obtained at Columbia[4]) by electronic methods for emulsion suggests that the polarization of the μ^+ beam used at Columbia is close to maximal. The two-component theory[9] predicts complete polarization and for it a $|a|$ value of $\leqslant \frac{1}{3}$; $-\frac{1}{3}$ is within the stated errors compatible with Columbia's results for graphite. The statistical accuracy of experiments both with muons from decays in flight and at rest will have to be greatly improved before any firm conclusions can be drawn.

ACKNOWLEDGMENTS

We wish to thank Mrs. R. Brooke, Mr. A. Lassaine, and Mr. J. Mott for their efficient work in scanning and measurement. We are grateful to Mr. T. Kinyon and Mrs. E. Garwin for their help with computations. Our thanks are due to Dr. R. Levi-Setti for criticism of the experimental techniques.

We also wish to acknowledge illuminating discussions concerning the depolarization problem with Dr. M. Cohen and Dr. F. Reif.

Note added in proof.—Detailed electronic experiments at Chicago (reference 10) have shown that even in substances yielding a low a value, μ^+ precess essentially without exhibiting relaxation over a period of 3 μsec. Hence the depolarization leading to the reduced a value must (1) take place during the slowing down process, presumably when $v_\mu \gtrsim c/137$; (2) act rapidly; (3) affect only a fraction of all muons. These conclusions would appear to support the muonium picture given in the text. At a velocity of the order of $c/137$, however, the muon loses little energy by e^- capture, and muonium (having almost the same velocity) would make $\sim 10^{16}$ collisions/sec in a solid. This may throw some doubt on the literal validity of the picture. On the other hand, the latter is not critically dependent on the assumed *binding* between electron and muon. Conceivably, thermalized muons can sufficiently couple to spins of bound electrons (or nuclei) to depolarize.—These points will be discussed in detail in a future publication.

PHYSICAL REVIEW VOLUME 106, NUMBER 6 JUNE 15, 1957

Information Obtainable on Polarization of μ^+ and Asymmetry of e^+ in Muonium Experiments*

G. BREIT AND V. W. HUGHES
Yale University, New Haven, Connecticut
(Received March 7, 1957)

The formation of the compound $\mu^+ + e^-$ (muonium) is considered as a tool for gaining information regarding the polarization of the μ^+ before their capture by the e^- and the asymmetry of e^+ emission in the disintegration $\mu^+ \rightarrow e^+ + \nu + \nu'$. The detection of asymmetry effects is supposed to take place through the counting of e^+. The effect of constant magnetic fields and of microwave-induced transitions among magnetic substates of muonium is calculated, with the conclusion that all of the experiments considered here determine in different ways the same combination of parameters describing the initial muon polarization and the asymmetry of e^+ emission in muon decay.

I. INTRODUCTION AND NOTATION

In order to answer the question of parity conservation raised by Lee and Yang,[1] experiments were performed by Wu, Ambler, Hayward, Hoppes, and Hudson[2] showing that parity is not conserved in beta decay of Co^{60} giving the first example of lack of conservation of parity in weak interactions. The experiment of Garwin, Lederman, and Weinrich[3] then showed that in the reactions

$$\pi^+ \rightarrow \mu^+ + \nu, \quad \mu^+ \rightarrow e^+ + \nu + \nu',$$

the μ^+ are strongly polarized along the line of motion and that there is a large asymmetry in the angular distribution of the e^+ with respect to the spin direction of μ^+. The question arises as to whether the degree of polarization of μ^+ and the asymmetry parameter in the e^+ emission, or some parameter connected with both, could be determined by observing the asymmetry of e^+ emission from the compound

$$\mu^+ + e^-,$$

the formation of which may be expected to take place at the end of the range of the μ^+ in suitable materials.[4] The present note is concerned with this question.[5] The

* This research was supported by the U. S. Air Force through the Air Force Office of Scientific Research of the Air Research and Development Command under Contract Nos. AF 18(600)-771 and AF 18(600)-1565.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956); T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[2] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

[3] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).

