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Role of Strong Interactions in Decay Processes

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An attack is made on the problem of determining which are the primary interactions that contribute to decay processes. It is necessary first to understand the rôle of the strong interactions in these processes; this has proven difficult in the past because of the appearance of infinities. It is shown that all infinities appearing in decay processes involving nucleons, pions, photons, and one lepton pair may be removed by renormalization to all orders in the strong and electromagnetic coupling constants. The necessary and sufficient condition for renormalizability is that the primary interactions that actually exist in nature form one of certain subclasses of a class of fourteen possible primary interactions. In particular, from this point of view it is incorrect to treat the π-meson decay as proceeding via Fermi interactions only. Two incidental results of this work are that the use of perturbation theory in computing the contribution of the pion decay interaction to μ-meson absorption is justified, and that the "principle of minimal electromagnetic coupling" is violated in the radiative tensor decay of the π meson.

I. INTRODUCTION

In setting up a field theory of elementary particles, one must first understand which of the particle interactions in nature are primary. These interactions will appear explicitly in the Hamiltonian. In spinor electrodynamics, for example, the only primary interactions are the current potential coupling and the mass- and field-renormalization counter terms. Processes such as Compton and Møller scattering proceed indirectly via the primary interactions, though they may be represented as virtual interactions among the particles involved.

But the bewildering number of particles and reactions now known has made it very hard to pick out the primary interactions. Even before the discovery of the strange particles, this problem arose in the study of decay processes involving pions, nucleons, and lepton pairs. This can be illustrated by considering the processes of μ-meson absorption and π-meson decay. Accepting the strong Yukawa interaction as primary, one must still decide among the following alternatives:

(a) A primary Fermi interaction exists among nucleons, μ mesons, and neutrinos, leading directly to the process $N+\mu\rightarrow N'+\nu$ (where N denotes a nucleon); π-meson decay is an indirect process proceeding in lowest order according to

$$\pi\rightarrow N+\bar{N}\rightarrow\mu+\nu. \tag{1}$$

(b) A primary Yukawa interaction exists among pions, μ mesons, and neutrinos, leading directly to the process $\pi\rightarrow\mu+\nu$; μ-meson absorption is an indirect process proceeding in lowest order according to

$$\mu+N\rightarrow\mu+\pi+N'\rightarrow\nu+N'. \tag{2}$$

(c) Both primary interactions exist.

Inspection of (1) and (2) will make it clear that progress in untangling the primary interactions cannot be made without a better understanding of the role of the strong interaction $\pi\rightleftarrows N+\bar{N}$. This is only one of many examples of the importance of strong interactions in decay processes. Consider also, for instance, the mesonic corrections to beta decay. Even if one assumes a primary Fermi interaction leading directly to $N\rightarrow N'+e+\nu$, one cannot *a priori* neglect reactions of a higher order in the strong-coupling constant, such as

$$N\rightarrow N'+\pi\rightarrow N''+e+\nu+\pi\rightarrow N'''+e+\nu. \tag{3}$$

The first obstacle to our understanding the role of strong interactions is the presence of infinities which are difficult to interpret physically. In the case of reaction (1) for example, the S-matrix integral will diverge quadratically (linearly) if the Fermi coupling is pseudoscalar (pseudovector). In reaction (3), furthermore, there appears a logarithmic divergence.

Two possible points of view can be taken toward these infinities. One might assume that local field theory breaks down at high energies, and proceed to cut off or "regulate" all integrals in some more or less arbitrary manner. This has been the approach used in previous investigations of decay processes,[1–5] although in accepting this point of view, one is confronted with an infinity of possible primary interactions and an infinity of possible cutoff procedures.

We shall, instead, follow the alternative course, which is to take seriously the implications of local relativistic field theory, and to make use of the renormalization method of Dyson.[6] When successful, this method has the advantage of allowing the unambiguous subtraction of infinities; in principle, only a small number of empirical parameters are required to obtain all numerical results.

