absorption process.) Using the value $G_1^2/4\pi=12$, we obtain $g_{P,\mathrm{eff}}=11.7\times10^{-49}$ erg cm^3, giving an absorption rate of 6 sec^{-1} in hydrogen. Verification or refutation of this prediction would be proof, one way or the other, of the presence or absence of any μ-meson weak couplings in addition to ($A2$) and ($P2$). It seems plausible that experiments will either agree with this figure of 6 sec^{-1} or will give a rate of about 140 sec^{-1}, the figure expected on the assumption that μ absorption occurs through a scalar Fermi coupling with $g_S=3\times10^{-49}$ erg cm^3. The major uncertainty in the figure 6 sec^{-1} stems from uncertainty in the coupling constant G_1. It is however reassuring to note that the approximations made in arriving at (18) will err in the same direction as the approximations made in the derivation of the Kroll-Ruderman theorem,[23,24] from which we derive the value of G_1.

We have here been concerned with the better known particles and decay processes. But it is hoped that the considerations presented will be of assistance in understanding the decays of the strange particles.

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[23] N. Kroll and M. Ruderman, Phys. Rev. **93**, 233 (1954).
[24] F. E. Low, Phys. Rev. **97**, 1392 (1955).

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Chirality of *K* Particle*

Satosi Watanabe
International Business Machines, Research Laboratory, Poughkeepsie, New York
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In the two-component theory, the neutrino that can exist in nature is characterized by one of the eigenvalues of the "chirality" operator, γ_5, which anticommutes with the parity operator. The chirality operator is generalized so that it can be applied also to bosons. The K particle that can exist in nature is characterized by a certain condition on the eigenvalues of the chirality operator. There is strong reason to believe that the chirality quantum number thus introduced is closely related to the strangeness quantum number.

1. INTRODUCTION

The series of theoretical efforts, which has originated from the tau-theta paradox, has culminated in a return to the once-abandoned two-component neutrino theory.[1] The present paper is intended to show that a unified point of view is possible in dealing with both problems.

A special mathematical formalism is used in this paper, so that an operator called "chirality," which anticommutes with the parity operator, can be applied to both fermions and bosons. In the case of a spinor particle, the eigenvalues of chirality are ±1, but they are good quantum numbers only when the mass is zero.[2] If we take one of the possible eigenvalues (say, -1 in the right-handed coordinate system), we obtain the well-known two-component theory of neutrinos. If the mass is finite, the chirality is indeterminate (zero on the average).[2]

In the case of a boson, the eigenvalues are ±2 and 0. They are good quantum numbers even if the mass is finite. The scalar particle can have only eigenvalues ±2. The eigenstates of chirality imply of course an indefinite parity. Conversely, a boson with a definite parity (such as pion) has an indefinite chirality (zero by convention). In view of the fact that the same K particle seems to be capable of decaying into two pions or three pions, it is proposed to assume the K particle to be in an eigenstate of chirality.[3] The tensorial rank of K particles is assumed to be zero, i.e., of the scalar type. Each of the two eigenstates of chirality (±2) provides further two eigenstates, corresponding to two possible charges. To accommodate the K particle and the anti-K particle (either charged or neutral), one thus has four possibilities to choose from. This leads to two alternative assignments of K particles to chiral eigenstates. It is still premature to decide which alternative is preferable.

According to the first assignment, the K particle (either positive or neutral) is identified, say in the right-

* The word "chirality" (pronounced as kirality) seems to have been coined by Kelvin and was extensively used by Eddington [A. S. Eddington, *Fundamental Theory* (Cambridge University Press, New York, 1949, p. 111]. The usage of this term here may be justified by two reasons: (1) Etymologically, it can mean "handedness." (2) Eddington used it also in the sense of the sign of γ_5 though in a different context.

[1] W. Pauli, *Handbuch der Physik* (Julius Springer, Berlin, 1933), Vol. 24, p. 226; A. Salam, Nuovo cimento **5**, 229 (1957); T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957); L. Landau, preprint, among others. Experimental tests, proposed by Lee and Yang [Phys. Rev. **104**, 254 (1956)], played a decisive role in this development.

[2] This is true only when one uses γ_5 as chirality operator. For further discussions of a chiral particle of spin $\frac{1}{2}$ with finite mass and finite charge, see S. Watanabe, Nuovo cimento (to be published).

[3] The assumption that the theta and the tau are the same particle naturally leads to a unique lifetime for two-pi and three-pi decay modes.

hand coordinate system, with chirality $+2$, while the anti-K particle (either negative or neutral) is identified with chirality -2. In this choice, the K particle with chirality -2 and the anti-K particle with chirality $+2$, in the right-hand coordinate system, are declared to be nonexistent in nature. According to the second assignment, both K particle and anti-K particle are supposed to have chirality $+2$, and corresponding particles with chirality -2 are considered as nonphysical.

In the first assignment, the K particle and the anti-K particle are the charge conjugate of each other. This reproduces thus, in the case of neutral K particles, an aspect of Gell-Mann and Pais' theory.[4] In this first assignment, the theory can be made invariant separately for charge-conjugation and for space-*and*-time inversion. In the second assignment, the theory can be made invariant separately for time reversal and for the combination of charge conjugation and space inversion. The latter is analogous to the two-component neutrino theory. In either assignment, a K particle, by space inversion, becomes a forbidden kind of particle. In other words, to describe the same K particle in the right-hand and left-hand coordinate systems, one has to use wave functions with opposite chiralities (but with the same charge).

In either assignment, an arbitrary number of K particles (and/or anti-K particles) can interact with an arbitrary number of pions.

Chirality as a physical quantity is perfectly well defined within the framework of the accepted concept of space, yet it has not been exploited up to the present. For this reason, it can be expected that this new variable may be capable of giving a meaningful interpretation to some empirical facts that have hitherto escaped explanation. In fact, in the first assignment, one-half the chirality turns out to be equal to the "strangeness" number of the K particle. In the second assignment, one-half the product of chirality and charge is equal to the strangeness. This might not be considered to be a trivial coincidence.

This significant situation encourages a further speculation that a hyperon is probably a compound particle composed of a nucleon and K particles, in such a way that the algebraic sum of the strangeness numbers of constituent K particles gives the strangeness number of the hyperon. Whether or not this particular scheme of composition corresponds to reality, a composite picture of hyperons gives a wide range of flexibility to the properties of hyperons. For instance, being a compound particle, a hyperon need not satisfy the Dirac equation rigorously, and it may exhibit complicated handedness in spite of its being a fermion with a finite mass. This might give a clue to a future explanation of the fact that the Λ-K production, for instance, can be a strong interaction.[5] In any event, it should be mentioned that at the present stage the theory proposed in this paper, is no more capable of explaining why a parity-violating interaction should be "weak" than is the two-component neutrino theory. Neither does it give a reason why strangeness, as derived from chirality, can give a measure of "weakness" of interaction. It is hoped, however, that the mathematical methods used in the present paper will prove to be a useful instrument in unraveling problems pertaining to the distinction between strong and weak interactions.

To avoid confusion of exposition, the first assignment exclusively is used in the body of the following text, while Appendix 2 is reserved for discussion of the consequences of the second assignment.

