Angular Correlations in the $F^{19}(p,\alpha\gamma)O^{16}$ Reaction*

H. J. MARTIN,† W. A. FOWLER, C. C. LAURITSEN, AND T. LAURITSEN
Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California

(Received March 11, 1957)

Angular correlation patterns in the $F^{19}(p,\alpha\gamma)O^{16}$ reaction, between alpha particles leading to the 6.13-Mev excited state of O^{16} and the corresponding gamma rays, have been measured for proton bombarding energies of 873, 935, 1250, 1280, 1346, and 1372 kev. The patterns confirm earlier spin and parity assignments and permit quantitative determination of the relative importance of high orbital angular momenta in the formation and decay of Ne^{20} states. In two cases, the higher l-values participate much more strongly than might be expected from elementary barrier considerations.

INTRODUCTION

THE reaction $F^{19}(p,\alpha)O^{16}$ has been the subject of a number of intensive investigations directed both toward a determination of the properties of the revelant excited levels of Ne^{20} and O^{16}, and toward a more general study of the course of complex nuclear reactions involving compound nucleus formation. The reaction shows numerous resonances, involving relatively well-defined states of Ne^{20} which decay by alpha-particle emission to various states of O^{16}; of particular interest here are those Ne^{20} states which yield alpha-particle groups, designated α_1, α_2, α_3, to the 6.14-Mev, $J=3^-$, 6.91-Mev, $J=2^+$, and 7.12-Mev, $J=1^-$ levels of O^{16}. In all three cases, the alpha-particle emission is followed by a γ transition directly to the ground state of O^{16}. A schematic excitation curve showing the principal $(p,\alpha\gamma)$ resonances from $E_p=0.8-1.4$ Mev is shown in Fig. 1.

A compound nuclear state characterized by a given total angular momentum and parity, $J_1\pi_1$, may be formed by various combinations of channel spin s and relative orbital angular momentum l. The decay to a final state, $J_2\pi_2$, may again involve various combinations of channel spin and orbital angular momentum. Thus, for example in the present reaction, a 2^- Ne^{20} state may be formed in channel spin $s=1$ by p- and f-wave protons, and may decay to the 3^- state of O^{16} via d- and g-wave alpha particles. The extent to which such admixtures occur depends not only upon the relative barrier factors, but also upon detailed properties of the nuclear states involved,[1] and a quantitative determination of the relative contributions of various angular momentum combinations should therefore be of considerable significance for the theory.

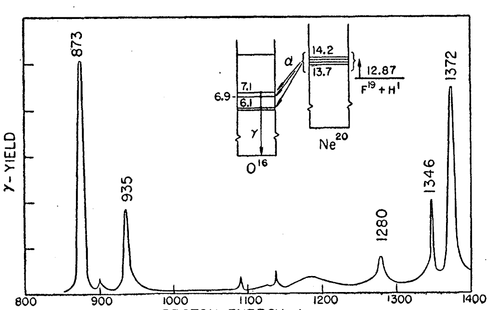

FIG. 1. Excitation curve (schematic) for $F^{19}+p$ reactions leading to the 6.1, 6.9, and 7.1-Mev states of O^{16}.

The most powerful method of obtaining information on the angular momentum changes in a nuclear reaction involves analysis of the angular distributions of emitted particles. In the present case, the pertinent quantities may include the distribution of each of the three alpha-particle groups with respect to the proton beam ($p-\alpha$ distributions), of the subsequent γ rays with respect to the proton beam ($p-\gamma$ distributions), and of the γ rays with respect to the alpha-particle directions ($\alpha-\gamma$ correlations). Three experiments which have appeared in the literature have particular bearing on the work reported in the present paper. The first of these was an exhaustive study of the ($p-\alpha$) and ($\alpha-\gamma$) correlations at proton energies of 340, 669, 873, and 935 kev, carried out by Seed and French[2]; this paper has been a most valuable guide in our work. In the second set of experiments, reported by Sanders,[3] the ($p-\gamma$) angular distributions, involving the 6.14-Mev γ ray (γ_1) and the combined 6.9- and 7.1-Mev γ rays ($\gamma_2+\gamma_3$) were determined at $E_p=873$, 935, and 1372 kev. Finally, Peterson *et al.*[4] have measured ($p-\alpha_2$) and ($p-\alpha_3$) (α particles leading to the 6.9- and 7.1-Mev O^{16} states, respectively) distributions at 873, 935, 1346, and 1372-kev bombarding energy. These experiments, together with elastic scattering studies[5] have served to establish the spins and parities of nearly all the Ne^{20} and O^{16} levels involved, and in several cases have yielded quantitative information on the mixing of orbital angular momenta.

* Supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

† Now at Physics Department, Indiana University, Bloomington, Indiana.

[1] R. F. Christy, Phys. Rev. **89**, 839 (1953).
[2] J. Seed and A. P. French, Phys. Rev. **88**, 1007 (1952). Earlier studies of ($\alpha-\gamma$) correlations at the 340-kev resonance were made by W. R. Arnold, Phys. Rev. **79**, 170 (1950); **80**, 34 (1950), and by Barnes, French, and Devons, Nature **166**, 145 (1950).
[3] J. E. Sanders, Phil. Mag. **43**, 630 (1952); **44**, 1302 (1953).
[4] Peterson, Fowler, and Lauritsen, Phys. Rev. **96**, 1250 (1954); R. W. Peterson, Ph.D. thesis, California Institute of Technology, 1954 (unpublished).
[5] Webb, Hagedorn, Fowler, and Lauritsen, Phys. Rev. **99**, 138 (1955); E. Baranger, Phys. Rev. **99**, 145 (1955).

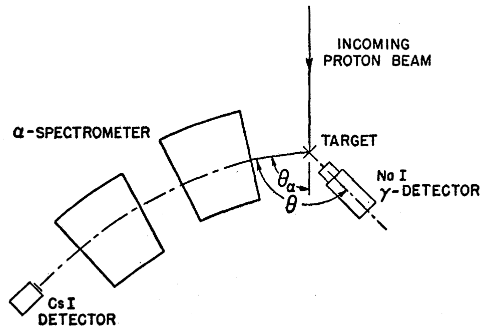

FIG. 2. Schematic diagram of experimental arrangement. The alpha spectrometer was set at either 90° or 95° to the incident beam in the center-of-mass system for all of the correlation patterns.

The present experiments serve to fill out and make more precise the information on several of these levels.

EXPERIMENTAL PROCEDURE

Protons accelerated in a 2-Mev electrostatic generator were passed through a 90-degree magnetic analyzer to give a beam homogeneous in energy to ±0.1%. The cross-sectional area of this beam was about one square millimeter and currents between 0.1 and 2 μa were used during the course of the experiments. Thin evaporated targets of CaF_2 on 0.005-in. copper backings, placed at an angle of 45° to the incident beam were used; target thicknesses were varied between 5 and 10 kev to the incident protons, depending upon the resonance being studied.

A schematic diagram of the target and detecting equipment is shown in Fig. 2. The target chamber was a 2.5-in. o.d. Lucite cyclinder of $\frac{1}{8}$-in. wall thickness. A conventional gamma-ray scintillation counter was mounted on a platform which rotated about the target chamber axis. This counter consisted of a 1.5-in. diameter, 1.5-in. long cylindrical NaI crystal, followed by a DuMont 6292 photomultiplier tube. The face of the crystal was located 2.25-in. from the target chamber axis. Alpha particles emerging from the target were analyzed by a 30-degree strong-focusing magnetic spectrometer[6] of 31.5-in. radius. This spectrometer was used with an energy resolution of 3.8% and a solid angle of about one-thousandth of a sphere. Detection of the alpha particles was accomplished with a CsI crystal followed by a DuMont 6291 photomultiplier tube.

In the present experiments the alpha particles were detected at a fixed angle, corresponding closely to 90° in the center-of-mass system, to the incident beam. The associated gamma rays were then measured at different positions of the gamma detector, in the plane of the alpha spectrometer and the incident proton beam. The angular position of the alpha detector as well as that of the gamma detector was measured relative to the incoming beam, the gamma-detector angle being determined to $\pm\frac{1}{4}$ degree and the alpha-detector angle to better than $\frac{1}{2}$ degree. The variation in sensitivity of the gamma detector as a function of angle was determined by examining the gamma rays from the 935-kev resonance in the $F^{19}(p,\alpha\gamma)O^{16}$ reaction. The angular distribution of these gamma rays is known[7] to be isotropic to ±0.3%. This distribution was measured several times during the present experiments and the maximum observed deviation from isotropy was 2.5%; the indicated corrections, attributable mainly to variations in absorption in the target backing, were made to the experimental data.