We shall demonstrate that all infinities appearing for processes involving nucleons, pions, photons, and one lepton pair may be removed by renormalization in

[1] L. I. Schiff, Phys. Rev. **76**, 303 (1949).
[2] M. Ruderman and R. Finkelstein, Phys. Rev. **76**, 1458 (1949).
[3] J. Steinberger, Phys. Rev. **76**, 1180 (1949).
[4] R. A. Finkelstein and S. A. Moszkowski, Phys. Rev. **95**, 1695 (1954).
[5] See, however, L. I. Schiff, Phys. Rev. **76**, 1266 (1949).
[6] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).

a reasonable and mathematically consistent manner. From a study of the conditions requisite for renormalizability we will obtain a partial solution and a general clarification of the problem of determining which are the actual primary interactions in nature. As a by-product of this work, some light will be thrown on the often speculated "principle of minimal electromagnetic coupling." All our results are entirely dependent on the adoption of the renormalization approach.

The fully relativistic theory of pseudoscalar mesons with pseudoscalar coupling will be the mathematical model we will use for the strong interactions. As experience has made amply clear, no weak coupling approximation for strong interactions can validly be applied in this model. In the mathematically similar problem of nucleon magnetic moments, use of the weak-coupling approximation for the pion-nucleon interaction yields numerically incorrect results.[7] The renormalization to be performed here must thus be correct to all orders in the pion-nucleon coupling constant. The particularly drastic approximation of neglecting strong interactions altogether is certainly not valid.

With weak interactions the reverse holds true; the weak-coupling constant is small enough for the application of the perturbative approximation. We are in fact compelled to use this approximation; no experimental data have thus far been available, and no adequate mathematical model is known, for high-order effects in the weak-coupling constants.

In attempting a numerical evaluation of the effects of strong interactions, we would again be faced with the same difficulty encountered in the theory of pion-nucleon scattering: we do not know how to calculate using all orders in the strong-coupling constant. However, we will prove a few simple "low-energy" theorems which allow the use of the perturbative approximation in some special cases.

II. RENORMALIZATION—GENERAL THEORY

Let us consider the calculation of the Dyson S matrix in a theory involving photons and any kinds of spinless mesons and spin-$\frac{1}{2}$ fermions. We use the usual rules for constructing momentum-space integrands from Feynman diagrams. A total interaction form (TIF) is defined as the sum (usually pictured as a black box) of all proper contributions to a Feynman diagram, or part of a diagram, specified by describing F_e external fermion lines and B_e external boson lines. Each TIF is represented by an S-matrix kernel, K, which depends on the momenta, spins, and charges of the F_e+B_e external lines. Primary interactions act within TIF's and can constitute complete contributions to TIF's in themselves. A primary interaction is partially specified by three numbers: f is the number of fermion field factors in the interaction Hamiltonian; b is the number of boson field factors in the interaction Hamiltonian, and s is the number of derivatives acting on the $f+b$ field factors. In momentum-space Feynman diagrams, a primary interaction is represented as a corner, which contributes s momentum factors to the integrand, and from which $f+b$ lines emerge.

Consider first a single primitive contribution to a TIF, having F_i+B_i internal lines and $N(f,b,s)$ corners of type (f,b,s), where f,b,s range over all possible values. The degree of divergence D of this diagram may be computed by counting all momentum factors, and is

$$D=3F_i+2B_i+\sum_{f,b,s} sN(f,b,s)-4\left[\sum_{f,b,s} N(f,b,s)-1\right]. \quad (4)$$

We use the topological relations:

$$2F_i+F_e=\sum_{f,b,s} fN(f,b,s), \quad (5)$$

$$2B_i+B_e=\sum_{f,b,s} bN(f,b,s), \quad (6)$$

and obtain[8]

$$D=4-\tfrac{3}{2}F_e-B_e+\sum_{f,b,s} (\tfrac{3}{2}f+b+s-4)N(f,b,s). \quad (7)$$