2. CHIRALITY AND CHIRAL CONJUGATION FOR SPINORS

By space inversion (mirage), a spinor $\psi(\mathbf{r},t)$ becomes $\gamma_4\psi(-\mathbf{r}, t)$, except for an arbitrary phase factor, i.e.,

$$M:\quad \psi(\mathbf{r},t)\rightarrow\gamma_4\Omega\psi(\mathbf{r},t)=\gamma_4\psi(-\mathbf{r}, t), \tag{2.1}$$

where Ω can be considered as a matrix $(\Omega=\Omega^{-1}=\Omega^T=\bar{\Omega})$[6]:

$$(\mathbf{r}'|\Omega|\mathbf{r}'')=\delta(\mathbf{r}'+\mathbf{r}''). \tag{2.2}$$

Thus, for instance, operated on the momentum operator $(x'|p_x|x'')=i\delta'(x'-x'')$, Ω leads to

$$\Omega:\quad \mathbf{p}\rightarrow\Omega\mathbf{p}\Omega=-\mathbf{p}. \tag{2.3}$$

The operator $\gamma_4\Omega$ is called the parity operator:

$$P\equiv\gamma_4\Omega,\quad P^2=1. \tag{2.4}$$

Its eigenvalues are therefore ±1. Any operator X that anticommutes with P,

$$[P,X]_+=0, \tag{2.5}$$

will interchange the two eigenfunctions of P, and such an X will be called a "parity conjugation operator."[7] The simplest X, which shows no preferential direction in space, is γ_5,

$$[P,\gamma_5]_+=0. \tag{2.6}$$

Hereafter, we shall put

$$X\equiv\gamma_5,\quad X^2=1, \tag{2.7}$$

and call it the "chirality operator." The eigenvalues of X are again ±1. Any operator Y that anticommutes with X,

$$[X,Y]_+=0, \tag{2.8}$$

will interchange the eigenstates of X, and will be called the "chiral conjugation operator." P is one of such Y's, but we do not specify for the moment which one of the Y's should be chosen as the suitable chiral conjugation

[4] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[5] As regards the property of the Λ particle toward space inversion, see reference 18.

[6] S. Watanabe, Revs. Modern Phys. **27**, 40 (1955), Eqs. (2.74) and (8.6).

[7] Parity conjugation was also considered in T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956). However, it was used only to interchange the members of a parity doublet, and its eigenstates were not considered there.

operator. Since the two eigenfunctions of X are interchanged by space inversion (which changes the handedness of a coordinate system), the name chirality for X seems to be appropriate.

Charge conjugation of a spinor is performed by

$$C:\quad \psi\rightarrow\bar{\psi}\gamma_4 C,\quad \bar{\psi}\rightarrow C^{-1}\gamma_4\psi, \tag{2.9}$$

where the bar means the complex (Hermitian) conjugate, and C is a matrix satisfying

$$\begin{gathered} C^{-1}\gamma_\mu C=-\gamma_\mu{}^T\quad (\mu=1,2,3,4),\\ C^T=-C,\quad \bar{C}=C^{-1}. \end{gathered} \tag{2.10}$$

It is easy to see that charge conjugation changes the parity in c-number theory. In q-number theory, however, charge conjugation leaves the expectation value of parity unchanged, on account of the interchange of ψ with $\bar{\psi}$ in (2.9). Similarly, charge conjugation interchanges the eigenvalues of chirality in c-number theory, but it leaves the expectation value of chirality unchanged in q-number theory.

The "helicity" ("spirality," according to Lee and Yang) operator $\eta(\mathbf{p})$ is defined by[8]

$$\eta(\mathbf{p})=i\gamma_4\gamma_5\gamma_a p_a/|\mathbf{p}|,\quad (a=1,2,3) \tag{2.11}$$

whose eigenvalues are ±1. $\eta(\mathbf{p})$ means twice the spin in the direction of the propagation vector.

The "para-helicity" operator $\lambda(\mathbf{p})$ is defined by

$$\lambda(\mathbf{p})=i\gamma_5\gamma_a l_a,\quad (a=1,2,3) \tag{2.12}$$

with

$$l_a l_a=1,\quad l_a p_a=0,\quad (a=1,2,3). \tag{2.13}$$

Its eigenvalues are again ±1. This quantity means the magnetic moment (in a suitable unit) in a direction perpendicular to the propagation vector.

It is easy to see that[9]

$$[P,\eta(\mathbf{p})]_+=0,\quad [X,\eta(\mathbf{p})]_-=0, \tag{2.14}$$

$$[P,\lambda(\mathbf{p})]_-=0,\quad [X,\lambda(\mathbf{p})]_+=0. \tag{2.15}$$

The last equation in (2.15) shows that $\lambda(\mathbf{p})$ is one of the possible chiral conjugation operators.

As an illustration, let us consider the linear momentum representation of a spinor field, in which the Hamiltonian,

$$H=i\gamma_4\gamma_a p_a+\gamma_4 m,\quad (a=1,2,3) \tag{2.16}$$

the linear momentum $\mathbf{p}$, and the helicity $\eta(\mathbf{p})$ are used to characterize eigenfunctions. If $m=0$, the chirality X commutes with H as well as with $\mathbf{p}$, and $\eta(\mathbf{p})$, and we have

$$\eta=-XH/|\mathbf{p}|,\quad m=0. \tag{2.17}$$

[8] Let us recall that a spiral is a plane-curve while a helix is a space-curve. Therefore, in a 3-dimensional space, the former has no handedness, while the latter has.

[9] Two views are possible regarding the behavior of the sign of l_a with regard to space inversion: It may change with the sign change of p_a, or remain unchanged. The second view is adopted in the first equation of (2.15).

We shall define eigenfunctions $u_i(\mathbf{k})$, $(i=1,2,3,4)$, where $\mathbf{k}$ is an eigenvalue of $\mathbf{p}$, as given in Table I. Expanding as usual in the form:

$$\begin{aligned}\psi(\mathbf{r},t)=V^{-\frac{1}{2}}\textstyle\sum_{\mathbf{k}}\{&[a_1(\mathbf{k})u_1(\mathbf{k})+a_2(\mathbf{k})u_2(\mathbf{k})]\\ &\times\exp(i\mathbf{k}\cdot\mathbf{r}-i|E|t)\\ &+[\bar{b}_1(\mathbf{k})u_3(-\mathbf{k})+\bar{b}_2(\mathbf{k})u_4(-\mathbf{k})]\\ &\qquad\times\exp(-i\mathbf{k}\cdot\mathbf{r}+i|E|t)\},\end{aligned} \tag{2.18}$$

we get

$$\int(\psi,\eta\psi)d\mathbf{r}=\textstyle\sum_{\mathbf{k}}[N_1(\mathbf{k})-N_2(\mathbf{k})+M_1(\mathbf{k})-M_2(\mathbf{k})], \tag{2.19}$$

$$\int(\psi,X\psi)d\mathbf{r}=\textstyle\sum_{\mathbf{k}}[-N_1(\mathbf{k})+N_2(\mathbf{k})-M_1(\mathbf{k})+M_2(\mathbf{k})],\quad (m=0) \tag{2.20}$$

where

$$\begin{gathered} N_{1,2}(\mathbf{k})=\bar{a}_{1,2}(\mathbf{k})a_{1,2}(\mathbf{k}),\\ M_{1,2}(\mathbf{k})=\bar{b}_{1,2}(\mathbf{k})b_{1,2}(\mathbf{k}). \end{gathered} \tag{2.21}$$

The charge conjugation brings $N_1(\mathbf{k})$ to $M_1(\mathbf{k})$, i.e., to a quantum with opposite charge but with the same helicity. Thus:

$$\begin{aligned} C:\quad &N_1(\mathbf{k})\rightleftarrows M_1(\mathbf{k}),\\ &N_2(\mathbf{k})\rightleftarrows M_2(\mathbf{k}). \end{aligned} \tag{2.22}$$

If the mass is finite, the helicity of a particle will depend on the relative velocity of the coordinate system used. However, in the case of a massless particle, the helicity can be considered as an intrinsic property of a particle. (The particle is a "helixon".) For this reason, the antiparticle (in the usual sense) of N_1 should be M_1, and that of N_2 should be M_2.