A conventional coincidence mixer having a resolving time of 0.35 μsec was used to detect $\alpha-\gamma$ coincidences; allowance was made for the time of flight of the alpha particles through the spectrometer (approximately 0.1 μsec). The gain of the gamma channel was monitored by two integral discriminators, one discriminator accepting only the high-energy peaks of the gamma spectrum while the other discriminator accepted most of the spectrum. The ratio of counts from the two discriminators provided a continuous check on the gamma channel gain.

The angular correlation patterns obtained at several different bombarding energies are shown in Figs. 3–8; a minimum of 1000 total coincidence counts were obtained at each angle and the true coincidence counts were then normalized to the number of alpha-particle counts at the angle. A 2.1-μsec delay line was placed in the alpha channel when only accidental coincidences were to be counted. Fifteen to thirty percent of all of the runs were accidental coincidence measurements.

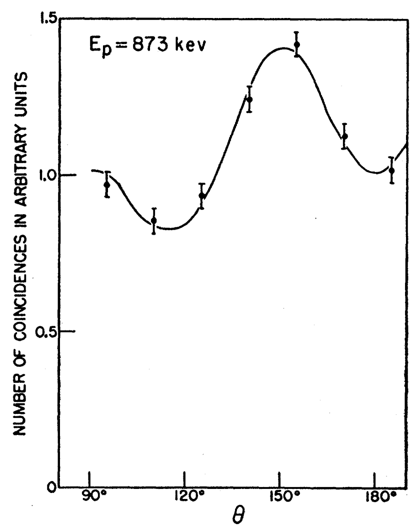

FIG. 3. $(\alpha_1-\gamma_1)$ correlation pattern at a proton bombarding energy of 873 kev; the alpha detector was at $\theta_\alpha=95°$ in the center-of-mass system. The theoretical curve for the parameters given in the text is shown.

[6] H. J. Martin and A. A. Kraus, Rev. Sci. Instr. **28**, 175 (1957).

[7] R. B. Day, Ph.D. thesis, California Institute of Technology, 1951 (unpublished); Phys. Rev. **80**, 131(A) (1950).

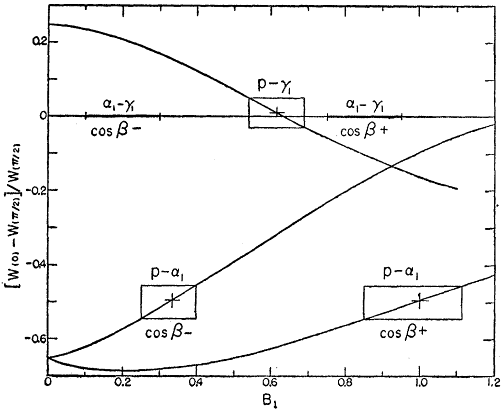

FIG. 4. Calculated angular distributions for α_1 particles and for the corresponding gamma rays at the 873-kev resonance, plotted as functions of B_1; the square of B_1 measures the relative contribution of $l'=4$ and $l'=2$ alpha particles in the disintegration leaving O^{16} in the $J=3^-$ state. The rectangles indicate the probable ranges of B_1 as determined from $(p-\gamma_1)$ and $(p-\alpha_1)$ distributions, while the results of the present experiment are indicated by the two bars in the base line.

EXPERIMENTAL RESULTS AND DISCUSSION

The $(\alpha-\gamma)$ angular correlation function may be written[2]:

$$W(\theta,\phi)\sim\sum_s\sum_{m_s}\Big|\sum_l\sum_{m_l}f_i(l)\langle s,m_s;l,m_l|J_1m_1\rangle$$

$$\times Y_l^{m_l}(\theta_\alpha,0)\sum_{l'}(2l'+1)^{\frac{1}{2}}f_e(l')\langle J_1,m_1|l',0;J_2m_2\rangle$$