Thus, the condition for renormalizability is that every primary interaction satisfies

$$\tfrac{3}{2}f+b+s-4\leqslant 0. \quad (8)$$

If this condition is fulfilled, then D may be given the value $D(F_e,B_e)$ [or more properly, the upper bound $D(F_e,B_e)$], where:

$$D(F_e,B_e)=4-\tfrac{3}{2}F_e-B_e. \quad (9)$$

Since $D(F_e,B_e)$ is independent of all N, the condition for a primitive divergent TIF is $D(F_e,B_e)\geqslant 0$. For such a TIF, the infinities in K may be separated in the usual manner by differentiation with respect to external momenta.[9] It is possible to express this separation in very general terms, by noting that infinities always contribute to K in the same manner as primary interactions. We obtain

$$K=\textstyle\sum_a g_a I_a+\sum_a C_a I_a+K_1. \quad (10)$$

(For example, in spinor electrodynamics we have $\Gamma_\mu=e\gamma_\mu+L\gamma_\mu+\Lambda_{1\mu}$, where in accordance with our notation for TIF's the vertex function Γ_μ is defined to include a factor e at the principal vertex.) Here I_a has the form of a primary interaction (without coupling constant) with $f_a=F_e$, $b_a=B_e$, and s_a momentum factors; the label a implies a specific dependence of I_a on spins, momenta, and charges. The function K_1 is finite but as yet ambiguous, and the infinite constants C_a have degree of divergence $D(f_a,b_a)-s_a$. The quanti-

[7] Nakabayasi, Sato, and Akiba, Progr. Theoret. Phys. Japan **12**, 250 (1954).

[8] A similar relation has been given by Sakata, Umezawa, and Kamefuchi, Progr. Theoret. Phys. Japan **7**, 377 (1952).

[9] See, e.g., reference 6, Eq. (66).

ties g_a are the unrenormalized coupling constants for the various primary interactions, except perhaps for irrelevant Z factors (see below). The first summation in (10) is over all primary interactions existing in nature that have the required external lines. The second summation in (10) is over all possible primary interactions satisfying the condition:

$$D(f_a,b_a)-s_a\geqslant 0, \tag{11}$$

with the following exceptions:

(A) If a general selection rule would be violated by the existence of I_a as a primary interaction, then the infinity C_a will not actually occur in the sum.

(B) It may be, because of the nonexistence of particular primary interactions, that no diagram with the prescribed external lines can be drawn, and in this event the TIF itself is zero. Generally, when this is the case, we dignify the situation by reference to a selection rule (such as light-particle conservation) and include it under (A).

(C) It may be, because of the nonexistence of particular primary interactions, that only improper diagrams can be drawn with the prescribed external lines. Again, in this case the TIF itself is zero.

(D) It may be that the only proper diagrams having the prescribed external lines always include corners with positive-definite $D(f,b)-s$ [see Eq. (7)] and some of the infinities C_a may not actually appear.

(E) It may be that the only proper diagrams having the prescribed external lines always include corners, some of whose s momentum factors are fixed external momenta. Again, in this case D will be lowered and some of the infinities C_a may not actually occur. [The possibilities (C), (D), and (E) are of course quite rare. Examples of (C) and (D) will be given in Sec. III.]

(F) Conceivably, an "accidental" cancellation in all orders would make some C_a vanish. However, there is no known occurrence of this case. Direct calculation in lowest order perturbation theory verifies that none of the infinities discussed in Sec. III vanish accidentally.

The infinities C_a are interpreted as renormalizations of the primary interaction strengths g_a, with the renormalized coupling constants g_{1a} given by

$$g_{1a}=g_a+C_a. \tag{12}$$

This is the essential point of the renormalization program; for this to be possible every term present in the second sum in (10) must have a corresponding term in the first sum. Thus, with the exceptions (A), (B), (C), (D), (E), and (F), *every primary interaction satisfying (11) must actually exist* in nature. [Of course the renormalized coupling constant g_{1a} may have an empirical value of zero, but the definition of the coupling constant depends on the particular mode of separation in Eq. (10). Only in such special cases as self-energy parts and electrodynamics vertices, where the kinematics are particularly simple, does there exist a canonical method of extracting the renormalized coupling constants.] *Comparison of (11) with (8) shows that the primary interactions satisfying (11) are renormalizable, and are the only renormalizable primary interactions.*