[4] The possible role of muonium in the Columbia and Chicago experiments has been discussed by J. I. Friedman and V. L. Telegdi, Phys. Rev. **105**, 1681 (1957); estimates of formation may be found in Vernon W. Hughes, Bull. Am. Phys. Soc. Ser. II, **2**, 205 (1957).

[5] After the present work was completed, there arrived a preprint of a paper by Lee, Oehme, and Yang, on "Remarks on

TABLE I. Inherent probabilities of spin directions of μ^+.

(F,M_F)	Spin function	Probability of α_μ	Probability of β_μ
(1,1)	$\alpha_\mu\alpha_e$	1	0
(1,0)′	$s\alpha_\mu\beta_e+c\beta_\mu\alpha_e$	s^2	c^2
(1, −1)	$\beta_\mu\beta_e$	0	1
(0,0)′	$c\alpha_\mu\beta_e-s\beta_\mu\alpha_e$	c^2	s^2

TABLE II. Relative numbers of muonium atoms and of spin directions after capture.

(F,M_F)	Number of muonium atoms	Number of μ^+ with spin orientations α_μ	β_μ
(1,1)	p	p	0
(1,0)′	$c^2+(s^2-c^2)p$	$s^2c^2+s^2(s^2-c^2)p$	$c^4+c^2(s^2-c^2)p$
(1, −1)	$1-p$	0	$1-p$
(0,0)′	$s^2+(c^2-s^2)p$	$c^2s^2+c^2(c^2-s^2)p$	$s^4+s^2(c^2-s^2)p$
	Totals	$2c^2s^2+[1+(c^2-s^2)^2]p$	$1+c^4+s^4-[1+(c^2-s^2)^2]p$

experiments considered will be seen to be incapable of determining the two parameters separately but are nevertheless able to give information regarding a combination of the two.

The symbols most frequently used are as follows:

F = fine-structure quantum number (total angular momentum of muonium).

M_F = weak-field magnetic quantum number with axis of quantization along the muon line of flight.

α_μ, β_μ = spin eigenfunctions of μ^+; α_μ corresponds to spin orientation along the line of flight, β_μ to the opposite spin direction.

α_e, β_e = similarly defined spin eigenfunctions of the captured e^-.

p = probability that μ^+ has its spin oriented along the beam direction just before capture.

In addition there will occur frequently a quantity a which is a measure of the asymmetry of e^+ emission. It is defined by counting e^+ emissions in two cones having angular openings 0 to θ and π to $\pi-\theta$, with θ standing for the angle between the direction of e^+ emission with respect to the muon spin. When one restricts oneself to these e^+, the probability that if an e^+ is observed its direction is in the first cone is denoted by $(1+a)/2$ and the probability that it should lie in the second cone is denoted by $(1-a)/2$.

II. CONDITION IN A STEADY FIELD ALONG THE LINE OF FLIGHT

The muonium will be supposed to be subjected to the action of a magnetic field $\mathcal{H}$ along the line of flight of the muons. A magnetic field orientation perpendicular to the line of flight results in equal probabilities of the μ^+ spin along and opposite to $\mathcal{H}$, and is thus not suitable for obtaining information regarding the original spin direction. The effect of the longitudinal magnetic field is to give the probabilities of the states α_μ, β_μ of μ^+ in the states (F,M_F) of muonium as in Table I. In this table the states (1,0)′ and (0,0)′ are the stationary states arising from (1,0), (0,0) if $\mathcal{H}$ is applied adiabatically. For $\mathcal{H}=0$, $s=c=1/\sqrt{2}$. In general s, c may be taken to be real numbers satisfying $s^2+c^2=1$. The notation is suggested by the fact that s and c are, respectively, the sine and cosine of an angle associated with the orthogonal transformation from the strong-field states to the actual ones. It will be supposed that if the muon spin is α_μ before capture, the relative number of muonium atoms formed in different states is proportional to the probability of α_μ, i.e., to the numbers in the next to the last column of Table I. If the muon spin is β_μ, the relative numbers of muonium atoms formed is similarly proportional to the probability of β_μ in the state into which capture takes place. This way of calculating the relative probability of formation amounts to the assumption that torques on the μ^+ spin during the formation have a negligible effect. This appears reasonable in view of the smallness of the magnetic moment of μ^+ and corrections for such effects could be worked out. In this way one obtains from Table I the relative numbers of muonium atoms produced in the various magnetic substates as well as the relative numbers of the μ^+ having different spin orientations α_μ, β_μ. These relative numbers are listed in Table II.