$$\times \mathbf{X}_{J_2}{}^{m_2}(\theta,\phi)\Big|^2. \quad (1)$$

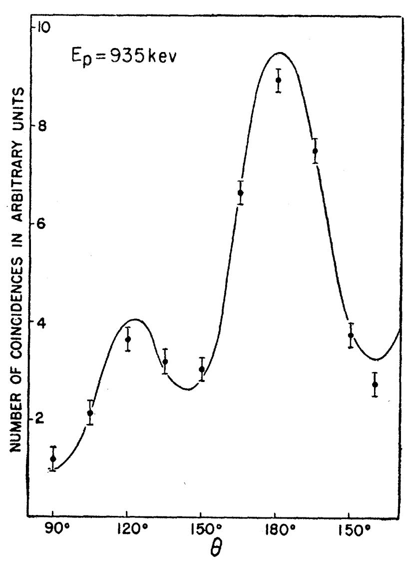

FIG. 5. $(\alpha_1-\gamma_1)$ correlation pattern at a proton bombarding energy of 935 kev; the alpha detector was at $\theta_\alpha=90°$ in the center-of-mass system. The theoretical curve includes an interference term resulting from the 873-kev resonance. The amplitude contributed by this resonance is taken to be -0.059 relative to the amplitude of the 935-kev resonance.

This formula describes the combination of a proton and F^{19} nucleus, in channel spin s (0 or 1), with projection m_s, and with relative orbital angular momentum l, into a compound Ne^{20} state with spin J_1, which subsequently decays to an O^{16} excited state of spin J_2 by emission of alpha particles of orbital angular momentum l'. The angle between the direction of emission of the alpha particle and the proton beam is θ_α; the gamma-ray direction is defined by the spherical coordinates (θ,ϕ) where the z axis lies along the alpha-particle direction and the (p,α) plane is the $\phi=0$ plane. $\mathbf{X}_{J_2}{}^{m_2}(\theta,\phi)$ is the radiation function. In the present experiments, $\theta_\alpha=\pi/2$, $\phi=0$, and θ was varied; in those of Seed and French, $\phi=\pi/2$.

The complex terms $f_i(l)$ and $f_e(l')$ contain the purely nuclear factors determining the course of the reaction.

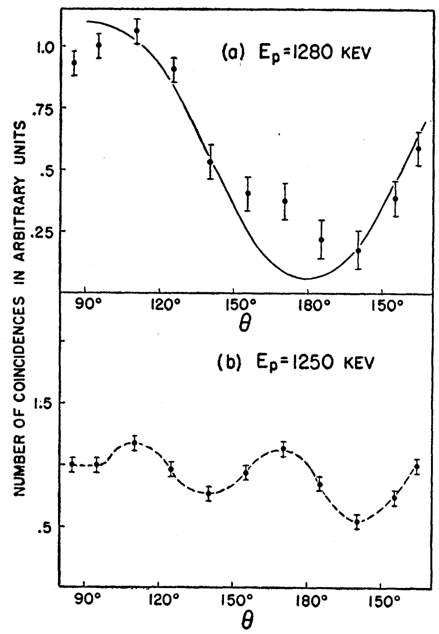

FIG. 6. $(\alpha_1-\gamma_1)$ correlation patterns at 1250 and 1280 kev; the alpha detector was at $\theta_\alpha=95°$ in the center-of-mass system. The solid curve at the 1280-kev resonance is calculated for a $J=3^+$ assignment.

In general, a given Ne^{20} state may be formed with protons of relative angular momentum l and $l+2$, and the decay may proceed via alpha particles of angular momentum l', $l'+2$, $l'+4$, $\cdots$. The relative contribution of the possible l-values may be presented by

$$Ae^{i\alpha}\equiv f_i(l+2)/f_i(l),\quad Be^{i\beta}=f_e(l'+2)/f_e(l'),$$

where α, β are the phase differences between the interfering channels, and A and B the relative amplitudes (contributions from $l'+4$ and higher are here ignored). The phase differences can be calculated, except for an additive term which is an integral multiple of π, from knowledge of the barrier factors.[2] For proton energies well below the barrier, α may be written

$$\alpha=\tan^{-1}\left(\frac{\eta}{l+2}\right)+\tan^{-1}\left(\frac{\eta}{l+1}\right)-\pi,$$

where $\eta=Z_1Z_2e^2/\hbar v$. An analogous expression may be written for β. For a given Ne^{20} state, there will be a unique A and α, but three values of B and β will occur, corresponding to the three alpha-particle groups; these will be distinguished by subscripts, as B_1, B_2, B_3, etc.

