Elastic Scattering of Alpha Particles with the Optical Model*†

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The scattering of medium-energy alpha particles by nuclei has been examined with the optical model. It is possible to reproduce the detailed features of the experimental data with such a model. An examination of the approximate models suggested by other authors is presented. Some features of the data can be understood on the basis of a one-parameter "black" nucleus model (i.e., the Blair model) interpreted quantum-mechanically. However, the requirement of detailed fit to the experimental data requires a model with more parameters. The "blackness" of nuclear matter to alpha particles causes the scattering cross section to be insensitive to the real part of the alpha-particle—nucleus interaction.

INTRODUCTION

IN recent years, reports have appeared in the literature of a series of experiments[1] on elastic scattering of alpha particles from nuclei in the intermediate energy range $20 \lesssim E \lesssim 40$ Mev. These data have been subjected to analysis by several authors[2-6] who have attempted to construct approximate interaction models which would explain or correlate the experimental data. It is the purpose of this paper to examine these interaction models on the basis of exact calculations assuming an optical or complex interaction potential between the alpha particle and the target nucleus.

The optical model is a phenomenological attempt to discuss the reflective, refractive, and absorptive characteristics of the particle-nucleus interaction. It has been used successfully in the analysis, for example, of neutron-nucleus[7] and proton-nucleus[8,9] elastic scattering where there appears to be some justification for its use based on the detailed consideration[10] of nuclear dynamics. Its use in the analysis of elastic scattering of alpha particles from nuclei has at present little or no comparable justification. We employ it here primarily in an attempt to analyze other phenomenological models which have direct relationship to the optical model. Although we have not attempted to fit all the experimental data with such a model, such an analysis has recently been reported.[11]

OPTICAL MODEL ANALYSIS

The nonrelativistic optical-model potential employed here is written as

$$V(r) = Vf(r) + iWg(r) + V_c. \qquad (1)$$

In Eq. (1), V and W, and $f(r)$ and $g(r)$ are the strengths and spatial forms of the real and imaginary parts of the alpha-nucleus potential, respectively, exclusive of the electrostatic potential V_c. For the work reported here, $f(r) = g(r)$ and is taken to have the Fermi form:

$$f(r) = \left[1 + \exp\left(\frac{r-R}{a}\right)\right]^{-1}. \qquad (2)$$

To compute V_c, a uniform sphere of charge of radius R is employed.[12] The parameter R in Eq. (2) is simply defined by $f(R) \simeq \frac{1}{2} f(0)$ for $R \gg a$; it is to be considered as the "radius" of the alpha-nucleus interaction. In the analysis reported here, we shall concentrate on the scattering of alpha particles from silver.

Figure 1 exhibits two attempts to fit the experimental data on $\sigma(\theta)/\sigma_R(\theta)$ (where $\sigma_R(\theta)$ is the differential cross section for scattering in a point Coulomb field) for 22-Mev alpha particles incident on silver. The set of parameters $V = -50$ Mev, $W = -20$ Mev, $R = 7.5$

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1 G. W. Farwell and H. E. Wegner, Phys. Rev. **95**, 1212 (1954); Wall, Rees, and Ford, Phys. Rev. **97**, 726 (1955); Wegner, Eisberg, and Igo, Phys. Rev. **99**, 825 (1955); Eisberg, Igo, and Wegner, Phys. Rev. **100**, 1309 (1956); E. Bleuler and D. J. Tendam, Phys. Rev. **99**, 1605 (1955).

2 J. S. Blair, Phys. Rev. **95**, 1218 (1954).

3 Wall, Rees, and Ford, Phys. Rev. **97**, 726 (1955).

4 N. Oda and K. Harada, Progr. Theoret. Phys. Japan **15**, 545 (1956).

5 C. B. O. Mohr and B. A. Robson, Proc. Phys. Soc. (London) **A69**, 365 (1956).

6 C. E. Porter, Phys. Rev. **99**, 1400 (1955).

7 See, for example: Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

8 See, for example: Melkanoff, Moszkowski, Nodvik, and Saxon, Phys. Rev. **101**, 507 (1956).

9 Glassgold, Cheston, Stein, Schuldt, and Erickson, preceding paper [Phys. Rev. **106**, 1207 (1957)].

10 F. L. Friedman and V. F. Weisskopf in *Niels Bohr and the Development of Physics*, edited by W. Pauli (Pergamon Press, London, 1955), Chap. VIII.