The definition and cancellation of Z factors, and the problems associated with external lines and overlapping divergences, may be handled as usual. It is possible to avoid all these problems by using the renormalized interaction representation,[10] in which infinities are compensated by counterterms as soon as they appear; self-energy parts are treated in exactly the same manner as other TIF's; and the quantities g_a are defined as:

$$g_a=\prod_p Z_p{}^{\frac{1}{2}n_{p,a}}g_{0a}, \tag{13}$$

where p labels a particular particle, $n_{p,a}$ is the number of p lines emerging from a corner of type a, Z_p is the field renormalization factor, and g_{0a} is the true unrenormalized coupling constant.

III. RENORMALIZATION—WEAK INTERACTIONS

The general theory outlined in Sec. II can be applied successfully to the five-field theory of protons, neutrons, charged and neutral pions, and photons. The primary interactions satisfying (11) and satisfying the accepted invariance principles are just those usually assumed to exist among these five fields. But the introduction of weak interactions is well known to cause serious trouble. Condition (8) is violated by the quadrilinear Fermi interaction ($f=4$, $b=s=0$) which we may take as an archetype of the weak interactions. However, we are restricting ourselves to lowest order in the weak coupling constants; in any diagram, only one weak corner will be present, and if it is a Fermi interaction the value of D will be lowered by one, exactly as if a boson were being emitted instead of a lepton pair. It will be convenient, in fact, to introduce five fictitious boson charged lepton-pair fields $l_\pm{}^r$, where "r" labels the five covariant classes, S, V, T, A, P, and the $\pm$ labels the charge. (It is interesting that in the original Yukawa theory of beta decay, real mesons exist corresponding closely to the l field. Even this theory is not completely renormalizable, since particles of spin 2 must be introduced to account for the tensor interaction.) We have[11]

$$l_-{}^r\equiv(\bar{\psi}_\nu\gamma^r\psi_{\mu,e}),\quad l_+{}^r\equiv(\bar{\psi}_{\mu,e}\gamma^r\psi_\nu). \tag{14}$$

The Fermi interactions can be written:

$$l_-{}^r(\bar{\psi}_n\gamma_r\psi_p)+l_+{}^r(\bar{\psi}_p\gamma_r\psi_n). \tag{15}$$

We catalog below all possible primary interactions among nucleons, pions, photons, and one lepton pair,

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[10] P. T. Matthews and A. Salam, Phys. Rev. **94**, 185 (1954).

[11] Except where indicated, our notation is identical with that of J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Press, Cambridge, 1955). In particular we use units with $\hbar=c=1$.

satisfying (11), with l counting as an external boson, and satisfying Lorentz invariance (including parity conservation), gauge invariance, charge conjugation invariance, charge conservation, the Lorentz condition, and Hermiticity. (See below for remarks on the use of parity conservation and charge conjugation invariance.) We omit the fourteen real coupling constants g_a in writing these interactions.

$$l_+^S(\bar{\psi}_p\psi_n)+l_-^S(\bar{\psi}_n\psi_p), \qquad (S1)$$

$$\phi_0(\phi l_+^S+\phi^* l_-^S), \qquad (S2)$$

$$l_+^\mu(\bar{\psi}_p\gamma_\mu\psi_n)+l_-^\mu(\bar{\psi}_n\gamma_\mu\psi_p), \qquad (V1)$$

$$\phi_0[(\partial_\lambda-ie_1A_\lambda)\phi l_+^\lambda-(\partial_\lambda+ie_1A_\lambda)\phi^* l_-^\lambda], \qquad (V2)$$

$$\partial_\lambda\phi_0(\phi l_+^\lambda-\phi^* l_-^\lambda), \qquad (V3)$$

$$l_+^{\mu\nu}(\bar{\psi}_p\gamma_{\mu\nu}\psi_n)+l_-^{\mu\nu}(\bar{\psi}_n\gamma_{\mu\nu}\psi_p), \qquad (T1)$$