Formulas generally similar to (1) may be written for $(p-\gamma)$ and $(p-\alpha)$ correlations as well.[2] Thus, if the spins and parities of intermediate and final states are known, combination of information from the three types of experiment determines A and the sign of $\cos\alpha$ for each Ne^{20} state, and B and the sign of $\cos\beta$ for each alpha-particle group from that state. In the following paragraphs, the various Ne^{20} states will be discussed in turn, and an attempt will be made to determine these parameters for each state, insofar as presently available experimental results permit.

$E_p=873$ kev.—The Ne^{20} level formed at a bombarding energy of 873 kev is well-established[2] as $J=2^-$; it can be formed with channel spin $s=1$ and orbital angular momentum $l=1$ or 3. It cannot be formed with $s=0$. Alpha-particle emission to the 7.1 Mev, $J=1^-$ level of O^{16} (α_3) can occur only through $l'=2$ so that $B_3=0$ and the $(p-\alpha_3)$ angular distribution gives a direct measure of A and $\cos\alpha$. For these parameters, the work of Peterson *et al.*[4] yields $A=0.06$ and $\cos\alpha=+0.438$, consistent with $A=0.1\pm0.1$ given by Seed and French.[2]

The emission of alpha particles (α_1) to the 6.1-Mev, $J=3^-$ state of O^{16} may occur through $l'=2$ or 4; the $(\alpha_1-\gamma_1)$ correlation will then involve A, $\cos\alpha$, B_1, and $\cos\beta_1$. On Fig. 3 are shown the experimental points obtained for this correlation, together with the distribution calculated from Eq. (1), using $A=0.06$, $\cos\alpha=+0.438$ and either $B_1=0.85\pm0.10$, $\cos\beta_1=+0.242$ or $B_1=0.2\pm0.1$, $\cos\beta_1=-0.242$. The curve shown includes corrections for the finite solid angle of the detectors; the probable errors indicated for B_1 have been determined by trial fitting of at least three neighboring curves. The $(\alpha_1-\gamma_1)$ pattern of Seed and French[2] (at $\phi=\pi/2$) is reported by them to yield $B_1=0.54$, $\cos\beta_1=-0.242$. It appears, however, that some considerable latitude may be allowed in this value of B_1, and furthermore that the first set of parameters indicated above, with $\cos\beta_1=+0.242$, also produces an acceptable fit to the data. The $(p-\alpha_1)$ distribution of Peterson *et al.*,[4] $W(\theta)=1-0.49\cos^2\theta$ also admits two values of B_1 : $B_1=0.34\pm0.1$ for $\cos\beta_1=-0.242$, and $B_1=1.0\pm0.15$ for $\cos\beta=+0.242$, where the errors indicated correspond to $\pm10\%$ latitude in the coefficient of $\cos^2\theta$. The $(p-\gamma_1)$ distributions, on the other hand, are independent of $\cos\beta_1$ and determine B_1 uniquely. The distribution reported by Sanders,[3] $W(\theta)=1+(0.01\pm0.04)\cos^2\theta$ gives $B_1=0.62\pm0.08$.

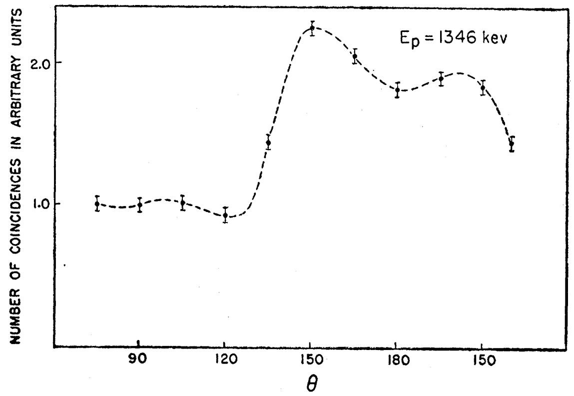

FIG. 7. $(\alpha_1-\gamma_1)$ correlation pattern at 1346 kev; the alpha detector was at $\theta_\alpha=90°$ in the center-of-mass system.