Space inversion P applied on the Hamiltonian results in a sign change of $\mathbf{p}$, and P anticommutes with momentum $\mathbf{p}$ and helicity $\eta(\mathbf{p})$ (and X). Thus:

$$\begin{aligned} M:\quad &N_1(\mathbf{k})\rightleftarrows N_2(-\mathbf{k}),\\ &M_1(\mathbf{k})\rightleftarrows M_2(-\mathbf{k}). \end{aligned} \tag{2.23}$$

The para-helicity operator $\lambda(\mathbf{p})$ commutes with H (without change of $\mathbf{p}$) and with $\mathbf{p}$, while it anticommutes with $\eta(\mathbf{p})$ and X. Hence, the para-helicity operator as a

TABLE I. The adopted assignment of the u's to the signs of H and $\eta(\mathbf{k})$. The column for X applies only when $m=0$.

	H	$\eta(\mathbf{k})$	X
$u_1(\mathbf{k})$	+	+	−
$u_2(\mathbf{k})$	+	−	+
$u_3(\mathbf{k})$	−	+	+
$u_4(\mathbf{k})$	−	−	−

chiral conjugation has the following effect:

$$\begin{aligned}\lambda:\quad & N_1(\mathbf{k})\rightleftarrows N_2(\mathbf{k}).\\ & M_1(\mathbf{k})\rightleftarrows M_2(\mathbf{k}).\end{aligned}\tag{2.24}$$

Thus, the para-helicity is a particularly desirable chiral conjugation, in the sense that it changes chirality ($m=0$) without changing the momentum.

In the case where $m=0$, it should be noticed that although u_3 and u_4, respectively, correspond to chiralities $+1$ and -1, the corresponding expectation values of chirality are -1 and $+1$, in Eq. (2.20), due to the emission operator instead of absorption operator standing in front of u_3 and u_4 in Eq. (2.18).

In the two-component neutrino theory, we separate out, from a general ψ, two eigenfunctions of chirality:

$$\begin{aligned}\tfrac{1}{2}(1+X)\psi=\psi_{(1)},\quad X\psi_{(1)}=+\psi_{(1)},\\ \tfrac{1}{2}(1-X)\psi=\psi_{(2)},\quad X\psi_{(2)}=-\psi_{(2)}.\end{aligned}\tag{2.25}$$

$\psi_{(1)}$ corresponds to N_2 and M_1, whereas $\psi_{(2)}$ corresponds to N_1 and M_2. Note that this is a classification by the eigenvalues of the u's, and not by the expectation values in the sense of Eq. (2.20). An important hypothesis of the two-component theory is that in nature there exists only one or the other of $\psi_{(1)}$ and $\psi_{(2)}$. Namely, according to a recent experiment, it seems that $\psi_{(2)}$ (i.e., N_1 and M_2), in the right-handed coordinate system, is favored by nature. In the terminology of the two-component theory, if N_1 is a neutrino, then M_2 is called an "antineutrino." However, it should be noticed that this antineutrino is not the antiparticle of a neutrino in the sense of (2.22). A combination of charge conjugation and chiral conjugation brings an allowed particle to another.

If the mass of a particle is not zero, the chirality is not a good quantum number,[2] and therefore the average of chirality can be expected to vanish. This can be seen most easily by using H, $\mathbf{p}$, and para-helicity $\lambda(\mathbf{p})$ to characterize eigenfunctions. Indeed, an eigenfunction $v(\mathbf{p})$ of $\lambda(\mathbf{p})$ will satisfy

$$(v,Xv)=(\lambda v,X\lambda v)=-(v,Xv),\tag{2.26}$$

by virtue of Eq. (2.15), showing that the average chirality is zero. Para-helicity cannot be used for the neutrino, for it anticommutes with X.

Time reversal (R) in field theory is given by

$$\begin{aligned}R:\quad & \psi(\mathbf{r},t)\rightarrow\bar{\psi}(\mathbf{r},-t)\gamma_5 C,\\ & \bar{\psi}(\mathbf{r},t)\rightarrow C^{-1}\gamma_5\psi(\mathbf{r},-t).\end{aligned}\tag{2.27}$$

The meaning of this is that if we have $\bar{\psi}(\mathbf{r},t)O\psi(\mathbf{r},t)$, then we first make $\psi(\mathbf{r},t)O^T\bar{\psi}(\mathbf{r},t)$ and then apply the above transformation. Space-inversion and charge-conjugation are the same in field theory as in the purely mathematical theory.

3. BOSON FIELD WITH INDEFINITE PARITY

We shall begin with a spinor-theoretical consideration. Let S be the usual transformation matrix for the spinors such that

$$\gamma_\mu a_{\mu\nu}=S\gamma_\nu S^{-1},\tag{3.1}$$

where $a_{\mu\nu}$ is the transformation tensor for the coordinates, including inversions. For later use, it should be mentioned that[10]

$$C^{-1}SC=\sigma_t(S^T)^{-1},\tag{3.2}$$

and

$$\gamma_4 S\gamma_4=\sigma_t(\bar{S})^{-1},\tag{3.3}$$

where σ_t is $+1$ or -1, according as the number of simple time-like reflections involved in S is even or odd.[11] We consider a four-four-matrix G with two Dirac indices which transforms as[12]

$$S:\quad G\rightarrow G'=SG\bar{S}.\tag{3.4}$$

If we define another kind of matrix F by

$$F=G\gamma_4,\quad G=F\gamma_4,\tag{3.5}$$

this will transform, by virtue of Eq. (3.3), as

$$S:\quad F\rightarrow F'=\sigma_t SFS^{-1}.\tag{3.6}$$

The transformation rules of G and F are, respectively, that of $\varphi\bar{\psi}$ and that of $\varphi\psi^\dagger$, where $\psi^\dagger=\bar{\psi}\gamma_4$, and the bar means the complex (Hermitian) conjugate.

Expanding an arbitrary G by the sixteen bases of the γ system in the form

$$G=(S+iP\gamma_5+iV_\mu\gamma_\mu+iA_\mu\gamma_5\gamma_\mu+iT_{\mu\nu}\gamma_{[\mu}\gamma_{\nu]})\gamma_4,\tag{3.7}$$

we can easily see, with the help of Eqs. (3.1) and (3.4) that S, P, V, A, and T, respectively, transform like a scalar (pseudoscalar of the second kind), a pseudoscalar (of the third kind), a vector (pseudovector of the second kind), an axial vector (pseudovector of the third kind), and a tensor (pseudotensor of the second kind).[13] The last term can of course be expressed as a pseudotensor (of the third kind) by interchanging space-space components with time-space components. Thus,

$$T_{\mu\nu}\gamma_{[\mu}\gamma_{\nu]}=T_{\kappa\lambda}'\gamma_5\gamma_{[\kappa}\gamma_{\lambda]},\quad T_{\kappa\lambda}'=-T_{\mu\nu}\epsilon_{\mu\nu\kappa\lambda},\tag{3.8}$$

where $\epsilon_{\mu\nu\kappa\lambda}$ is $+1$ or -1 according as (μ,ν,κ,λ) is an even or odd permutation of (1, 2, 3, 4).