$$i\mathfrak{F}_{\mu\nu}(\phi l_+^{\mu\nu}+\phi^* l_-^{\mu\nu}), \qquad (T2)$$

$$l_+^{5\mu}(\bar{\psi}_p\gamma_5\gamma_\mu\psi_n)+l_-^{5\mu}(\bar{\psi}_n\gamma_5\gamma_\mu\psi_p), \qquad (A1)$$

$$(\partial_\mu-ie_1A_\mu)\phi l_+^{5\mu}+(\partial_\mu+ie_1A_\mu)\phi^* l_-^{5\mu}, \qquad (A2)$$

$$l_+^5(\bar{\psi}_p\gamma_5\psi_n)+l_-^5(\bar{\psi}_n\gamma_5\psi_p), \qquad (P1)$$

$$\phi l_+^5+\phi^* l_-^5, \qquad (P2)$$

$$\phi^*\phi(\phi l_+^5+\phi^* l_-^5), \qquad (P3)$$

$$\phi_0\phi_0(\phi l_+^5+\phi^* l_-^5), \qquad (P4)$$

$$(\Box^2+m_\pi^2-2ie_1A^\mu\partial_\mu-e_1^2A^2)\phi l_+^5 + (\Box^2+m_\pi^2+2ie_1A^\mu\partial_\mu-e_1^2A^2)\phi l_-^5. \qquad (P5)$$

Here $\mathfrak{F}_{\mu\nu}$ is the dual electromagnetic field-strength tensor, ϕ is the charged pion field, and ϕ_0 the neutral pion field. If we ignore all electromagnetic effects and require charge symmetry as well as charge conjugation invariance, then the interaction ($S2$) is forbidden.[12]

The fourteen listed interactions are of varying familiarity. The first in each class—($S1$), ($V1$), ($T1$), ($A1$), ($P1$)—are of course the usual Fermi interactions. Interactions ($P2$), ($P5$), ($A2$), and ($T2$) lead directly to ordinary or radiative pion decay; ($T2$) will be discussed below. Interactions ($S2$), ($V2$), and ($V3$) lead directly to the process $\pi^\pm\rightarrow\pi^0+e^\pm+\nu$ conjectured by Feenberg and Primakoff,[13] with the possibility of extra photons being omitted. Interactions ($P3$) and ($P4$) appear at present to be of only technical importance.

In constructing the above catalog we have used the requirements of parity conservation and charge-conjugation invariance. It might be argued that our conclusions are therefore incorrect, since current experimental evidence shows that these invariance principles do not hold in decay processes.[14,15] However, even if these invariance principles hold only for strong interactions, and provided they are violated *in the same way* in all weak primary interactions, no changes need be made in the above catalog; we need only redefine the pair field l. In the theory of Lee and Yang[16] the violation of parity conservation arises from the structure of the neutrino, and must therefore be the same in all weak interactions of the sort discussed in this paper. If it should develop that charge-conjugation invariance is violated in different manners for different neutrino processes, then it will only be necessary to correct the relative phases of the first and second term in interactions ($S1$), $\cdots$, ($P5$).

Considering the results of Sec. II, we note the following interesting properties of the primary interactions ($S1$), $\cdots$, ($P5$):

(i) These interactions are all renormalizable.

(ii) These are the only renormalizable interactions with one l-field factor. (Since we have considered diagrams with only one weak corner this is not strictly accurate; the existence of other interactions such as $\phi^4 l$ would increase—but leave finite—the number of primitive divergents. However, if we allow the possibility of many external lepton pairs, then only the fourteen cataloged interactions lead to a finite number of primitive divergents.)

(iii) These are the only interactions that will be needed as counterterms.