11 G. Igo and R. M. Thaler, Phys. Rev. **106**, 126 (1957).

12 This approximation is inconsistent with the form factor $f(r)$. However, a similar approximation in proton-nucleus scattering has little effect on the resultant angular distributions (see reference 9). The same situation should apply here. The difference in the values of the electrostatic potential at the radius of interaction R for a uniform charged sphere and for a charge density with a form factor approximating Eq. (2) is small compared to the value of $V(R)$. Although the derivatives of the charge density at R are quite different in the two cases, the average value of the derivative of the charge density (which is large in the region of R only) is proportional to the electrostatic potential at the origin. With the values of the optical-model parameters necessary to approximate the experimental data in the medium-energy range, the scattering cross section is insensitive to the value of the electrostatic potential at the origin.

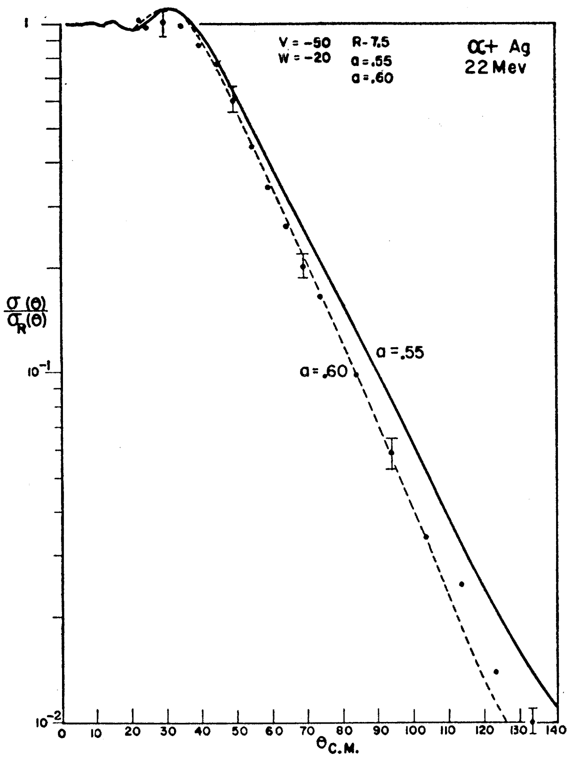

FIG. 1. $\sigma(\theta)/\sigma_R(\theta)$ for 22-Mev alpha particles incident on silver. The set of optical parameters $V=-50$ Mev, $W=-20$ Mev, $R=7.5\times10^{-13}$ cm, $a=0.60\times10^{-13}$ cm gives adequate fit to the data. The experimental uncertainties indicated are representative of the uncertainties at other angles.

$\times10^{-13}$ cm, $a=0.60\times10^{-13}$ cm gives adequate fit to the data. The same set of parameters used in the analysis of the scattering of 40-Mev alpha particles from silver is illustrated in Fig. 2. To obtain the fit shown in Fig. 2, a change in the normalization of the experimental data of approximately 20% was employed. The fit for the higher energy data is apparently unsatisfactory for $\theta\gtrsim60°$. However, the spacing of the angles at which data were taken beyond 60° is of the same order of magnitude as the separation of adjacent maxima and minima predicted by the optical model. Consequently, the apparent absence of structure in the data beyond 60° cannot at this stage be considered as contradicting the predictions of the optical model. The same set of parameters gives adequate fit in the case of copper at 40 Mev.

In attempting to ascertain whether the set of parameters leading to Fig. 1 was in any sense unique, the real part of the potential, V, was decreased to -150 Mev. The result of this large change in V, as well as the effects produced by alteration of W and R at a large real well depth, is illustrated in Fig. 3. It is evident that the set of optical model parameters $V=-150$ Mev, $W=-20$ Mev, $R=7.09\times10^{-13}$ cm, and $a=0.6\times10^{-13}$ cm gives equally good fit to the experimental data on silver at 22 Mev. This agreement is maintained to a certain extent for 40-Mev alpha particles on silver as is illustrated in Fig. 4. Small changes in W and/or R will better the agreement at 40 Mev and cause little change at 22 Mev. It appears, therefore, that the optical model cross section for the scattering of medium-energy alpha particles from silver is insensitive to large changes in the depth of the real part of the potential.