The Hermitian conjugate of G is

$$\bar{G}=(\bar{S}+i\bar{P}\gamma_5+i\bar{V}_\mu\gamma_\mu+i\bar{A}_\mu\gamma_5\gamma_\mu+i\bar{T}_{\mu\nu}\gamma_{[\mu}\gamma_{\nu]})\gamma_4.\tag{3.9}$$

[10] S. Watanabe, Revs. Modern Phys. **27**, 40 (1955), Eqs. (2.32) and (2.33).

[11] S. Watanabe, Phys. Rev. **84**, 1008 (1951), Eqs. (A.4) and (A.7).

[12] The use of spinor matrices to represent tensorial quantities originates from É. Cartan, Bull. Soc. Math. France **41**, 53 (1913). For an application of this instrument in the field theory of bosons, see S. Watanabe, Sci. Papers Inst. Phys. Chem. Research (Tokyo) **39**, 157 (1941).

[13] See, for instance, Table III of reference 11. Alterations ensuing from the field theory pertain only to time reversal, and must be the same as indicated in Table VI in S. Watanabe, Revs. Modern Phys. **27**, 26 (1955).

The imaginary unit is attached in some of the terms in Eq. (3.7), so that G may become Hermitian if S, P, V, A, and T are all real. This, however, implies by no means that an electrically neutral field can be expressed by a Hermitian G. We shall later see the reason for this.

From (3.4) we see that $\bar{G}$ transforms exactly in the same way as G itself. Hence, $\bar{G}\gamma_4$ transforms as an F matrix:

$$S:\quad \bar{G}\gamma_4 \to \sigma_t S(\bar{G}\gamma_4)S^{-1}. \tag{3.10}$$

Writing

$$\mathrm{sp}A = \tfrac{1}{4}\sum_i A_{ii}, \tag{3.11}$$

we see from Eq. (3.6) that the spur of a product of n F matrices transforms as

$$S:\quad \mathrm{sp}(F_1F_2\cdots F_n) \to (\sigma_t)^n\,\mathrm{sp}(F_1F_2\cdots F_n), \tag{3.12}$$

i.e., this is a scalar, except for the factor $(\sigma_t)^n$. By virtue of Eq. (3.10), we see also that

$$S:\quad \mathrm{sp}(\bar{G}\gamma_4G\gamma_4) \to \mathrm{sp}(\bar{G}\gamma_4G\gamma_4), \tag{3.13}$$

showing that this quantity is a regular scalar. Expanding G and $\bar{G}$ as in Eqs. (3.7) and (3.9), we have indeed

$$\mathrm{sp}(\bar{G}\gamma_4G\gamma_4) = \bar{S}S - \bar{P}P - \bar{V}_\mu V_\mu + \bar{A}_\mu A_\mu + \bar{T}_{\mu\nu}T_{\mu\nu}. \tag{3.14}$$

As far as the scalar part (S and P) is concerned, it is sometimes more convenient to use

$$\mathrm{sp}(\bar{G}G) = \bar{S}S + \bar{P}P, \tag{3.15}$$

which is not only a regular scalar, but also positive-definite.

The tensorial components involved in Eq. (3.7) can be divided into two classes, one with positive parity and the other with negative parity:

$$G = [(G+\gamma_4G\gamma_4)/2] + [(G-\gamma_4G\gamma_4)/2] = G^{(+)} + G^{(-)}, \tag{3.16}$$

with

$$G^{(+)} = \gamma_4G^{(+)}\gamma_4, \quad G^{(-)} = -\gamma_4G^{(-)}\gamma_4. \tag{3.17}$$

The parity here considered refers to each component, and the parity of a field is, as is well-known, defined by the parity of a certain component belonging to that field. Decomposition of G in five terms in Eq. (3.7) corresponds to a classification with respect to the rank and the field parity.

If a field satisfies

$$\gamma_5G = c_1G, \quad (c_1 = \pm 1), \tag{3.18}$$

we say that G has "first chirality" equal to c_1, which is $+1$ or -1. Any arbitrary G can be decomposed into two parts, one with positive c_1 and the other with negative c_1:

$$G = [(1+\gamma_5)/2]G + [(1-\gamma_5)/2]G = {}_{(+)}G + {}_{(-)}G, \tag{3.19}$$

with

$$\gamma_{5\,(+)}G = {}_{(+)}G, \quad \gamma_{5\,(-)}G = -{}_{(-)}G. \tag{3.20}$$

Applying Eq. (3.19) on Eq. (3.7), we have

$$\begin{aligned}{}_{(\pm)}G = [&(S\pm iP) + i(P\mp iS)\gamma_5 + i(V_\mu \pm A_\mu)\gamma_\mu \\ &+ i(A_\mu \pm V_\mu)\gamma_5\gamma_\mu + i(T_{\mu\nu} \pm T_{\mu\nu}{}')\gamma_{[\mu}\gamma_{\nu]}]\gamma_4/2.\end{aligned} \tag{3.21}$$

In a similar way, we define the "second chirality" c_2 by

$$G\gamma_5 = c_2G. \tag{3.22}$$

In lieu of Eqs. (3.19) through (3.21), we have here

$$G = G[(1+\gamma_5)/2] + G[(1-\gamma_5)/2] = G_{(+)} + G_{(-)}, \tag{3.23}$$

with

$$G_{(+)}\gamma_5 = G_{(+)}, \quad G_{(-)}\gamma_5 = -G_{(-)}, \tag{3.24}$$

and

$$\begin{aligned}G_{(\pm)} = [&(S\mp iP) + i(P\pm iS)\gamma_5 + i(V_\mu \pm A_\mu)\gamma_\mu \\ &+ i(A_\mu \pm V_\mu)\gamma_5\gamma_\mu + i(T_{\mu\nu} \mp T_{\mu\nu}{}')\gamma_{[\mu}\gamma_{\nu]}]\gamma_4/2.\end{aligned} \tag{3.25}$$

When a given G does not satisfy Eq. (3.18), we say that $c_1 = 0$, as we did in the case of a spinor. Similarly, if c_2 in Eq. (3.22) is indefinite, we say that $c_2 = 0$. We define the total chirality by

$$c = c_1 - c_2, \tag{3.26}$$

which could be ± 2, ± 1 or 0. By comparing Eq. (3.21) with Eq. (3.25), we can easily see that if one of c_1 and c_2 is definite (i.e., $+1$ or -1) then the other is also definite, in each of the scalar, vector, and tensor parts. More precisely, we get for the scalar and tensor parts $c_1 = -c_2$, and for the vector part $c_1 = c_2$. Hence, for a field with definite chirality, the total chirality is ± 2 in the scalar and tensor cases, and 0 in the vector case. To deal with a vector field, it might be more convenient to define the total chirality by $c = c_1 + c_2$ instead of (3.26).

It should be noted that the parity transformation (multiplication by γ_4 from both sides) changes the sign of c_1 as well as that of c_2. Similarly, any transformation that anticommutes with γ_5, such as λ of the last section, changes the sign of both c_1 and c_2. Such a transformation is a chiral conjugation for boson fields.

