P, k) = f(k). \quad (16)$$

Hence

$$\xi(k) = f(k) \sum_{P=-R}^R a^*(P) a(P+k). \quad (17)$$

With Eq. (17) we find

$$[\xi(k), \xi^*(k')] = f(k) f^*(k') \sum_{P=-R}^R \{ a^*(P) a(P+k-k') - a^*(P+k') a(P+k) \}.$$

Changing variables in the second term gives

$$\sum_{-R}^R a^*(P+k') a(P+k) = \sum_{-R+k'}^{R+k'} a^*(P) a(P+k-k').$$

Using the conditions of (14) and the commutation relations, we find

$$\begin{aligned} \sum_{-R+k'}^{R+k'} a^*(P) a(P+k-k') \\ = -k' \delta(k, k') + \sum_{-R}^R a^*(P) a(P+k-k'). \end{aligned}$$

Hence

$$[\xi(k), \xi^*(k')]_- = |f(k)|^2 k \delta(k, k').$$

Therefore the condition (13) yields<sup>4</sup>

$$f(k) = 1/\sqrt{k},$$

and thus<sup>5</sup>

$$\xi(k) = \frac{1}{\sqrt{k}} \sum_P a^*(P) a(P+k). \quad (18)$$

#### IV. SPIN ZERO FROM A FOUR-COMPONENT THEORY

An analogous procedure can be followed here. Instead of Eq. (11) an arbitrary combination of terms of the form of Eq. (10) may be tried. A general solution is

<sup>4</sup> An arbitrary multiplicative phase factor is certainly always possible.

<sup>5</sup> This is identical with the one-dimensional solution obtained by Max Born and N. S. Nagendra Nath, Proc. Indian Acad. Sci. A3, 318 (1936).

easily obtained. Let

$$\Phi_1(k) = \frac{1}{\sqrt{k}} \sum_P a_1^*(P) a_1(P+k), \quad (19a)$$

$$\Phi_2(k) = \frac{1}{\sqrt{k}} \sum_P a_2^*(P) a_2(P+k), \quad (19b)$$

$$\Phi_3(k) = \frac{1}{(2k)^{\frac{1}{2}}} \sum_P \{ a_1(P) a_2(k-P) + a_1^*(P) a_2^*(-P-k) \}. \quad (19c)$$

A general solution<sup>6</sup> is

$$\xi(k) = \alpha_1 \Phi_1(k) + \alpha_2 \Phi_2(k) + \alpha_3 \Phi_3(k), \quad (20)$$

provided

$$|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 = 1 \quad (21a)$$

and

$$\alpha_1 \alpha_3^* + \alpha_1^* \alpha_3 + \alpha_2 \alpha_3^* + \alpha_2^* \alpha_3 = 0. \quad (22a)$$

Two distinct cases arise:

(1)  $\alpha_3 = 0$ . The condition of Eq. (22a) is automatically fulfilled and we only have the normalization condition

$$|\alpha_1|^2 + |\alpha_2|^2 = 1. \quad (21b)$$

(2)  $\alpha_3 \neq 0$ . Since the choice of phase of  $\xi$  is always arbitrary we can choose it so that  $\alpha_3$  is real. In addition to the condition (21a), we then have the reality condition

$$\alpha_1 + \alpha_1^* + \alpha_2 + \alpha_2^* = 0. \quad (22b)$$

It should be noted that the solution for the "two-component theory" obtained in the previous section merely corresponds to the special case of Eq. (20) with  $\alpha_2 = \alpha_3 = 0$  and  $\alpha_1 = 1$ .

This relationship may be used to determine the parity properties of the field described by Eq. (18). Under spatial reflection, the operators  $a_{1,2}$  of the four-component theory transform so that

$$a_1 \rightarrow a_1' = a_2, \quad a_2 \rightarrow a_2' = a_1.$$

Hence  $\Phi_3 \rightarrow +\Phi_3' = -\Phi_3$  and describes a pseudoscalar field.  $\Phi_1$  and  $\Phi_2$  are merely interchanged under reflection. Instead of these operators, we can introduce the combinations

$$\begin{aligned} \Phi_s(k) &= \{ \Phi_1(k) + \Phi_2(k) \} / \sqrt{2}, \\ \Phi_{ps}(k) &= \{ \Phi_1(k) - \Phi_2(k) \} / \sqrt{2}. \end{aligned}$$

