Sum Rules for Photodisintegration of H^3 and He^3†

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We apply the sum rules of Levinger and Bethe to calculate the dipole bremsstrahlung weighted cross section ($\sigma_b = \int_0^\infty (\sigma/W)dW$) and the integrated cross section ($\sigma_{\rm int} = \int_0^\infty \sigma dW$) for H^3 and He^3 using Irving's wave function. On assuming charge symmetry and neglecting Coulomb repulsion, we find that $\sigma_b(H^3) = \sigma_b(He^3)$ and $\sigma_{\rm int}(H^3) = \sigma_{\rm int}(He^3)$.

In a recent paper Rustgi and Levinger[1] have extended the sum-rule calculations of Levinger and Bethe[2] to include two-body Heisenberg forces. We shall in this note apply these sum rules to calculate the dipole bremsstrahlung weighted cross section ($\sigma_b = \int_0^\infty (\sigma/W)dW$) and the integrated cross section ($\sigma_{\rm int} = \int_0^\infty \sigma dW$) for H^3 and He^3 nuclei. We use a two-body spin dependent Yukawa potential:

$$V(r_{ij}) = -V_0(\exp(-Kr_{ij})/Kr_{ij}) \times [1-(y+z)+(y+z)P_{ij}^B] \quad (1)$$

and Irving's wave function.[3] Here r_{ij} is the relative distance between two nucleons i and j, and V_0 and K have the standard values for the two-body triplet potential[3]:

$$V_0 = 67.3 \text{ Mev and } 1/K = 1.17\times 17^{-13} \text{ cm.} \quad (2)$$

$(y+z)$ is the fraction of Heisenberg plus Bartlett exchange with operator P^B. The ratio (singlet depth/triplet depth) $=1-2(y+z)$ is called q by Irving and has the numerical value 0.69 chosen to fit the two-body system.

According to Irving,[3]

$$\psi = N \exp[-\sqrt{2}\alpha(\rho^2+r^2)^{\frac{1}{2}}], \quad (3)$$

$$\boldsymbol{\rho} = \tfrac{1}{2}[(\mathbf{r}_2-\mathbf{r}_1)+(\mathbf{r}_3-\mathbf{r}_1)],$$
$$\mathbf{r} = \tfrac{1}{2}\sqrt{3}(\mathbf{r}_2-\mathbf{r}_1); \quad \mathbf{R} = \tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3), \quad (4)$$

$$N = (\alpha^6 2^9/\pi^3 5!)^{\frac{1}{2}}. \quad (5)$$

Here $\mathbf{r}_1$ is the position vector of the proton, and $\mathbf{r}_2$ and $\mathbf{r}_3$ are the position vectors for the neutrons in an H^3 nucleus. The variation parameter α has been adjusted by Irving to give the lowest energy, which is in good agreement with experiment.

Levinger and Bethe evaluate the $E1$ bremsstrahlung weighted cross section σ_b as

$$\sigma_b = (4\pi^2/3)(e^2/\hbar c)[(\mathbf{r}_1-\mathbf{R})^2]_{00}, \quad (6)$$

$$\sigma_b = (14\pi^2/9)(e^2/\hbar c)(1/\alpha^2) = 1.32 \text{ mb}, \quad (7)$$

where we have used $\alpha = 0.92\times 10^{13}$ cm from Table I in Irving.[3] We obtain the same bremsstrahlung weighted cross section for He^3, because

$$(\mathbf{r}_1-\mathbf{R}) = -[(\mathbf{r}_2-\mathbf{R})+(\mathbf{r}_3-\mathbf{R})]. \quad (8)$$

Equation (6) shows that σ_b for H^3 is proportional to the mean square radius. Irving's wave function for He^4 gives too small values[1] for both σ_b and the mean square radius as compared to photonuclear and electron scattering experiments, respectively. Also, Irving's wave function for H^3 gives too high an H^3-He^3 Coulomb energy difference, showing that for H^3 Irving's mean square radius is too small. Experiments on the photoeffect and electron scattering from H^3 are not available for comparison with our numerical result in Eq. (7).

According to Rustgi and Levinger,[1] the integrated cross section $\sigma_{\rm int}$ for H^3 is given by

$$\sigma_{\rm int} = \int \sigma dW = \frac{4\pi^2 e^2\hbar}{3Mc}\left\{1-\frac{M(x+\frac{1}{2}y)}{2\hbar^2} \times \int \psi_0^* \sum_i \sum_j V(r_{ij}) r_{ij}^2 P_{ij}^M \psi_0 d\tau\right\}, \quad (9)$$

where we have used $(P_{ij}^B)_{\text{spin average}} = \frac{1}{2}$. The $\frac{1}{2}y$ term comes from the Heisenberg exchange; i denotes proton and j neutron, the double sum being taken over all pairs of neutrons and protons; x is the fraction of Majorana exchange force; $V(r_{ij})$ is the neutron-proton potential and P_{ij}^M is the Majorana exchange operator.