Charge conjugation of G should be defined by

$$C:\quad G \to -(C^{-1}\gamma_4\bar{G}\gamma_4C)^T, \tag{3.27}$$

where the superscript T refers only to the Dirac indices. The quantity $(C^{-1}\gamma_4\bar{G}\gamma_4C)^T$ is the only known quantity, except $\bar{G}$, that transforms exactly as G itself. For this reason alone, it is already plausible that the transformation (3.27) (except for the sign) is the right choice. A more convincing argument is provided by the assumption that G transforms like $\varphi\bar{\psi}$, not only for the S transformation, but also for charge conjugation which was given in (2.9). This assumption yields (3.27), the sign being determined by the anticommutation relation of spinors. (This sign, however, does not play any important role in the following).

A field with definite parity can be expressed by any one of the five terms in Eq. (3.7). Such a field has necessarily a vanishing chirality, for we see from Eqs. (3.21) and (3.25) that we have to mix a regular part and a pseudo part to engender a definite chirality. For instance, a pseudoscalar field (pion) can be expressed by

$$G_\pi = ib\gamma_5\gamma_4, \tag{3.28}$$

which obviously satisfies neither Eq. (3.18) nor Eq. (3.22). By our usage of the word, the pion thus has chirality zero.

If we apply charge conjugation (3.27) on Eq. (3.28), we obtain

$$C:\quad ib\gamma_5\gamma_4 \rightarrow i\bar{b}\gamma_5\gamma_4. \tag{3.29}$$

Thus the antiparticle of G_π is given by

$$G_{\bar{\pi}} = i\bar{b}\gamma_5\gamma_4. \tag{3.30}$$

This of course has the same parity, and we see, further, that we need only take the complex (Hermitian) conjugate of b to obtain the antipion. In the case of a real field, G_π and $G_{\bar{\pi}}$ are the same.

A scalar field with definite chirality has to be expressed as a special superposition of S and P, as specified by Eqs. (3.21) and (3.25). For definiteness, let us assign (say, in a right-handed coordinate system) to the K particle the matrix:

$$G_K = a(1+\gamma_5)\gamma_4/2, \quad c_1 = -c_2 = 1, \quad c = 2. \tag{3.31}$$

By charge conjugation (3.27), this expression becomes

$$G_{\bar{K}} = \bar{a}(1-\gamma_5)\gamma_4/2, \quad c_1 = -c_2 = -1, \quad c = -2, \tag{3.32}$$

which should be called anti-K particle. The names "particle" and "antiparticle" are interchangeable, but we shall stick to Eqs. (3.31) and (3.32) for definiteness.

By space inversion, the matrix (3.31) will transform as

$$M:\quad a(1+\gamma_5)\gamma_4/2 \rightarrow a(1-\gamma_5)\gamma_4/2, \tag{3.33}$$

where the sign-change of $\mathbf{r}$ in the argument of a is omitted for simplicity. Any chiral conjugation follows the pattern of (3.33). In analogy to the neutrino theory, we assume that G_K and $G_{\bar{K}}$ given in Eqs. (3.31) and (3.32) are favored by nature, and their chiral conjugates,

$$a(1-\gamma_5)\gamma_4/2, \quad c_1 = -c_2 = -1, \quad c = -2, \tag{3.34}$$

and

$$\bar{a}(1+\gamma_5)\gamma_4/2, \quad c_1 = -c_2 = 1, \quad c = 2, \tag{3.35}$$

do not exist in nature.[14]

One of the most important features of the charge conjugation of a K particle is that not only the field component a passes to $\bar{a}$, but also the mixing ratio of S and P is changed. If G_K represents a positive K particle then $G_{\bar{K}}$ should represent a negative K particle. A naïve theory would describe the anti-K by $\bar{a}$ when the K is described by a. In the present theory, such a simple interchange of a and $\bar{a}$ is the result of a combination of charge conjugation and chiral conjugation, and the resulting matrix becomes a forbidden one given in (3.34), (3.35). This is a different situation, compared with the neutrino theory, in which a combination of charge conjugation and chiral conjugation transformed an allowed particle into another allowed particle.

Since (3.31) and (3.34) on one hand, and (3.32) and (3.35) on the other, would generate the same electric current, the foregoing rule can also be formulated as follows: A positive K particle with chirality -2, and a negative K particle with chirality $+2$ are forbidden, while the other alternatives are allowed.

Let us note that $\bar{G}_K = G_{\bar{K}}$. Hence, G_K, which is an eigenstate of chirality, can never be Hermitian, no matter what condition may be imposed on a. We observe also that the neutral K field and the neutral anti-K field can never be the same field.[15]

It is true that it is, in principle, possible to assign to the K and anti-K the following matrices instead of (3.31) and (3.32):

$$\begin{aligned} G_K &= a(1+\gamma_5)\gamma_4/2, \quad c_1 = -c_2 = 1, \quad c = +2, \\ G_{\bar{K}} &= \bar{a}(1+\gamma_5)\gamma_4/2, \quad c_1 = -c_2 = 1, \quad c = +2. \end{aligned} \tag{3.36}$$

In this assignment, the allowed particles are characterized by $c=2$, while the forbidden particles are characterized by $c=-2$. The combination of charge conjugation and chiral conjugation transforms an allowed particle into another, as in the neutrino theory. However, this assignment seems to be unnatural for it destroys the conformity with other charged fields in regard to charge conjugation. In the following, unless otherwise noted, we use the original assignment (3.31) and (3.32). The assignment (3.36), which has also certain merits, will be discussed separately in Appendix 2.

The field theoretical transformation of G for space inversion (M) and time reversal (R) can be inferred from the analogy with $\varphi\bar{\psi}$, with the help of Eqs. (2.1) and (2.28).

$$\begin{aligned} M:&\quad G(\mathbf{r}) \rightarrow \gamma_4 G(-\mathbf{r})\gamma_4, \\ R:&\quad G(t) \rightarrow \gamma_5 (C^{-1}\bar{G}(-t)C)^T\gamma_5. \end{aligned} \tag{3.37}$$

The space inversion is the same as in the mathematical spinor theory. In the time reversal, we have to reverse

[14] In the usual way of expressing tensorial (boson) fields, one might write $S+P$ and $S-P$ (or $S+iP$ and $S-iP$) to denote two different chiralities. However, by such expressions, one can never tell which one has a positive chirality and which one has a negative chirality. Here lies another advantage of the present formalism, which also has the merit of bridging the boson case with the fermion case.

[15] In this point, the present theory is similar to Gell-Mann and Pais' idea: M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955). However, the present theory has more specific information about the behavior of K particles with regard to space inversion. The present theory has also some similitude with Schwinger's theory, insofar as a combination of S and P is considered. J. Schwinger, Phys. Rev. **104**, 1164 (1956). However, the basic philosophy is different, since Schwinger's theory intends to preserve the invariance under space inversion, whereas the vital assumption of the present theory lies precisely in the abandonment of invariance under space inversion.

the order of the factors, if a product is involved. The superscript T in Eq. (3.37) means here again the transpose with respect to the Dirac indices only. By the combination of space inversion, time reversal, and charge conjugation, the components of G become[16]

$$C\times R\times M:\quad \begin{cases} S, P, T\to S, P, T; \\ V, A\to -V, -A. \end{cases} \tag{3.38}$$

Similarly, G_K returns to its original form by $C\times R\times M$.