These clearly have the transformation properties suggested by the subscripts:  $\Phi_s$  describes a scalar field and  $\Phi_{ps}$  a pseudoscalar field. Expressing the field described by Eq. (18) in terms of these operators gives

$$\xi(k) = \{ \Phi_s(k) + \Phi_{ps}(k) \} / \sqrt{2}. \quad (23)$$

<sup>6</sup> The relative phases of the two terms in Eq. (19c) could be chosen arbitrarily. However, this merely mirrors the lack of uniqueness of operators defined only by the anticommutation relation (8).

From this we can conclude that the spinless field resulting from a two-component theory describes a particle with "mixed parity." It is a superposition of equal amounts of a scalar and a pseudoscalar field.

### V. FIELDS OF SPIN $ms$

Given a two-component field of spin  $s$ , we can now ask whether we can construct the field operators for particles of integral spin  $ms$  by using  $m$  of the original operators. Thus for  $m=2$ ,  $s=\frac{1}{2}$  we would have a "two-component neutrino theory of light" and for  $m=4$ ,  $s=\frac{1}{2}$  we would have a "two-component neutrino theory of gravitation." This we will show to be impossible, in the sense that the transformation properties discussed in Sec. II determine the form of the possibilities we can try to such an extent that the relevant commutation relations cannot be satisfied.

The minimum requirement we can ask is to form the appropriate operators for a two-component theory of spin  $ms$  by using  $m$  spin- $s$  operators. Thus let us try to construct the absorption operator  $\xi$  for a particle of momentum  $\mathbf{k}\mathbf{n}$  which under rotation about  $\mathbf{n}$  behaves as  $e^{-ims\theta}$ . (We shall omit the index  $k$  since the impossibility can be proved by using only one  $k$ .)

The same arguments as in Sec. II shows  $\xi$  must be a superposition of terms of the form

$$a(P_1)a(P_2)\cdots a(P_m)\delta(P_1+P_2+\cdots+P_m, k); \quad (24)$$

$-\infty < P_1, P_2, \dots, P_m < \infty.$

Hence the most general possibility for  $\xi$  within the present context is

$$\xi = \sum_{P_1 P_2 \dots P_m} F(P_1, P_2, \dots, P_m) a(P_1) a(P_2) \cdots a(P_m). \quad (25)$$

(Tacitly we know that  $F=0$  unless  $P_1+P_2+\cdots+P_m = k$ , but we shall not need this.) Since the  $a$ 's all anticommute, it is clearly no restriction to assume

$$(1) F=0 \quad \text{if any } P_i=P_j, \quad i \neq j; \quad (26a)$$

$$(2) F(P_1, P_2, \dots, P_m) = \pm F(P_1', P_2', \dots, P_m') \quad (26b)$$

if  $P_1', P_2', \dots, P_m'$  is merely some permutation of  $P_1, P_2, \dots, P_m$ . [The + (-) sign holds if this is an even (odd) permutation.] Let us see if the commutation relation

$$[\xi, \xi^*]_- = 1$$

can be satisfied by an appropriate choice of  $F$ . This requires that the commutator be independent of operators  $a$ .

For the commutator, we have

$$\begin{aligned} [\xi, \xi^*]_- &= \sum_{P_1 P_2 \dots P_m} \sum_{P_1' P_2' \dots P_m'} F(P_1, P_2, \dots, P_m) \\ &\quad \times F^*(P_1', \dots, P_m') [a(P_1) a(P_2) \cdots a(P_m), \\ &\quad \times a^*(P_m') a^*(P_{m-1}') \cdots a^*(P_1')]. \quad (27) \end{aligned}$$

Let us restrict our attention to those terms in (27) which are not trivially zero and are diagonal in the spin- $s$  field occupation numbers. These arise only from terms with  $P_1', P_2', \dots, P_m'$ , some permutation of  $P_1, P_2, \dots, P_m$ . By using the properties expressed by Eq. (26), these can all be rearranged so that  $P_1'=P_1, P_2'=P_2, \dots, P_m'=P_m$ . Therefore

$$\begin{aligned} [\xi, \xi^*]_- (\text{diagonal}) &= m! \sum_{P_1 P_2 \dots P_m} |F(P_1, P_2, \dots, P_m)|^2 \\ &\quad \times [a(P_1) a(P_2) \cdots a^*(P_m) a^*(P_{m-1}) \cdots]_- . \quad (28) \end{aligned}$$

