Helium Excitation Cross Sections near Threshold*

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The cross section for excitation by incident electrons of the $^3S_1(1s,2s)$ metastable level of He has been fitted by using a Breit-Wigner one-level formula. The fit is very good. The significance of this result and its bearing on the energy dependence of the cross section near threshold are discussed.

I. INTRODUCTION

RECENTLY Schulz and Fox[1] have measured the cross sections for excitation by incident electrons of the $^3S_1(1s2s)$ and $^1S_0(1s2s)$ metastable levels of He. These energy levels are diagrammed on the left of Fig. 1. The observed cross sections are shown in Fig. 2 of their paper.[1] The spectroscopic values for the excitation thresholds, as listed in Fig. 1, are located on their energy scale by the arrows in Fig. 2^1. Figure 2^1 is a plot of the total cross section for metastable excitation; i.e., above the $2\,^1S_0$ threshold is plotted the sum of the cross sections for excitation of the $^3S_1$ and $^1S_0$ metastables. We make no use of the data above the $2\,^3P$ threshold, where they become increasingly difficult to interpret. Below the $2\,^1S_0$ threshold Fig. 2^1 shows the cross section for the sole energetically possible inelastic process—excitation of the $2\,^3S_1$ level. According to Schulz and Fox the value of the $^3S_1$ cross section at its maximum is 4×10^{-18} cm^2$=0.045\pi a_0^2\pm30\%$, a_0 being the Bohr radius. The tailing near threshold presumably results from the approximately 0.1-ev energy spread of the incident electron beam.[1] Between 19.8 ev and 20.1 ev the experimental points deviate imperceptibly from a straight line. This line (dashed) intercepts the energy axis at 19.75 ev, only 0.07 ev removed from the spectroscopic value of the $^3S_1$ threshold.

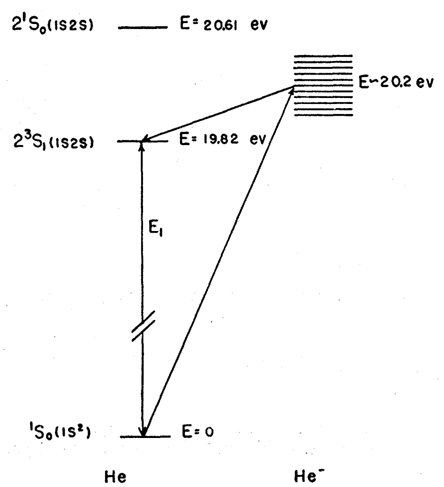

FIG. 1. Energy levels of atomic He and postulated unstable level of the negative ion He$^-$. The ionic energy is measured relative to the ground state energy of atomic He, i.e., relative to the total energy of the system consisting of a He atom in its ground state and of an electron with zero kinetic energy at infinity.

Wigner's[2] very general theory of the energy dependence of cross sections near threshold predicts that the $2\,^3S_1$ excitation cross section should be proportional to $(\Delta E)^{\frac{1}{2}}$, where ΔE is the energy excess above threshold. It is evident that a $(\Delta E)^{\frac{1}{2}}$ dependence is incompatible with Fig. 2^1 except possibly in the very immediate neighborhood of 19.75 ev. There the incident energy spread so distorts the excitation curve from the shape which would be observed with monoenergetic electrons that a $(\Delta E)^{\frac{1}{2}}$ dependence cannot be ruled out. The observations also can be compared with the theoretical cross sections computed by Massey and Moiseiwitsch.[3] They find a very sharp resonant peak in the $2\,^3S_1$ cross section, height $7\times10^{-2}\pi a_0^2$, at about 0.1 ev above threshold. As these authors note, if resonance actually occurs the height at resonance and the resonant energy probably depend very sensitively on fine details of the electronic wave function, and are not likely to be accurately predicted even by their elaborate calculations. Their theoretical results do point to the possibility of fitting the $^3S_1$ cross section, in the energy range 19.75–20.6 ev of Fig. 2^1, to a Breit-Wigner resonance curve.