There is little *a priori* reason to prefer a shallow or deep well for the alpha-particle—nucleus interaction. In one limit, if the nucleons in the alpha particle interacted with the nucleus as if they were free, then one would expect as the upper limit to the real part of the alpha-particle—nucleus potential ≈160 Mev, if one uses the "traditional" nuclear radius, or 240 Mev, if one uses as the radius of the nucleus the value suggested by the analysis of proton-nucleus scattering.[9] Tolhoek and Brussaard[13] have recently examined this limit and have demonstrated that analysis of alpha-decay lifetimes of heavy nuclei, in this limit, yield nuclear radii in close agreement with the radii obtained by electromagnetic means. If we take the parameters used in this alpha-decay analysis (these parameters are $V=-134$ Mev, $W=0$, $R=6.95\times10^{-13}$ cm, $a=0.455\times10^{-13}$ cm) and examine the optical-model cross section they produce, we note in Fig. 5 that poor agreement with the experimental scattering data is obtained. Increasing W to -15 and -30 Mev damps the large amplitude oscillations satisfactorily; however, it is

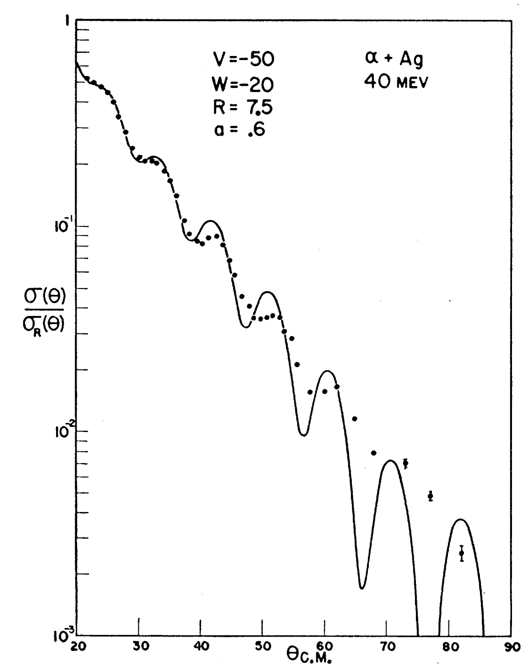

FIG. 2. $\sigma(\theta)/\sigma_R(\theta)$ for 40-Mev alpha particles incident on silver. The optical model parameters are the same as those giving adequate fit to the data at 22 Mev illustrated in Fig. 1.

[13] H. A. Tolhoek and P. J. Brussaard, Physica **21**, 449 (1955).

necessary to increase a and R also before agreement with the scattering data is reached.

In another limit, we might envisage the nucleons in the alpha particle as having lost all memory of their Fermi character and that, as a consequence, the alpha particle in nuclear matter acts as a perfect boson. In this limit, the existence of alpha-particle states of low excitation in heavy nuclei would imply a very shallow boson-nucleus well since at low nuclear temperature, the alpha particles would lie close to the bottom of this well.[14] As a consequence of the apparent multiplicity of the sets of optical model parameters which will give adequate fit to the data, we have not attempted to find unique fits to all the available alpha-particle scattering data.

APPROXIMATE INTERACTION MODELS

Blair[2] has suggested that the observed form of $d\sigma(\theta)/dE$ at $E\approx 20$ Mev can be understood if one assumes that the nucleus is "black" to alpha particles whose angular momenta are less than or at most equal to a critical value l' and "transparent" to alpha particles

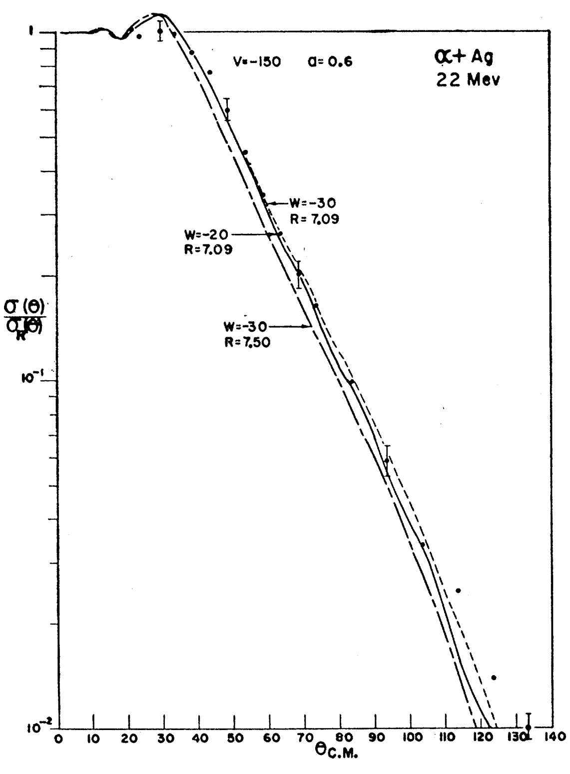

FIG. 3. $\sigma(\theta)/\sigma_R(\theta)$ for 22-Mev alpha particles incident on silver. The real part of the optical model potential is -150 Mev in contrast to the -50-Mev well depth used in Fig. 1 and 2.