noninvariance under time reversal and charge conjugation" which increases the interest in the question of the determination of the spin direction of the muon on account of its bearing on the question of conservation of light particles.

The directions of α and β will now be referred to by R and L (for right and left), and the fractional excess of R over L spin orientations by b so that

$$b=(R-L)/(R+L), \tag{1}$$

where R and L stand for the number of μ^+ spins having spin orientations along R and L. For the conditions applying to Table II, one has then

$$b=[(c^2-s^2)^2+1](p-\tfrac{1}{2}). \tag{2}$$

The observable asymmetry of the e^+ is

$$A=[R(e^+)-L(e^+)]/[R(e^+)+L(e^+)], \tag{3}$$

where $R(e^+)$ and $L(e^+)$ are the relative numbers of e^+ emitted along R and L, respectively. From the definition of a, one has

$$\begin{aligned} R(e^+)&=(1+a)R/2+(1-a)L/2,\\ L(e^+)&=(1-a)R/2+(1+a)L/2, \end{aligned} \tag{4}$$

and hence from (1) and (3),

$$A=ab. \tag{5}$$

Thus

$$A=[(c^2-s^2)^2+1](p-\tfrac{1}{2})a. \tag{6}$$

From $\mathcal{H}$ one can calculate c and s and hence a measurement of A through (6) determines $(p-\frac{1}{2})a$.

By applying a microwave magnetic field with a frequency corresponding to the energy difference between (1,1) and (0,0)′, it should be possible to equalize the relative numbers of muonium atoms in these two states making each of them $s^2/2+(1+c^2-s^2)p/2$. For this saturated condition of (1,1) and (0,0)′ states, the the asymmetry of electron emission is obtainable by the same procedure as that used in obtaining (6). The result is

$$A_1=(3c^4-2c^2+1)(p-\tfrac{1}{2})a \quad [(1,1) \text{ and } (0,0)' \text{ equalized}]. \tag{7}$$

Similarly one finds

$$A_2=(1-2s^2+3s^4)(p-\tfrac{1}{2})a \quad [(1,-1) \text{ and } (0,0)' \text{ equalized}]. \tag{8}$$

In all cases the measurement of the asymmetry of the electron emission gives the combination $(p-\frac{1}{2})a$ and does not suffice for the determination of p and a separately. The occurrence of the factor $(2p-1)a$ in the result is in agreement with expectation because a change from p to $1-p$ together with a change from a to $1-a$ replaces every R muon by an L muon and also reverses the sign of the inherent e^+ emission asymmetry. The value of A cannot be affected by these two changes as is in fact the case in (6), (7), and (8). The occurrence of $(p-\frac{1}{2})a$ in A could in fact have been predicted from this consideration because the only way of combining expressions containing no powers of a and p higher than the first into a form invariant to changing p to $1-p$ and a to $-a$ is to form a multiple of $(p-\frac{1}{2})a$. One may conclude therefore that the experiments just considered can give the parameter $(p-\frac{1}{2})a$ in several ways but not p and a separately.

III. ESTABLISHMENT OF EQUILIBRIUM

The question arises as to whether an observation of the rate at which equilibrium is established as a result of the application of a circularly polarized microwave field can distinguish between p and a. If it were possible to observe the reaction on the microwave cavity produced in transitions between (1,1) and (0,0)′, one would know the value $p(1+s^2-c^2)-s^2=s^2(2p-1)$ and the value of p would be determinable. The number of muonium atoms available is too small, however, to make such an observation of the absorption of microwave energy practical. It is necessary therefore to use the observation of the disintegration e^+ exclusively. The possibilities of experiments making use of such observations appear as follows.