The commutator under the sum in Eq. (28) is

$$\begin{aligned} [ , ] &= a(P_1) a(P_2) \cdots a(P_{m-1}) a^*(P_{m-1}) \cdots a^*(P_1) \\ &\quad - a^*(P_m) a(P_1) a(P_2) \cdots a(P_{m-2}) a^*(P_{m-2}) \cdots \\ &\quad \times a^*(P_1) + a^*(P_m) a^*(P_{m-1}) a(P_1) a(P_2) \cdots \\ &\quad \times a(P_{m-3}) a^*(P_{m-3}) \cdots a^*(P_1) - \cdots . \quad (29) \end{aligned}$$

(It has been assumed that  $m$  is even.) On inserting Eq. (12a) into (28), we shall successively interchange the names of the summation variables. Thus corresponding to the second line of (29) we interchange  $P_m$  and  $P_{m-1}$ , corresponding to the third line we interchange  $P_m$  and  $P_{m-2}$ , etc. Equation (28) remains the same but now the commutator under the summations sign is

$$\begin{aligned} [ , ] &= a(P_1) a(P_2) \cdots a(P_{m-1}) a^*(P_{m-1}) \cdots a^*(P_1) \\ &\quad - a^*(P_{m-1}) a(P_1) a(P_2) \cdots a(P_{m-1}) a^*(P_{m-2}) \cdots \\ &\quad \times a^*(P_1) + a^*(P_{m-1}) a^*(P_{m-2}) a(P_1) a(P_2) \cdots \\ &\quad \times a(P_{m-1}) a^*(P_{m-3}) \cdots a^*(P_1) - \cdots . \quad (30) \end{aligned}$$

If we consider only terms of highest order in the occupation number operator ( $N_1 N_2 \cdots N_{m-1}$ ), we can freely commute the operators in Eq. (30) so that for these terms all the rows are equivalent to the first, i.e.,

$$\begin{aligned} [ , ] &\sim m a(P_1) a(P_2) \cdots \\ &\quad \times a(P_{m-1}) a^*(P_{m-1}) \cdots a^*(P_1). \quad (31) \end{aligned}$$

This last expression can in turn be expressed in terms of the occupation number operators,

$$[ , ] \sim -m N(P_1) N(P_2) \cdots N(P_{m-1}), \quad (32)$$

where the equivalence is understood to mean an equality only so far as terms of the form  $N_1 N_2 \cdots N_{m-1}$  are concerned. Finally then

$$\begin{aligned} [\xi, \xi^*]_- &\sim -m(m!) \sum_{P_1 P_2 \dots P_m} |F(P_1, P_2, \dots, P_m)|^2 \\ &\quad \times N(P_1) N(P_2) \cdots N(P_{m-1}). \quad (33) \end{aligned}$$

Hence, if these operators are not to occur in  $[\xi, \xi^*]_-$  we must have

$$F(P_1, P_2, \dots, P_m) \equiv 0. \quad (34)$$

Then  $\xi$  vanishes identically.

Putting  $m=2$  and 4, respectively, we see that neither a two-component neutrino theory of light using bilinear

expressions in neutrino operators nor a two-component neutrino theory of gravitation using quadrilinear expression in neutrino operators is possible.

## VI. CONCLUSION

The situation can be summarized as follows:

(1) Using a four-component theory of half-integral spin fields, we can form three essentially independent fields of zero spin using bilinear expressions.

(2) The restriction to a two-component theory reduces these to one spin-zero field. This field is a parity mixture with equal amounts of scalar and pseudoscalar.

(3) Expressions of  $m$ th order in the operators of a two-component theory of spin  $s$  cannot produce operators for spin  $ms$  ( $m$  even).

This situation may be compared with that starting with an integral spin- $s$  field. Bilinear operators of either a two- or four-component theory cannot be constructed which will describe either spin 0 or spin  $2s$ .