The values and ranges of B_1 derived from these three experimental results are exhibited in Fig. 4, where, for the $(p-\alpha_1)$ and $(p-\gamma_1)$ distributions, the calculated values of $a\equiv[W(0)-W(\pi/2)]/W(\pi/2)$ are shown as a function of B_1. The blocks on the diagram show the reported values and ranges of the quantity a (vertical scale) and the resulting values and estimated probable errors in B_1 (horizontal scale). The values of B_1 determined in the present experiment are indicated on the base line. It is evident that the results from $(p-\alpha_1)$ and $(\alpha_1-\gamma_1)$ distributions agree reasonably well, but the fit of the $(p-\gamma_1)$ datum leaves something to be desired. Taking the probable error estimates at face value, a value of B_1 near 0.8 is indicated, with the lower alternative, near 0.2, much less likely. Alpha particles (α_2) to the $J=2^+$ state of O^{16} occur through $l'=1$ or 3. Peterson *et al.* give $B_2=0.32$, $\cos\beta_2=-0.250$.

$E_p=935$ kev.—This level has $J=1$ and even parity and is formed mainly by s-wave protons.[2–4] Because

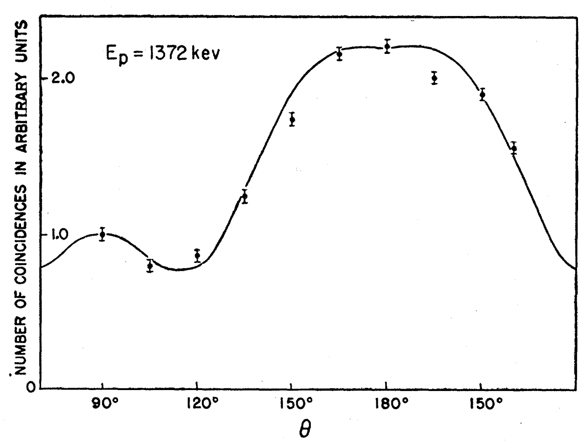

FIG. 8. $(\alpha_1-\gamma)$ correlation pattern at 1372 kev; the alpha detector was at $\theta_\alpha=90°$ in the center-of-mass system. The theoretical curve for the parameters given in the text is shown.

TABLE I. Angular momentum admixtures for three states of Ne^{20}.

E_{res}	J,π	A ($l_p=3/l_p=1$)	$\cos\alpha$	B_1 ($l'=4/l'=2$)	$\cos\beta_1$	B_2 ($l'=3/l'=1$)	$\cos\beta_2$
873	2^-	0.06	+0.438	0.85[a]	+0.242[a]	0.32	−0.250
1346	2^-	0.04	+0.595			0.43	−0.156
1372	2^-	0.05	+0.605	0.92	−0.319	0.58	−0.15

[a] The values $B_1=0.2$, $\cos\beta_1=-0.242$ are also possible, though less probable.

TABLE II. Reduced-width ratios.

E_{res}	$\gamma_p^2(3)/\gamma_p^2(1)$	$\gamma_{\alpha 1}^2(4)/\gamma_{\alpha 1}^2(2)$	$\gamma_{\alpha 2}^2(3)/\gamma_{\alpha 2}^2(1)$
873	1.2	10.9±2.5[a]	1.3
1346	2.8		1.5
1372	0.36	8.5±1.3	2.6

[a] The probable errors indicated result from the uncertainty in B_1; reduction of the assumed interaction radius by 10% would increase these ratios by about 40%.

of the small value of $\cos\alpha=0.033$, the patterns are relatively insentitive to d-wave admixture, and because no l'-mixtures are possible in the alpha decays, all three B's are zero. The $(\alpha_1-\gamma_1)$ pattern, shown in Fig. 5 serves mainly as a check on the angular resolution of the apparatus. The solid curve in the figure is calculated from Eq. (1) with $A=B=0$, again modified by the detector geometry. The slight asymmetry about $\theta=180°$ appears to be real, and is ascribed to interference with the 2^- state at $E_p=873$ kev. The amount of interference required is consistent with the known width of the 2^- state.

E_p=1280 kev.—Angular distribution studies at this resonance are complicated by the overlap of neighboring resonances, principally one[8] at 1190 kev with $\Gamma\cong 110$ kev. The $(\alpha_1-\gamma_1)$ correlations observed at $E_p=1280$ kev and at 1250 kev, well below the resonance ($\Gamma=19$ kev), are shown in Fig. 6. Because of the evident complexity of the background, no detailed analysis can be made, but the calculated distribution for $J=3^+$, an assignment suggested by earlier $(p-\alpha)$ and $(p-\gamma)$ distributions[4,7,9] appears to give an acceptable fit at resonance [solid curve in Fig. 6(a)]. Assignments of 1^+ or 2^- would lead to a maximum at $\theta=180°$ and can be excluded.