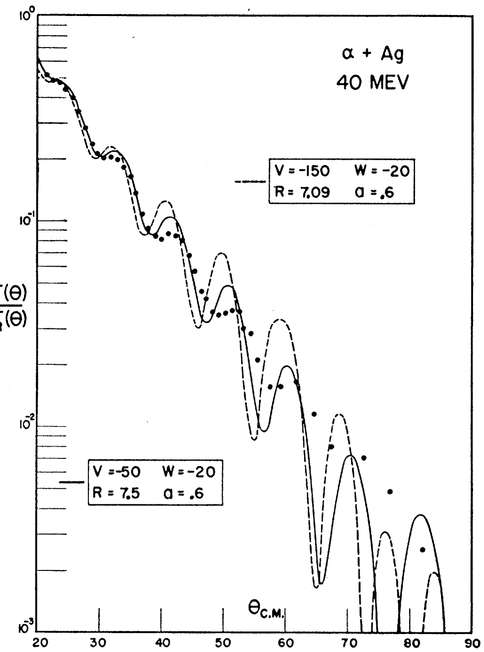

FIG. 4. $\sigma(\theta)/\sigma_R(\theta)$ for 40-Mev alpha particles incident on silver. The optical model parameters are $V=-150$ Mev, $W=-20$ Mev, $R=7.09\times10^{-13}$ cm, $a=0.6\times10^{-13}$ cm.

whose angular momenta are greater than l'. In other words, the amplitude of all outgoing partial waves with $l\leq l'$ is set equal to zero, whereas, for $l>l'$, the amplitude and phase are set equal to those appropriate to a pure Coulomb field. The critical value of l is determined by equating the classical turning point of the motion in a Coulomb field to the radius of interaction R of the

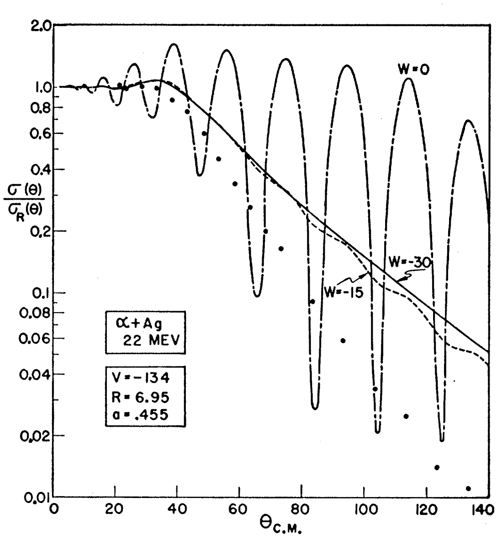

FIG. 5. $\sigma(\theta)/\sigma_R(\theta)$ for 22-Mev alpha particles incident on silver. V, R, and a determined by the analysis of alpha-decay rates of heavy nuclei.[13]

[14] The authors are indebted to Professor D. Peaslee for pertinent comments on this point.

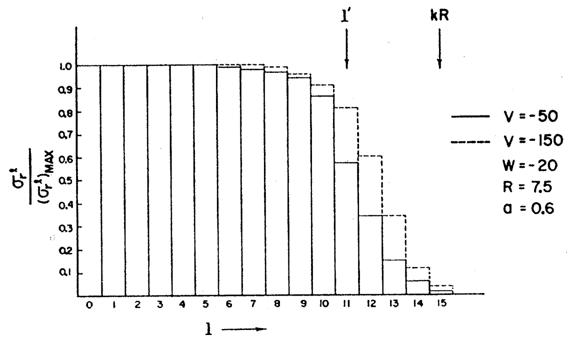

FIG. 6. The reaction cross section σ_r^l for partial waves up to $l=15$ divided by $(\sigma_r^l)_{\max}=(2l+1)\pi\lambda\!\!\!^{-2}$ plotted against l for 22-Mev alpha particles on silver using the optical model parameters $V=-50$ and -150 Mev, $W=-20$ Mev, $R=7.5\times10^{-13}$ cm, $a=0.60\times10^{-13}$ cm. For 22-Mev alpha particles on silver, $kR\approx15$ and $l'\approx11$.

alpha-nucleus interaction,[15] i.e.:

$$E_\alpha=\frac{2Ze^2}{R}+\frac{l'(l'+1)\hbar^2}{2M_\alpha R^2}. \qquad (3)$$

This model produces a $\sigma(\theta)/\sigma_R(\theta)$ which is somewhat too large in the region of the rise at forward angles and oscillates badly for $\theta\gtrsim80°$. According to Blair, both of these features may be attributed to the sharp-cutoff nature of the model.