The combined effect of absorptions and stimulated emissions is to give rates of change of populations N_a, N_b of two states a, b as follows:

$$dN_a/dt=-\lambda N_a+N_b, \quad dN_b/dt=-\lambda N_b+\lambda N_a,$$

where λ is determined by the strength of the microwave field and the transition matrix element. These rates of change leave N_a+N_b time-independent and give an exponential decay of N_a-N_b with time constant $1/(2\lambda)$. At the time t,

$$\begin{aligned} N_a(t)&=\tfrac{1}{2}(1+u)N_a(0)+\tfrac{1}{2}(1-u)N_b(0),\\ N_b(t)&=\tfrac{1}{2}(1-u)N_a(0)+\tfrac{1}{2}(1+u)N_b(0), \end{aligned} \tag{9}$$

where

$$u=e^{-2\lambda t}. \tag{9′}$$

Referring to the states (1,1), (1,0)′, (1, −1), (0,0)′ as 1, 2, 3, 4, the relative numbers of muonium atoms in the four states may be written as

$$c_i+d_ip, \quad (i=1,2,3,4). \tag{10}$$

These quantities are listed in succession in the second column of Table II. The quantities occurring in the third and fourth columns of Table I will be called q_i and $1-q_i$, respectively. The inherent probability of α_μ in state i is thus q_i and that of β_μ is $1-q_i$. The relative numbers of μ^+ with spin orientations along and opposite to the beam direction are therefore $q_i(c_i+d_ip)$ and $(1-q_i)(c_i+d_ip)$, respectively. Hence

$$b=\textstyle\sum_i (2q_i-1)(c_i+d_ip)/\sum_i (c_i+d_ip), \tag{11}$$

and, on account of the normalization used in Table II

$$\textstyle\sum_i c_i=2, \quad \sum_i d_i=0. \tag{11′}$$

Thus b is linear in p as has been observed previously. According to (9), the effect of a microwave transition is to change a pair c_k+d_kp, c_l+d_lp to

$$\begin{aligned} c_k'+d_k'p&=\tfrac{1}{2}(1+u)(c_k+d_kp)+\tfrac{1}{2}(1-u)(c_l+d_lp),\\ c_l'+d_l'p&=\tfrac{1}{2}(1-u)(c_k+d_kp)+\tfrac{1}{2}(1+u)(c_l+d_lp). \end{aligned} \tag{12}$$

The contribution of states k, l to $2b$ is, according to (11) and (12),

$$\begin{aligned} (2q_k-1)(c_k'+d_k'p)+(2q_l-1)(c_l'+d_l'p) &=(2q_k-1)(c_k+d_kp)+(2q_l-1)(c_l+d_lp)\\ &\quad+(1-u)(q_k-q_l)[c_l-c_k+(d_l-d_k)p]. \end{aligned}$$

Thus during the transition the value of b is

$$b'=b+\tfrac{1}{2}(1-u)(q_k-q_l)[c_l-c_k+(d_l-d_k)p]. \tag{13}$$

Since b is a multiple of $2p-1$, and since the same applies to b' when $u=0$, the value of b' for other u is also a multiple of $2p-1$. Inspection of the second column of Table II shows in fact that $c_l-c_k+(d_l-d_k)p$ is a multiple of $2p-1$ for any pair of states. The observation of the rate of transitions thus gives information about λ but not about the values of a or p separately.

IV. CONCLUSIONS

Measurements of the asymmetry in the emission of e^+ yields several ways of determining the quantity $(2p-1)a$ which can be of value in testing assumptions regarding the formation of muonium and the applicability of simple theory to its structure. Steady magnetic fields along the line of flight and microwave transitions between magnetic substates give formulas for e^+ asymmetries containing the same parameter $(2p-1)a$ as a factor.

It is desired to acknowledge helpful discussions with Professor T. D. Lee.