E_p=1346 kev.—The $(\alpha_1-\gamma_1)$ correlation pattern is shown in Fig. 7. The evident asymmetry about $\theta=180°$ indicates strong interference from a neighboring state of opposite parity, presumably that at $E_p=1280$ kev. Alpha-particle distributions[4] $(p-\alpha_2)$, $(p-\alpha_3)$ and elastic scattering studies[5] are consistent with $J=2^-$. Peterson *et al.*[4] find $A=0.14$, $\cos\alpha=+0.595$, $B_2=0.43$, $\cos\beta_2=-0.156$.

E_p=1372 kev.—Elastic scattering of protons at this resonance[5] indicates $J=1^-$ or 2^-. The angular distribution $(p-\gamma_2+\gamma_3)$ reported by Sanders[3] is consistent with $J=2^-$ and excludes $J=1^-$. The $(\alpha_1-\gamma_1)$ correlation is shown in Fig. 8, where the solid curve represents the theoretical fit using the parameters $J=2^-$, $A=0.05$, $\cos\alpha=+0.605$, $B_1=0.92\pm0.07$, $\cos\beta_1=-0.319$. A value of B_1 nearly 4.5 times larger must be used to obtain an acceptable fit with $\cos\beta_1=+0.319$. The $(p-\gamma_1)$ distribution[3] is relatively insensitive to the choice of A and leads to $B_1=0.95\pm0.08$, in excellent agreement with the present value. Some difficulty exists, however, in the $(p-\alpha_3)$ distribution, which depends only upon A and $\cos\alpha$. The predicted distribution with $A=0.05$, $\cos\alpha=+0.605$ is

$$W(\theta)=1+0.54\cos^2\theta+0.46\cos^4\theta,$$

whereas Peterson *et al.*[4] found the distribution to be approximately isotropic. Further measurements made to check this point revealed an anisotropy of $I(160°)/I(90°)=1.33$, but there remains some discrepancy with the predicted value. A satisfactory fit to Peterson's $(p-\alpha_2)$ distribution can be obtained with $B_2=0.58$, $\cos\beta_2=-0.15$. It may be noted that interference effects appear to play a strong role in both the α- and γ-ray distributions[4,7] at this resonance, and more complete experimental information is needed before definite conclusions can be drawn.

The present results, together with those from other cited works, are summarized in Table I. To the extent to which these resonances may be regarded as isolated, one may relate the observed widths $\Gamma_p(l)$ and reduced (or intrinsic) widths $\gamma_p^2(l)$, both in energy units, through the formula

$$\Gamma_p(l)=2kRP_l\gamma_p^2(l),$$

where $P_l=[F_l^2(kR)+G_l^2(kR)]^{-1}$ is the penetration factor. Then A is related to the ratio of reduced-widths for l and $l+2$ entering protons by

$$A^2=\frac{\gamma_p^2(l+2)}{\gamma_p^2(l)}\left(\frac{P_{l+2}}{P_l}\right). \tag{2}$$

A similar expression may be written for B and the corresponding alpha-particle reduced-widths. The reduced-width ratios so calculated, using a radius $R=1.40(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})\times10^{-13}$ cm, are exhibited in Table II. It is of interest to observe that for protons and for α_2, the ratios are near unity, as might be expected for levels of relatively great complexity, but for α_1 it would appear that the higher angular momentum considerably outweighs the lower in importance. In fact, for the two cases studied here, the absolute values of the reduced-widths for g-wave alpha emission amount to about ten percent of the single-particle limit, $3\hbar^2/2ma^2$. While the evidence here presented can in neither instance be said to be conclusive, the implications for nuclear theory are sufficiently striking to warrant further exploitation of the technique, with a view to

[8] S. E. Hunt and K. Firth, Phys. Rev. 99, 786 (1955).
[9] C. Y. Chao, Phys. Rev. 80, 1035 (1950).

application in other light nuclei. It may be remarked that two other examples of large alpha-particle reduced-widths are known[5] in the same region of Ne^{20}, at $E_p=340$ ($J=1^+$) and 598 kev ($J=2^-$). In both cases, it is again the α_1 group, leading to the 3^- state of O^{16} which shows the effect.