The Blair model may be interpreted in the following manner: the alpha-particle—nucleus interaction is explicitly velocity-dependent in such a way that the amplitudes of the outgoing spherical waves vanish for all $l\leq l'$ whereas for $l>l'$ the alpha-particle—nucleus interaction is pure Coulombic. With such an interpretation, the Blair model correctly takes into consideration the reflection of waves from the nuclear surface and other wave-mechanical features such as barrier penetration. The optical-model potential used for adequate fit to the data at 22 Mev reproduces effects which are quite similar to those of the Blair model.

In Fig. 6 are plotted the ratios of the partial reaction cross section σ_r^l to the maximum value

$$(\sigma_r^l)_{\max}=(2l+1)\pi\lambda\!\!\!^{-2}$$

as a function of the orbital angular momentum l. For an imaginary part of the potential of -20 Mev, two values of the real part have been used, $V=-50$ Mev and $V=-150$ Mev. For both cases, the partial reaction cross sections for $l\lesssim l'$ have essentially their maximum value. This corroborates one of the basic assumptions of Blair's model,[16] i.e., the nucleus is "black" for orbital angular momenta less than l'.

[15] This model was introduced by A. Akhiezer and I. Pomeranchuk, J. Phys. (U.S.S.R.) **9**, 471 (1945) for the nuclear scattering of charged particles.

[16] The prescription of Eq. (3) for predicting the critical value of l' gives erroneous results if one uses an R determined from the optical model. For example, Eq. (3) yields a value of $l'\simeq11$ for 22-Mev alpha particles incident on silver only if one takes an $R\simeq9.5\times10^{-13}$ cm; the optical-model value of R which yields adequate fit to the data is $\simeq7\times10^{-13}$ cm.

Figure 6 shows that for $l=l'$, $\sigma_r^l\simeq\frac{1}{2}(\sigma_r^l)_{\max}$. In addition it is seen that the partial reaction cross sections fall off gradually over a range of l in the neighborhood of l'. If we define a surface thickness $t=(l_2-l_1)k^{-1}$, where $\sigma_r^{l_2}=0.1(\sigma_r^{l_2})_{\max}$ and $\sigma_r^{l_1}=0.9(\sigma_r^{l_1})_{\max}$, and k=wave number of the alpha particle, we find that $t\simeq2.5\times10^{-13}$ cm. This is in agreement with the assumed value, $t=4.40a$ or $t=2.64\times10^{-13}$ cm. Wall, Rees, and Ford[3] attempted to modify Blair's model along these lines by assigning to the partial wave with the critical orbital angular momentum l' an amplitude of one-half the Coulomb amplitude and phase equal to the Coulomb phase. This modification removed some of the discrepancy at forward angles but had little effect at back angles.

The reason for the insensitivity of alpha-particle scattering to the real part of the potential is now quite clear. The absorption is so strong that effectively most partial waves do not experience the real part of the potential lying within the nucleus. Thus in Fig. 6 the only difference in the partial reaction cross sections produced by a change of V from -50 to -150 Mev occurs for large $l\approx l'$. Furthermore $l'(\approx11)$ is somewhat less than $kR(\approx15)$ and partial waves with $l'\lesssim l\lesssim kR$ are not strongly absorbed. These statements imply that the properties of the nuclear surface are important for the elastic scattering of alpha particles, and also lend credence to surface interaction models used to understand specific inelastic alpha-particle reactions such as (α,p) and (α,n) (and, by detailed balance arguments, their inverses). A one-parameter model such as Blair's does not reproduce the details of the scattering cross section which are necessary for the ascertaining of the radius of the alpha-particle—nucleus interaction. These details are sensitive to the properties of the nuclear surface.