4. FREE AND INTERACTION LAGRANGEANS FOR K PARTICLES

The invariant free Lagrangean can be written, with the help of Eq. (3.15), in the form:

$$\mathfrak{L}_F\sim-\int \mathrm{sp}(\bar{G}WG)d\mathbf{r}, \tag{4.1}$$

with

$$W=(-\partial_\mu\partial_\mu+m^2)I=(\overleftarrow{\partial}_\mu\partial_\mu+m^2)I, \tag{4.2}$$

where $\overleftarrow{\partial}_\mu$ means the differentiation of the factor standing to its left. (See Appendix 1 for the quantization of G_K.) We do not use Eq. (3.14) here, for it vanishes if we put (3.31). The ensuing wave equation is

$$WS=0,\quad \text{and}\quad WP=0; \tag{4.3}$$

therefore also

$$Wa=0. \tag{4.4}$$

The free Hamiltonian is then

$$H_F\sim\int \mathrm{sp}[\bar{G}(-2\overleftarrow{\partial}_4\partial_4+W)G)]d\mathbf{r}. \tag{4.5}$$

It is of importance to notice that since γ_5 commutes with $(2\partial_4\partial_4+W)$, the chirality is a good quantum number even if $m\neq 0$.

The current vector becomes

$$i_\mu\sim i\,\mathrm{sp}[\bar{G}(\overleftarrow{\partial}_\mu-\partial_\mu)G]. \tag{4.6}$$

Putting G_K, (3.31), in Eq. (4.6), one obtains

$$i_\mu\sim i[(\partial_\mu\bar{a})a-\bar{a}(\partial_\mu a)]. \tag{4.7}$$

This expression changes its sign if we put $G_{\bar{K}}$, (3.32), instead of G_K, (3.31). It would vanish if a were real (Hermitian), but we do not assume this even for the neutral field, for which i_μ will then bear the meaning of chiral current density (see Appendix 1). In the case of a charged field, i_μ means both electric and chiral currents. Equations (3.31) and (3.34) would give the same value for (4.7).

As far as interaction is concerned, it is not the intention of the present paper to investigate exhaustively all the decay modes or to calculate the lifetimes. To get a glimpse of the behavior of the interaction terms toward charge conjugation, time reversal, and space inversion, let us limit ourselves to the K-pi interaction.

To express an interaction Lagrangean among several K particles, anti-K particles, and pions, we notice first that the spur of a product of several G matrices does not form a scalar. See Eq. (3.4). However, by virtue of Eq. (3.12), we can use a product of F matrices. The F matrices corresponding to Eqs. (3.31), (3.32), and (3.28) are

$$F_K=a(1+\gamma_5)/2,\quad F_{\bar{K}}=\bar{a}(1-\gamma_5)/2, \tag{4.8}$$

$$F_\pi=ib\gamma_5. \tag{4.9}$$

The transformation rules of an F matrix for charge conjugation, space inversion, and time reversal are

$$C:\quad F\to\gamma_4(C^{-1}\bar{F}C)^T\gamma_4, \tag{4.10}$$

$$M:\quad F(\mathbf{r})\to\gamma_4F(-\mathbf{r})\gamma_4, \tag{4.11}$$

$$R:\quad F(t)\to\gamma_5(C^{-1}\bar{F}(-t)C)^T\gamma_5. \tag{4.12}$$

Then the Lagrangean involving n K's, l anti-K's, and m pions would be of the form:

$$\mathfrak{L}_I=\int L_I d\mathbf{r};$$

$$L_I=f\,\mathrm{sp}(F_KF_K\cdots F_{\bar{K}}F_{\bar{K}}\cdots F_\pi F_\pi\cdots)+\text{Hermitian conjugate}. \tag{4.13}$$

However, since the product of F_K and $F_{\bar{K}}$ vanishes, coexistence of F_K and $F_{\bar{K}}$ in Eq. (4.13) is impossible.[17] Let us then take the case of n K's and m pions.

$$L_I=f\,\mathrm{sp}(F_KF_K\cdots F_\pi F_\pi\cdots)+\text{H.c.}, \tag{4.14}$$

$$=fa^n(ib)^m/2+\text{H.c.},\qquad \text{if}\quad n\neq 0, \tag{4.15}$$

$$=f(ib)^m[1+(-1)^m]/2+\text{H.c.},\qquad \text{if}\quad n=0. \tag{4.16}$$

Since the transformation (4.10) applied on Eq. (4.9) results just in replacement of b by $\bar{b}$, no distinction between a pion and an antipion has been made in (4.14) for the sake of simplicity. There can actually be neutral and positive K particles and pions of different kinds of charge in Eq. (4.14), insofar as the total charge is conserved.

Equation (4.16) simply means that an odd number of pions alone cannot interact among themselves. If a K (or K's) intervenes, the parity of number of pions

[16] The results in Eq. (3.38) can also be derived directly by multiplying three signs, ρ_C, ρ_R, and ρ_M listed in Table III of reference 11. This is also in agreement with the rule given in Pauli's article in *Niels Bohr and Development of Physics* (edited by Pergamon Press, London, 1955). See also Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957). If we insert phase-factors, $\exp(i\alpha)$, $\exp(i\beta)$, and $\exp(i\gamma)$ in C, R, and M, respectively, the relation in (3.38) will remain valid only if $\beta=\alpha+\gamma$, mod. 2π. G_K of Eq. (3.31) becomes $G_{\bar{K}}$ of Eq. (3.32) by space-and-time inversion only when $\beta=\gamma$, mod. 2π. Similar remarks can be made regarding other transformations too.

[17] This by no means implies that an interaction like $(\pi-K-\bar{K})$ is forbidden. Such an interaction is possible through a Lagrangean of the type: $\mathrm{sp}F_K\,\mathrm{sp}(F_{\bar{K}}F_\pi)+\mathrm{sp}F_{\bar{K}}\,\mathrm{sp}(F_{\bar{\pi}})$.

involved does not matter any longer. Since Eq. (4.14) is an invariant, it naturally comes back to itself by space inversion (4.11). Thus, the left-right asymmetry of the K particle disappears formally from the surface. However, if a K particle is represented as (3.31) in a coordinate system, it is supposed to be represented as (3.34) in the space-inverted coordinate system, although this becomes immaterial in an actual calculation.

It can easily be seen that the Hermitian conjugate of the term explicitly written out in Eq. (4.14) can be written

$$f^* \, \mathrm{sp}(F_{\bar{K}}F_{\bar{K}}\cdots F_{\bar{\pi}}F_{\bar{\pi}}\cdots). \tag{4.17}$$

By charge conjugation (4.10), the term in (4.14) will become the term in (4.17). If the interaction constant f is real, the invariance for charge-conjugation is assured. By time reversal (4.12), each F does not become the F of the antiparticle, but, through the process of diagonal summation, the term in (4.14) becomes the term in (4.17) by time reversal. Here again, on the assumption that f is real, invariance is guaranteed.

The above conclusion is based on the assignment (3.31), (3.32). For discussions of the assignment (3.36), see Appendix 2.