ACKNOWLEDGMENTS

We are indebted to Dr. C. A. Barnes and Dr. R. F. Christy for many valuable comments and suggestions during the course of this work. Thanks are also due to Mr. C. Cook for his assistance during much of the experimental work.

PHYSICAL REVIEW VOLUME 106, NUMBER 6 JUNE 15, 1957

New Empirical Equation for Atomic Masses*

Harris B. Levy
University of California Radiation Laboratory, Livermore, California
(Received March 4, 1957)

A new empirical equation for atomic masses has been developed. This equation has been successfully applied to atomic masses of nuclides heavier than nickel. The form used is that of an expression for the mass defect, $\Delta M = M - A$, as a function of Z and A:

$$\Delta M(A,Z)=\alpha_0+\alpha_1 A+\alpha_2 Z+\alpha_3 AZ+\alpha_4 Z^2+\alpha_5 A^2+\delta.$$

Because of the effects of nuclear shell structure a different set of coefficients is necessary for the different nuclear shell regions.

Atomic masses calculated from this equation agree with experimental mass values to within ±0.5 millimass units in 75% of the 340 nuclides studied and agree to within ±1.5 millimass units in 95% of the nuclides. Beta-decay energies were calculated with the new equation and checked against a total of 179 experimental values. Agreement of calculated values with experiment was better than ±0.5 Mev in 95% of the cases and within ±0.25 Mev in 84%.

I. INTRODUCTION

A GOOD equation for atomic masses can be a highly useful tool in many problems of nuclear physics where an accurate estimate of such quantities as nuclear binding energies, alpha- and beta-decay energies, Q values of nuclear reactions, etc. are desired. These quantities all involve differences between atomic masses. A mass equation that accurately reproduces known atomic masses is perhaps the most convenient means of estimating atomic masses of nuclides that have so far defied measurement. To facilitate calculations it is desirable that such an equation should be as simple as possible in form.

The mass equation that is probably most widely used at present is the Fermi-Weizsäcker semiempirical mass equation,[1] hereafter referred to as the FW mass equation. The form of this equation was dictated by theoretical considerations, with the numerical values of the coefficients being obtained by fitting the equation to known masses. The FW mass equation contains terms with fractional powers, and calculations with it are not simple. Metropolis and Reitwiesner have compiled a table of atomic masses using the FW mass equation,[2] thereby making calculations much easier. However, in many regions of the periodic table the FW equation deviates very markedly from experimentally measured atomic masses and becomes unsatisfactory for many calculations unless empirical corrections, which are sometimes elaborate, are made.

Recently, Green has proposed an empirical function to describe the over-all behavior of the mass surface.[3,4] Green's equation is simpler and agrees with known masses somewhat better than the FW mass equation, but the disagreement with experimental masses is still fairly large in some places, especially near "magic number" nuclei.

Much of the discrepancy between experimental masses and the Green and FW equations arises because of the latter having ignored the effects of nuclear shell structure on the mass surface. Green obtains improved agreement when he adds to his simplified equation a set of empirical functions[4,5] to correct for shell-structure effects. However, Green's equation thereby loses much of its ease of handling for calculations, while some annoying disagreements remain.

In this paper will be presented a new empirical equation developed through a new approach. The problem of nuclear shell structure was met by treating each shell region individually. (Justification for this is taken up in the next section.) The result is a simple

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] C. F. von Weizsäcker, Z. Physik **96**, 431 (1905), and E. Fermi, *Nuclear Physics*, notes by Orear, Rosenfeld, and Schluter (University of Chicago Press, Chicago, Illinois, 1950).

[2] N. Metropolis and G. Reitwiesner, U. S. Atomic Energy Commission Report NP-1980, 1950 (unpublished).

[3] A. E. S. Green and N. A. Engler, Phys. Rev. **91**, 40 (1953).

[4] A. E. S. Green, *Nuclear Physics* (McGraw-Hill Book Company, Inc., New York, 1955), pp. 244–270.

[5] A. E. S. Green and D. F. Edwards, Phys. Rev. **91**, 46 (1953).