If one takes a value of R needed to obtain a partial fit to the data on silver from the Blair model in its original or modified form, a value of $l'\approx11$ is obtained from Eq. (3). In the optical-model analysis, contributions from l as high as 19 had to be included at 22 Mev in order that the higher partial waves would contribute less than 5% to the predicted cross section. For 40-Mev alpha particles, the above comments are even more pertinent. The cut-off angular momentum on the Blair model at this energy is $l'\approx21$. The oscillations in the experimental data for 40-Mev alpha particles on silver correspond to a factor ≈2 in relative values of $\sigma(\theta)/\sigma_R(\theta)$ for adjacent maxima and minima. If the optical-model calculations were terminated at $l'=21$, an error of a factor of ≈2 would be made in the values of $\sigma(\theta)/\sigma_R(\theta)$ at maxima and minima, indicating that a completely erroneous positioning of these extrema would ensue.

The optical model has been applied in a special form by Oda and Harada[4] who have set $V=0$ and let $g(r)=\exp\{(R-r)b^{-1}\}$. With this form of the optical-model potential, the Schrödinger equation appropriate to the problem may be solved in closed form. They were able to obtain approximate fit to the 22-Mev silver data at forward angles with $W=-7$ Mev, $R=9.67\times10^{-13}$ cm, and $b=0.86\times10^{-13}$ cm. The oscillations they obtained in $\sigma(\theta)/\sigma_R(\theta)$ for $\theta>55°$ are undoubtedly caused by the cutting-off of the calculation at $l=11$, the critical value of l on the Blair model, and by setting $V=0$. The optical model has also been employed by Mohr and Robson[5] in Born approximation. The parameters they found which fitted the 22-Mev silver data are at variance with exact calculations in two respects: (1) $W_{\mathrm{Born}}\approx3W_{\mathrm{exact}}$ (i.e., the absorption predicted by the Born approximation is too large); (2) the Born approximation is somewhat insensitive to V as long as $V<W$, in contrast to the insensitivity of the exact results upon V for $V>W$.

Finally, there is the classical model of Porter[6] which attempts to obtain agreement with experimental data by attributing the features of $\sigma(\theta)/\sigma_R(\theta)$ to a pure absorption mechanism. In this model, the alpha particle travels on classical Rutherford trajectories "outside" the nucleus and suffers attenuation by collisions "inside" the nucleus. One of the results of Porter's model is the increased rate of drop-off of $\sigma(\theta)/\sigma_R(\theta)$ with increasing depth of the imaginary part of the well (decreasing absorption mean free path). The optical-model predictions do not agree with this result of Porter's semiclassical calculation. One example of the behavior of $\sigma(\theta)/\sigma_R(\theta)$ with W at 22 Mev predicted by the optical model is contained in Fig. 5 with $W=-15$ and -30 Mev. Similar behavior at 40 Mev is contained in Fig. 7. The behavior of $\sigma(\theta)/\sigma_R(\theta)$ with changes in W depends not only on the variation of the absorption mean free path with W but also on the variation of the reflectivity of the nuclear "surface" with W. Optical-model calculations seem to indicate that the latter effect is more important than the former, as long as the absorption mean free path is less than R. As has been indicated above, the main features of the alpha-particle—nucleus scattering cross section can be understood as arising from the quantum-mechanical properties of the surface of a nucleus which is "black" to angular momenta lying below a certain critical value. The fact that the classical model of Porter gives an absorption mean free path which is in approximate agreement with that calculated from the optical model may, therefore, be considered as fortuitous.

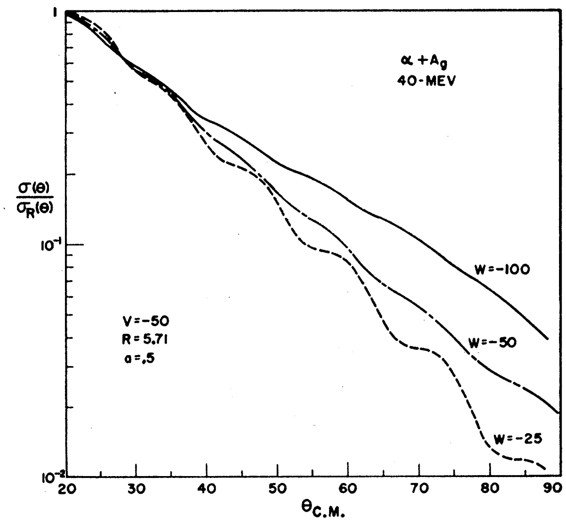

FIG. 7. An example of the behavior of $\sigma(\theta)/\sigma_R(\theta)$ for 40-Mev alpha particles on silver with changes in the absorptive part of the alpha-particle—nucleus interaction.

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