For an interaction in which a boson F_1 and a spinor particle ψ_1 disappear and another spinor particle ψ_2 appears, one may write

$$\mathrm{sp}(F_1F_3), \tag{4.18}$$

with

$$F_3=\psi_1\psi_2^{\dagger}. \tag{4.19}$$

This expression automatically selects the invariant combination of the components of F_1, ψ_1, and $\psi_2^{\dagger}$. In a similar way, one can write an interaction Lagrangean for a more complicated process.[18]

5. POSSIBLE RELATION BETWEEN CHIRALITY AND STRANGENESS

The matrix expression of a boson field is suggestive of a fusion-theoretical interpretation of a boson as being made out of two spinor particles.[19] However, unless we know the nature of the cohesive force, we cannot even write the Bethe-Salpeter equation. There is no guarantee that the chirality of a boson as defined in the foregoing can be derived by some kind of addition of the chirality of each constituent spinor particle. This would evidently depend on the nature of the cohesive field. For this reason, we do not indulge in any further speculation about the structure of a K particle. Instead, we shall try to picture all the particles as heavy as, or heavier than the pion as composite particles built out of nucleons and already-existing K particles.[20] The guiding principle in determining the composition of each particle is to reproduce its established isotopic spin, the third component of its isotopic spin, and its heavy-particle number by the smallest possible number of constituent particles, using the established values of these quantities of the nucleon and the K particle.

TABLE II. All particles are assumed to be composed of nucleons, antinucleons, K particles, anti-K particles, in such a way that isotopic data and conservation of heavy particles are satisfied. The algebraic sum of chiralities of constituent particles turn out to be twice the "strangeness."

Particle	Composition	I	I_3	Q	Chirality
$K^0(\bar{K}^0)$		$\frac{1}{2}$	$-\frac{1}{2}(+\frac{1}{2})$	0	$+2(-2)$
$K^+(K^-)$		$\frac{1}{2}$	$+\frac{1}{2}(-\frac{1}{2})$	$+(-)$	$+2(-2)$
$n(\bar{n})$		$\frac{1}{2}$	$-\frac{1}{2}(+\frac{1}{2})$	0	0
$p(\bar{p})$		$\frac{1}{2}$	$+\frac{1}{2}(-\frac{1}{2})$	$+(-)$	0
π^+	$p\bar{n}$	1	$+1$	$+$	0
π^-	$\bar{p}n$	1	-1	$-$	0
π^0	$\bar{n}n$, $\bar{p}p$	1	0	0	0
$\Lambda^0(\bar{\Lambda}^0)$	$n\bar{K}^0$, pK^- ($\bar{n}K^0$, pK^+)	0	0	0	$-2(+2)$
$\Sigma^+(\bar{\Sigma}^+)$	$p\bar{K}^0(\bar{p}K^0)$	1	$+1(-1)$	$+(-)$	$-2(+2)$
$\Sigma^-(\bar{\Sigma}^-)$	$nK^-(\bar{n}K^+)$	1	$-1(+1)$	$-(+)$	$-2(+2)$
$\Sigma^0(\bar{\Sigma}^0)$	$n\bar{K}^0$, pK^- ($\bar{n}K^0$, $\bar{p}K^+$)	1	0	0	$-2(+2)$
$\Xi^-(\bar{\Xi}^-)$	$n\bar{K}^0K^-$, pK^-K^- ($\bar{n}K^0K^+$, $\bar{p}K^+K^+$)	$\frac{1}{2}$	$-\frac{1}{2}(+\frac{1}{2})$	$-(+)$	$-4(+4)$
$\Xi^0(\bar{\Xi}^0)$	$n\bar{K}^0\bar{K}^0$, $pK^-\bar{K}^0$ (nK^0K^0, $\bar{p}K^+K^0$)	$\frac{1}{2}$	$+\frac{1}{2}(-\frac{1}{2})$	0	$-4(+4)$

First, the pion can be considered as a particle similar to the K's, but only in an eigenstate of parity instead of in an eigenstate of chirality. The pion can also be pictured as an isotopic triplet composed of a nucleon and an antinucleon.

The Λ particle is an isotopic singlet consisting of a nucleon and an anti-K particle. The Σ particle is an isotopic triplet of a nucleon and an anti-K particle. The Ξ particle is an isotopic singlet, composite of a nucleon and two anti-K particles. After having made such an assignment, we calculate the algebraic sum of the chiralities of K particles and anti-K particles involved. The chirality of a nucleon is zero according to Sec. 2. The total chirality thus computed turns out to be exactly twice the strangeness quantum number. See Table II.

It must be admitted that the above-sketched composition scheme may not be a unique one, and that the

[18] It seems natural to assume that strong interactions are parity-conserving. In order to make the present formalism compatible with this assumption, one has only to assign a special parity-property to the partner of the K particle in its "associated" production. Consider, for instance, the production process: pion $+$nucleon$\rightarrow K+\Lambda$. The parity-property of Λ should then be equivalent to that of a hypothetical compound system of (nucleon $+K'$), where K' (chirality -2) is a hypothetical (forbidden) particle of Eq. (3.34), which is the space-inverted image of the K particle (chirality $+2$). The negative parity of the pion can be absorbed in the orbital motion of the $(K+\Lambda)$ system. Then, the parity conserves. If one wants to give a realistic meaning to this interpretation, the relation $\Lambda^0=n\bar{K}^0$ of Table II should be replaced by $n\bar{K}^{0\prime}$, which has also chirality -2. However, the K' need not exist as an independent particle in nature.

[19] L. De Broglie, Compt. rend. **198**, 135 (1934), and subsequent writings; W. Heisenberg, Z. Naturforsch. **5a**, 251 (1950), and subsequent papers. See also S. Watanabe, Phys. Rev. **91**, 771 (1953).

[20] A similar but not exactly the same view was proposed by M. Goldhaber, Phys. Rev. **101**, 433 (1956). Goldhaber's "dionic charge" is given a physical meaning in the present paper.

algebraic addition of chirality numbers has no solid justification. However, the fact alone that a new variable has been introduced on a solid mathematical basis, having opposite values for the K and anti-K (whose strangeness quantum numbers are ± 1) is already suggestive of some connection between chirality and strangeness.

Strangeness, so far, has received no explanation, except by Pais and Gell-Mann's theory and d'Espagnat and Prentki's theory.[21] According to d'Espagnat and Prentki (and also Racah), the strangeness is connected with the symmetry property in the isotopic spin space. If the present theory and the theory of d'Espagnat and Prentki are both correct in their essence, then it would mean that there exists a close relationship between the symmetry property in the outer space and that in the inner (isotopic) space. It is interesting to note that Yukawa, from an entirely different approach, also suspected a relationship of similar nature.[22]

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APPENDIX 1

Physical quantities Q (energy-momentum density, current density, etc.) have the form:

$$Q \sim \mathrm{sp}(\bar{G}OG). \tag{A.1}$$

If G here represents the K wave function, then $\bar{G}$ represents the anti-K wave function. Indeed, from Eqs. (3.31) and (3.32) we see that

$$\bar{G}_K = G_{\bar{K}}. \tag{A.2}$$

In the case of pions, if an expression like (A.1) is used for the charged field, we should take one half of the expression for the neutral field. In the case of K particles, however, we have to use the same expression for both charged and neutral fields, for the neutral K and the neutral anti-K are two different particles.