(iv) If a cataloged member of one of the covariant classes S, V, T, A, or P exists, then all other cataloged interactions in that class must also exist, since they will be required as counterterms. [Exceptions to this rule: If interaction ($P2$) exists—or if ($A2$) or ($P5$) exists and we ignore all electromagnetic effects—then the other primary interactions of that class need not exist (case (C)). If interactions ($S2$) or ($A2$) exist, then ($S1$) or ($A1$) need not exist (case (D)).]

Rather interestingly, the so-called "law" of minimal electromagnetic coupling is violated by interactions ($T2$); the electromagnetic field enters as $\mathfrak{F}_{\mu\nu}$ and not in a $J_\mu A^\mu$-type coupling. This violation is simply a reflection of the fact that our theory cannot be renormalized to all orders in the weak-coupling constant. It is easy to prove that *in any completely renormalizable theory the law of minimal electromagnetic coupling is a consequence of Lorentz invariance (without inversions) and gauge invariance.*

Since experimental information makes it fairly certain that ($T1$) exists for electrons, it follows that ($T2$) must exist as a counterterm, and might eventually be observed.

It would of course be possible to make explicit the extraction of infinities in (10) for all eight primitive divergent TIF's: $np l$, πl, $\pi^0\pi l$, $\pi\pi\pi l$, $\pi^0\pi^0\pi l$, $\pi l\gamma$, $\pi^0\pi l\gamma$, $\pi l\gamma\gamma$. It will be convenient to do this here for only the first three of these. We define the "radiative correction"

[12] A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952).

[13] E. Feenberg and H. Primakoff, Bull. Am. Phys. Soc. Ser. II, **2**, 39 (1957).

[14] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

[15] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957); J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957).

[16] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

function K_1 [see (10)] to be zero for npl at equal free-particle nucleon momenta, neglecting the nucleon mass difference; for πl at free-particle pion momenta; and for $\pi^0\pi l$ at equal free-particle pion momenta, neglecting the pion mass difference. This definition makes the respective renormalized coupling constants g_{1a} [see (12)] unambiguous.

It is hoped, of course, that some of the renormalized coupling constants will turn out to be exactly zero. At present it appears that for electrons the renormalized coupling constants for ($P2$) and probably ($A2$) are very small (see below). This result says nothing about the magnitude of the beta-decay interactions ($P1$) and ($A1$) since the rate for ordinary pion decay is a function of the renormalized coupling constants for ($P2$) and ($A2$) only.

IV. APPLICATIONS

The main point of this work is that decay processes must be analyzed in terms of a fairly large but sharply limited set of empirical coupling constants. Processes such as $\pi\rightarrow\mu+\nu$, $\pi\rightarrow\pi+e+\nu$, and $\pi\rightarrow\gamma+e+\nu$ cannot be discussed in terms of Fermi couplings alone, as has been done in the past.[1,2,3,13,17,18] Unavoidably we must consider the contribution of ($A2$), ($P2$), ($P5$) for $\pi\rightarrow\mu+\nu$; ($S2$), ($V2$), ($V3$) for $\pi\rightarrow\pi+e+\nu$; and ($T2$), ($A2$), ($P5$) for $\pi\rightarrow\gamma+e+\nu$. In particular, it would not be too surprising if experiments should show that the pseudoscalar beta decay coupling ($P1$) is large and yet does not lead to a $\pi-e$ decay, or that even though the vector beta-decay coupling ($V1$) is small a vector radiative pion decay does take place.

Quantitative predictions can be obtained from this theory by using a perturbative approximation. This approximation is valid only when a "low-energy" theorem can be proven; that is, when the S-matrix element can be written as the sum of a small number of irreducible diagrams, in each of which the "radiative" correction functions K_1 of the primitive divergent TIF's inserted in the diagrams can be shown to be negligible.

The simplest example of a low-energy theorem in decay processes occurs for nonradiative pion decay. Only one irreducible diagram contributes (see Fig. 1) and the corrections to the πl TIF vanish exactly. The rate for $\pi\rightarrow(\mu$ or $e)+\nu$ is given by

$$1/\tau=[(m_\pi^2-m_{\mu,e}^2)^2/8\pi m_\pi^3]g'^2, \qquad (16)$$

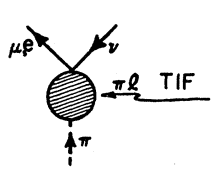

FIG. 1. The irreducible diagram for charged pion decay.