## Parity Mixing Effects in $\tau$ - $\theta$ Decay\*

S. B. TREIMAN AND H. W. WYLD, JR.

*Palmer Physical Laboratory, Princeton University, Princeton, New Jersey*

(Received March 4, 1957)

The recent results on nonconservation of parity in decay processes involving neutrinos do not provide an unambiguous solution of the  $\tau$ - $\theta$  puzzle. In fact the  $2\pi$  and  $3\pi$  decay modes of  $\theta$  and  $\tau$  involve no neutrinos, whereas the Lee-Yang two-component neutrino theory attributes the nonconservation of parity to special properties of the neutrino. However, even if the  $\tau$  and  $\theta$  are different particles with opposite parities, the neutrino decay modes  $(\mu, \nu)$ ,  $(e, \pi, \nu)$ , and  $(\mu, \pi, \nu)$ , which presumably violate parity conservation, will cause mixing of the two particles  $\tau$  and  $\theta$ —in the sense that the states with definite lifetimes will be certain linear combinations,  $K_1$  and  $K_2$ , of  $\tau$  and  $\theta$ . Both  $K_1$  and

$K_2$  will then decay into both  $2\pi$  and  $3\pi$  as well as the neutrino modes, but with different lifetimes. An explanation of the apparent equality of lifetimes of  $\tau$  and  $\theta$  may be that under present experimental conditions only the long-lived component  $K_2$  is observed. If this is the case, interference effects between the  $K_1$  and  $K_2$  components should be found in experiments performed at shorter times. Conversely, absence of such effects would constitute strong evidence that  $\tau$  and  $\theta$  are identical.

Phenomenological expressions are derived by the Wigner-Weisskopf method for the decay rates, including interference effects; and various experimental possibilities are discussed.

## I. INTRODUCTION

THE recent spectacular developments concerning parity nonconservation in  $\beta$  decay and  $\pi$ - and  $\mu$ -meson decay<sup>1-5</sup> have not as yet led to a clear understanding of the familiar problem that motivated them, namely, the  $\tau$ - $\theta$  puzzle. It is of course now possible to suppose that the puzzle has vanished: that  $\tau$  and  $\theta$  are one and the same particle, which may decay, with violation of parity conservation, into both the  $2\pi$  and  $3\pi$  modes observed in  $K$ -meson disintegrations. But in the processes where parity nonconservation has been established experimentally, neutrinos are always involved among the decay products; and it appears that the parity conservation law is violated here in a very special way "attributable" to special properties of the neutrino.<sup>3</sup> No equally natural and compelling picture for parity nonconservation in decay processes not involving neutrinos has as yet been put forward. It is

thus still conceivable that the  $\tau$  and  $\theta$  mesons are different particles. If this is so, one is still faced with the familiar problem of understanding, among other things, the apparent equality in their lifetimes.<sup>6</sup>

It was in fact this problem that represented one of the major difficulties with the parity-doublet scheme introduced by Lee and Yang.<sup>7</sup> We may now, however, re-examine this scheme in the light of the apparent fact that parity conservation is always violated in decay processes involving neutrinos.<sup>4,5</sup> Suppose, as did Lee and Yang, that  $\tau$  and  $\theta$  are members of a parity doublet, and that between them they account for all the decay modes observed among  $K$  mesons; for the charged  $K$  mesons:  $K_{2\pi}$ ,  $K_{3\pi}$ ,  $K_{\mu\nu}$ ,  $K_{e\pi\nu}$ ,  $K_{\mu\pi\nu}$ . In the original scheme parity conservation was of course assumed. Let us now abandon this conservation law, *but only for those processes involving neutrinos*. Even in the original scheme, it was always conceivable that  $\tau$  and  $\theta$  could both contribute to neutrino decay modes, but they could do so only to states of opposite parity. Now, however, it appears that both  $\tau$  and  $\theta$  could contribute to neutrino processes involving the same states. If this were the situation, it would give rise to a "parity"

\* This work was supported by the Office of Naval Research and the U. S. Atomic Energy Commission.

<sup>1</sup> T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

<sup>2</sup> Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).

<sup>3</sup> T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

<sup>4</sup> Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

<sup>5</sup> Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).

<sup>6</sup> See, e.g., *Report of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York), Sec. V, p. 2.

<sup>7</sup> T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956).