Putting Eqs. (3.31) and (3.32) in (A.1), one obtains

$$\mathrm{sp}(G_{\bar{K}}OG_K) = \tfrac{1}{4}\bar{a}Oa. \tag{A.3}$$

Therefore, we can use the usual method of quantization for a and $\bar{a}$. In other words, if we expand G_K as

$$G_K(\mathbf{r},t) = \sum_{\mathbf{k}} (2V|E|)^{-\frac{1}{2}}[\alpha(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{r} - i|E|t) + \bar{\beta}(\mathbf{k}) \exp(-i\mathbf{k}\cdot\mathbf{r} + i|E|t)](1+\gamma_5)\gamma_4/2, \tag{A.4}$$

and $G_{\bar{K}}$ as the Hermitian conjugate of (A.4), we can interpret $N(\mathbf{k}) = \bar{\alpha}(\mathbf{k})\alpha(\mathbf{k})$ and $M(\mathbf{k}) = \bar{\beta}(\mathbf{k})\beta(\mathbf{k})$ as K particles and anti-K particles, respectively, having momentum $\mathbf{k}$.

If we assume a in (3.31) to be Hermitian, then β will become identical with α. In order to differentiate the neutral K from the neutral anti-K, we have to assume a not to be Hermitian. The quantity (4.7) will be proportional to $N-M$, which in the case of a neutral field may be interpreted as the current of chirality, for N and M correspond to the opposite chiralities. In the case of a charged field, the quantity (4.7) means both electric and chiral currents. To avoid an electric interaction of the neutral field, one needs only omit the electric interaction term from the Lagrangean. Incidentally, in the neutrino theory, the current expression can be interpreted as a current of helicity.

APPENDIX 2

The alternative assignment (3.36) has a welcome consequence: the form of physical laws remains unchanged by time reversal.

Thus, we shall study in this Appendix some of the results that ensue from the assumption that the allowed K particles and anti-K particles are represented by

$$G_K = a(1+\gamma_5)\gamma_4/2, \quad c=2, \qquad G_{\bar{K}} = \bar{a}(1+\gamma_5)\gamma_4/2, \quad c=2, \tag{A.5}$$

while

$$G = a(1-\gamma_5)\gamma_4/2, \quad c=-2, \qquad G = \bar{a}(1-\gamma_5)\gamma_4/2, \quad c=-2, \tag{A.6}$$

are forbidden, say in the right-handed coordinate system.

This choice, (A.5), is analogous to the two-component neutrino theory in the sense that a combination of charge conjugation, (3.27), and chiral conjugation, such as (3.37), can convert (by a suitable choice of phase-factors) an allowed particle to another allowed particle. For instance, starting from G_K one reaches $G_{\bar{K}}$ by $M\times C$, as one has

$$\begin{aligned} C:&\quad G_K = a(1+\gamma_5)\gamma_4/2 \rightarrow \bar{a}(1-\gamma_5)\gamma_4/2, \\ M:&\quad \bar{a}(1-\gamma_5)\gamma_4/2 \rightarrow \bar{a}(1+\gamma_5)\gamma_4/2 = G_{\bar{K}}. \end{aligned} \tag{A.7}$$

In view of the general rule given in (3.38), this means that an allowed particle becomes another allowed particle also by time reversal. This can also be verified directly by applying (R), (3.37), on (A.5).

In the same way that the so-called antineutrino in the two-component theory is not the antiparticle, in the proper sense, of the neutrino, the anti-K particle $G_{\bar{K}}$, according to (A.5), is not the antiparticle, in the usual sense, of the K particle G_K. In this respect, the present assignment deviates from the Gell-Mann-Pais theory.[4]

In terms of F matrices, the assumption (A.5) is equivalent to:

$$F_K = a(1+\gamma_5)/2, \qquad F_{\bar{K}} = \bar{a}(1+\gamma_5)/2. \tag{A.8}$$

[21] A. Pais, Physica **19**, 869 (1953); M. Gell-Mann, *Proceedings of the Glasgow Conference* (Pergamon Press, London, 1955). B. d'Espagnat and J. Prentki, Phys. Rev. **99**, 328 (1955); **102**, 1684 (1956). G. Racah, Nuclear Phys. **1**, 302 (1956).

[22] H. Yukawa, *Proceedings of the International Conference on Theoretical Physics, Seattle, 1956* (to be published).

We have here, instead of (A.2),

$$\bar{F}_K = F_{\bar{K}}. \tag{A.9}$$

Hence, $\mathrm{sp}(F_{\bar{K}} F_K) = \bar{a}a/2$ is not only invariant but also positive-definite. Thus, we can write the free Lagrangean in the form:

$$L_F \sim -\int \mathrm{sp}(F_{\bar{K}} W F_K) d\mathbf{r}, \tag{A.10}$$

in lieu of (4.1). Considerations regarding the Hamiltonian, the current vector, etc., follow the same procedure as in Sec. 4, except that the F's are substituted for the G's.

The interaction Lagrangean involving n K's, l anti-K's, and m pions may be written in the form:

$$\begin{aligned} L_I &= f\,\mathrm{sp}(F_K F_K \cdots F_{\bar{K}} F_{\bar{K}} \cdots F_\pi F_\pi \cdots) + \mathrm{H.c.} \\ &= f a^n \bar{a}^l (ib)^m/2 + \mathrm{H.c.} \quad (n \neq 0,\ l \neq 0). \end{aligned} \tag{A.11}$$

Creation of pairs of K and $\bar{K}$ is included. In particular, if $n=l$, we have only pairs created. The Hermitian conjugate omitted in (A.11) is

$$\begin{aligned} \mathrm{H.c.} &= (-1)^m f^* \,\mathrm{sp}(F_{\bar{K}} F_{\bar{K}} \cdots F_K F_K \cdots F_{\bar{\pi}} F_{\bar{\pi}} \cdots) \\ &= f^* \bar{a}^n a^l (-i\bar{b})^m/2. \end{aligned} \tag{A.12}$$

By space inversion, the term in (A.11) returns to itself. By charge conjugation, as well as by time reversal, the term in (A.11) passes to the term in (A.12), and vice versa. By these transformations, each F is transformed according to (4.10), (4.11), (4.12).

The quantization of the K field can be done by expanding F_K as

$$\begin{aligned} F_K(\mathbf{r},t) = \textstyle\sum_{\mathbf{k}} (2V|E|)^{-\frac{1}{2}} [&\alpha(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{r} - i|E|t) \\ &+ \bar{\beta}(\mathbf{k}) \exp(-i\mathbf{k}\cdot\mathbf{r} + i|E|t)](1+\gamma_5)/2, \end{aligned} \tag{A.13}$$

and $F_{\bar{K}}$ as the Hermitian conjugate of (A.13). Assuming that F_K is not Hermitian, even for the neutral K field, we can interpret $N = \bar{\alpha}\alpha$ and $M = \bar{\beta}\beta$ as particle numbers of the K and of the anti-K, respectively.

We can now interpret the current

$$i_\mu \sim \mathrm{sp}[F_{\bar{K}} (\overleftarrow{\partial}_\mu - \partial_\mu) F_K] \tag{A.14}$$

as the current of "strangeness." This amounts to defining the strangeness of the particles N as one-half their chirality ($c=2$), and defining the strangeness of the antiparticles M as one-half the negative of their chirality ($c=2$). In the case of charged field, we can say that strangeness is one-half the product of chirality and charge. By this definition, the strangeness of the K-particle and that of the anti-K particle become, respectively, $+1$ and -1. This definition is not as direct as in the original assignment, (3.31) and (3.32), yet it is mathematically meaningful.

The compound picture of heavy particles explained in Sec. 5 is still feasible in this alternative assignment; we need only substitute twice the strangeness defined in the preceding paragraph for the chirality in Table II.