[17] S. B. Treiman and H. W. Wyld, Jr., Phys. Rev. **101**, 1552 (1956).
[18] M. Ruderman, Phys. Rev. **85**, 157 (1952).

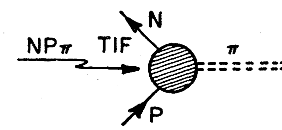

FIG. 2. The irreducible diagram for μ-meson absorption.

where we have set

$$g'=g_{1(P2)}+m_{\mu,e}g_{1(A2)}, \qquad (17)$$

the quantities $g_{1(P2)}$ and $g_{1(A2)}$ being the renormalized coupling constants for primary interactions ($A2$) and ($P2$). Using the observed pion-decay rates[19,20] we obtain $g_{\pi\mu}'=1.62\times10^{-7}$, $g_{\pi e}'\lesssim5\times10^{-10}$. It should be emphasized that Eq. (16) is exact, following directly from the definition of the renormalization and does not depend on any theoretical assumptions about the mechanism of the decay process.

Neutron beta decay furnishes another example of a low-energy theorem. The ratios m_e/m_π and $(m_n-m_p)/m_\pi$ are sufficiently small so that corrections to the npl TIF are negligible. Nuclear beta decay is made considerably more complicated by theoretical uncertainties in bound state problems, but it seems reasonable to suppose that "radiative" corrections to nuclear beta decay are also quite small, and that therefore the renormalized coupling constants are the quantities measured in beta decay experiments.

It has been speculated[21] that μ mesons differ from electrons in that μ mesons have primary interactions with bosons only. Accepting this hypothesis for the time being, let us assume that the only weak μ-meson interactions are ($A2$) and ($P2$). It is well known[22] that with this hypothesis the rate of μ-meson absorption in hydrogen can be easily calculated by using lowest-order perturbation theory for weak and strong interactions, and comes out in fair agreement with the rate derived from experiments on complex nuclei. It is now possible to justify the use of the perturbative approximation, by noting that only one irreducible diagram contributes to μ absorption. (See Fig. 2.) To the extent that is is permissible to neglect terms of order m_π^2/m_n^2 and m_μ^2/m_n^2, we can ignore corrections to the $np\pi$ TIF and the pion propagator, and thus obtain an effective pseudoscalar Fermi-type coupling constant for μ absorption of

$$g_{P,\text{eff}}=g'G_1\sqrt{2}/(m_\pi^2+q^2), \qquad (18)$$

where q is the pion momentum in Fig. 2 and $G_1\sqrt{2}$ is the renormalized $np\pi$ coupling constant. (The effective coupling constant $g_{P,\text{eff}}$ is the constant that would be used to account for μ absorption if it were assumed that only the pseudoscalar Fermi coupling contributes to the

[19] Durbin, Loar, and Havens, Phys. Rev. **88**, 179 (1952).
[20] S. Lokanathan and J. Steinberger, Phys. Rev. **98**, 240(A) (1955).
[21] See, e.g., J. Schwinger, Phys. Rev. **104**, 1164 (1956).
[22] Lee, Rosenbluth, and Yang, Phys. Rev. **75**, 905 (1949).

absorption process.) Using the value $G_1^2/4\pi=12$, we obtain $g_{P,\mathrm{eff}}=11.7\times10^{-49}$ erg cm^3, giving an absorption rate of 6 sec^{-1} in hydrogen. Verification or refutation of this prediction would be proof, one way or the other, of the presence or absence of any μ-meson weak couplings in addition to ($A2$) and ($P2$). It seems plausible that experiments will either agree with this figure of 6 sec^{-1} or will give a rate of about 140 sec^{-1}, the figure expected on the assumption that μ absorption occurs through a scalar Fermi coupling with $g_S=3\times10^{-49}$ erg cm^3. The major uncertainty in the figure 6 sec^{-1} stems from uncertainty in the coupling constant G_1. It is however reassuring to note that the approximations made in arriving at (18) will err in the same direction as the approximations made in the derivation of the Kroll-Ruderman theorem,[23,24] from which we derive the value of G_1.