Substituting $V(r_{ij})$ from Eq. (1) and carrying out the integrations using the transformation $\rho = R\cos\theta$ and $r = R\sin\theta$, we obtain

$$\sigma_{\rm int} = \frac{4\pi^2 e^2\hbar}{3Mc}\left\{1+\frac{M(x+\frac{1}{2}y)2^4(6)^{\frac{1}{2}}V_0(1+q)}{2\hbar^2\pi K\alpha} \times \int_0^1 \frac{(1-u^2)^{\frac{1}{2}}u^3 du}{(1+Cu)^7}\right\}, \quad (10)$$

where $u = \sin\theta$ and $C = (K/6^{\frac{1}{2}}\alpha)$. The value of the integral in Eq. (10) is found to be

$$\int_0^1 \frac{(1-u^2)^{\frac{1}{2}}u^3 du}{(1+Cu)^7} = \frac{1}{720}\left\{\frac{96+741C^2+120C^4-12C^6}{(1-C^2)^5} - \frac{315C(1+2C^2)\cos^{-1}C}{(1-C^2)^{11/2}}\right\}. \quad (11)$$
$$= 2.58\times 10^{-2}.$$

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[1] M. L. Rustgi and J. S. Levinger, Phys. Rev. **106**, 530 (1957); M. L. Rustgi, Ph.D. thesis, Louisiana State University, January, 1957 (unpublished).
[2] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).
[3] J. Irving, Phil. Mag. *42*, 338 (1951).

We therefore obtain

$$\sigma_{\text{int}} = \frac{4}{3}\frac{\pi^2 e^2 \hbar}{Mc}[1+0.55(x+\tfrac{1}{2}y)] \quad (12)$$

$$=40[1+0.55(x+\tfrac{1}{2}y)] \text{ Mev-mb.}$$

Because of the assumption of charge symmetry, σ_{int} is the same for H^3 and He^3.

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Neutron Spectrum from $T+t$†

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The energy spectrum of neutrons from the $T+t$ reactions has been measured at 0° for a triton energy of 1.48 Mev. The energies of the neutrons extend from 0 to 12 Mev. A peak at the high-energy end at 11.3 Mev is ascribed to the $T(t,n)He^5$ reaction. No other peaks are observed within the counting statistics of the experiment.

INTRODUCTION

DETERMINATION of the energy distribution of the neutrons from the triton bombardment of tritium provides a means of investigating the details of the reactions which take place. Most of the possible reactions are many-body disintegrations, each of which results in a complex spectrum of neutrons. The reactions which are energetically possible are listed below:

$T(t,n)He^5(n)He^4$, $Q_1=10.37$ Mev, $Q_2=0.95$ Mev,

$T(t,n)He^{5*}(n)He^4$,

$T(t,n,n)He^4$, $Q=11.32$ Mev.

$T(t,{}_0n^2)He^4$,

Each of these reactions should produce a characteristic neutron energy spectrum. The unstable product nuclei He^5 and ${}_0n^2$ (dineutron) are of fundamental interest, and past evidence regarding their nature has not been completely consistent. It is generally accepted and borne out by numerous experiments that He^5 and its mirror nucleus Li^5 are unstable and very short-lived ($\sim 10^{-21}$ sec) but nevertheless act as though formed in definite states, the lowest-lying or ground state being a rather broad $P_{\frac{3}{2}}$ state. The next higher state or first excited state is thought to be a broad $P_{\frac{1}{2}}$ state, but as to its position and width conflicting evidence has appeared. In a previous determination of the neutron energy spectrum from the $T+t$ reaction in which one of the present authors participated,[1] a group of neutrons was observed which was attributed to the formation of He^5 in a state 2.6 Mev above the ground state. Titterton and Brinkley[2] likewise found a group of neutrons from the reaction $Li^6(\gamma,n)$ which they attributed to an excited state in Li^5, 2.5 Mev above the ground state. On the other hand, Allen *et al.*,[3] in their study of neutrons from the $T+t$ reactions at low energies, did not find a second group of neutrons. Likewise, other investigators in studying other reactions involving He^5 and Li^5 have not observed well-defined particle groups other than those attributed to the ground state and conclude that the excited state is either very improbable, very broad, or both.[4–7] Analyses of n-He^4 scattering and p-He^4 scattering[8,9] also are not in agreement as to the position and width of the expected $P_{\frac{1}{2}}$ state. However, the more recent of the analyses,[8] which is based on more extensive data, finds that a best fit is obtained with a level splitting of 6 Mev and a reduced width of 29.3 Mev for the $P_{\frac{1}{2}}$ level. The $P_{\frac{3}{2}}$, $P_{\frac{1}{2}}$ states presumably may form a doublet based on spin-orbit coupling. In view of the significance of the reaction, it seemed worthwhile to attempt to resolve some of the discrepancies in the experimental information by remeasuring the neutron spectrum from the $T+t$ reaction, utilizing improved apparatus now available.

APPARATUS AND PROCEDURE

A 1-inch-long gas target with a 0.0002-inch aluminum window was filled with tritium gas to 30-cm pressure.

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[1] W. T. Leland and H. M. Agnew, Phys. Rev. **82**, 559 (1951).
[2] E. W. Titterton and T. A. Brinkley, Proc. Phys. Soc. (London) **A64**, 232 (1951).
[3] Allen, Almqvist, Dewan, Pepper, and Sanders, Phys. Rev. **82**, 262 (1951).
[4] G. M. Frye, Jr., Phys. Rev. **93**, 1086 (1954).
[5] Good, Kuntz, and Moak, Phys. Rev. **94**, 87 (1954).
[6] Almqvist, Allen, Dewan, and Pepper, Phys. Rev. **91**, 1022 (1950).
[7] C. D. Moak, Phys. Rev. **92**, 383 (1954).
[8] D. C. Dodder and J. L. Gammel, Phys. Rev. **88**, 520 (1952).
[9] R. K. Adair, Phys. Rev. **86**, 155 (1952).