We have here been concerned with the better known particles and decay processes. But it is hoped that the considerations presented will be of assistance in understanding the decays of the strange particles.

It is a pleasure to thank Professor S. B. Treiman for his helpful advice and encouragement, and Professor F. J. Dyson for a valuable discussion. This work was performed during the tenure of a National Science Foundation predoctoral fellowship.

[23] N. Kroll and M. Ruderman, Phys. Rev. **93**, 233 (1954).
[24] F. E. Low, Phys. Rev. **97**, 1392 (1955).

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Chirality of *K* Particle*

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In the two-component theory, the neutrino that can exist in nature is characterized by one of the eigenvalues of the "chirality" operator, γ_5, which anticommutes with the parity operator. The chirality operator is generalized so that it can be applied also to bosons. The K particle that can exist in nature is characterized by a certain condition on the eigenvalues of the chirality operator. There is strong reason to believe that the chirality quantum number thus introduced is closely related to the strangeness quantum number.

1. INTRODUCTION

THE series of theoretical efforts, which has originated from the tau-theta paradox, has culminated in a return to the once-abandoned two-component neutrino theory.[1] The present paper is intended to show that a unified point of view is possible in dealing with both problems.

A special mathematical formalism is used in this paper, so that an operator called "chirality," which anticommutes with the parity operator, can be applied to both fermions and bosons. In the case of a spinor particle, the eigenvalues of chirality are ±1, but they are good quantum numbers only when the mass is zero.[2] If we take one of the possible eigenvalues (say, -1 in the right-handed coordinate system), we obtain the well-known two-component theory of neutrinos. If the mass is finite, the chirality is indeterminate (zero on the average).[2]

In the case of a boson, the eigenvalues are ±2 and 0. They are good quantum numbers even if the mass is finite. The scalar particle can have only eigenvalues ±2. The eigenstates of chirality imply of course an indefinite parity. Conversely, a boson with a definite parity (such as pion) has an indefinite chirality (zero by convention). In view of the fact that the same K particle seems to be capable of decaying into two pions or three pions, it is proposed to assume the K particle to be in an eigenstate of chirality.[3] The tensorial rank of K particles is assumed to be zero, i.e., of the scalar type. Each of the two eigenstates of chirality (±2) provides further two eigenstates, corresponding to two possible charges. To accommodate the K particle and the anti-K particle (either charged or neutral), one thus has four possibilities to choose from. This leads to two alternative assignments of K particles to chiral eigenstates. It is still premature to decide which alternative is preferable.

According to the first assignment, the K particle (either positive or neutral) is identified, say in the right-

* The word "chirality" (pronounced as kirality) seems to have been coined by Kelvin and was extensively used by Eddington [A. S. Eddington, *Fundamental Theory* (Cambridge University Press, New York, 1949, p. 111]. The usage of this term here may be justified by two reasons: (1) Etymologically, it can mean "handedness." (2) Eddington used it also in the sense of the sign of γ_5 though in a different context.

[1] W. Pauli, *Handbuch der Physik* (Julius Springer, Berlin, 1933), Vol. 24, p. 226; A. Salam, Nuovo cimento **5**, 229 (1957); T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957); L. Landau, preprint, among others. Experimental tests, proposed by Lee and Yang [Phys. Rev. **104**, 254 (1956)], played a decisive role in this development.

[2] This is true only when one uses γ_5 as chirality operator. For further discussions of a chiral particle of spin $\frac{1}{2}$ with finite mass and finite charge, see S. Watanabe, Nuovo cimento (to be published).

[3] The assumption that the theta and the tau are the same particle naturally leads to a unique lifetime for two-pi and three-pi decay modes.