In the following sections comparisons of the observations with the Breit-Wigner formula are described and discussed. As will be seen, a very good fit is obtained with parameters (e.g., level width) whose magnitudes are quite reasonable. Our ability to account for this He $2\,^3S_1$ electron excitation cross section in terms of a single resonance level is we think provocative, and bears importantly on the applicability (to excitation by electrons) of Wigner's threshold theory.

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[1] G. J. Schulz and R. E. Fox, preceding paper [Phys. Rev. **106**, 1179 (1957)]. We consistently refer to Fig. 2 of their paper simply as "Fig. 2^1."

[2] E. P. Wigner, Phys. Rev. **73**, 1002 (1948).

[3] H. S. W. Massey and B. L. Moiseiwitsch, Proc. Roy. Soc. (London) **A227**, 38 (1954).

Wu[4] estimates that the $1s2s^2$ level of He$^-$ lies within a few tenths ev of the He $2\,^3S_1$ state. It is tempting to assert that the observed $^3S_1$ cross section results from resonance with this He$^-$ level, and to assert further that this resonance explains the very large slow electron cross section for deexcitation[5] to $^3S_1$ of the $2\,^1S_0$ metastables. However, as amplified below these assertions do not account for all known information about the cross sections. This is not surprising; a glance at the spacing of the $1sns$ levels of He shows the ionic compound states are not likely to be widely spaced compared to the 1-ev level width inferred from our fit to the $^3S_1$ excitation curve. Nonetheless, these many-level complications do not negate the possible utility of a one-level approximation in appropriately selected atomic problems. Wigner's[6] discussion of the development of the resonant or compound model for nuclear reactions suggests that a resonant picture should be sought whenever an observed cross section shows sharp maxima and minima as a function of energy.

II. APPLICATION OF ONE-LEVEL THEORY

We attempt first to fit solely the $^3S_1$ excitation cross section, i.e., that portion of Fig. 2[1] lying below the $2\,^1S_0$ threshold. The transition from the ground to the $2\,^3S_1$ state does not involve a transfer of orbital angular momentum to the atom from the incident electron, and at energies near threshold the outgoing very slow electron must be in an s state. Thus near threshold the reaction proceeds mainly via incoming and outgoing s electrons going (we postulate) through a compound $^2S_{\frac{1}{2}}$ state of He$^-$ (Fig. 1). Under these circumstances the $^3S_1$ excitation cross section is given by[7]

$$\sigma=\frac{\pi}{k_0^2}\frac{\Gamma_0\Gamma_1}{[(E-E_a)^2+\frac{1}{4}\Gamma^2]},\tag{1}$$

where E is the kinetic energy of the incident electron; $k_0=(2mE)^{\frac{1}{2}}/\hbar$; E_a is the energy of the compound state[8] measured relative to the ground state of the neutral atom; $\Gamma=\Gamma_0+\Gamma_1$ is the (energy-dependent) width of the compound state; Γ_0 is the partial width for disintegration of the compound state into an electron and a neutral He atom in its ground $^1S_0$ state; and Γ_1 is the partial width for breakup leaving the neutral atom in its $2\,^3S_1$ state. For each incident energy E, the values of Γ_0 and Γ_1 are

$$\Gamma_0=2k_0\gamma_0^2,\quad \Gamma_1=2k_1\gamma_1^2,\tag{2}$$

with $k_1=[2m(E-E_1)]^{\frac{1}{2}}/\hbar$, E_1 being the threshold energy (Fig. 1); γ_0^2 and γ_1^2 are energy-independent quantities, the so-called Wigner reduced widths.

The solid line of Fig. 2 of this paper is a plot of the best fit obtained with Eq. (1) using $E_1=19.79$ ev, $E_a=20.19$ ev, $\gamma_0^2=2.2\times10^{-9}$ ev cm, and $\gamma_1^2=2.5\times10^{-10}$ ev cm. The crosses of Fig. 2 are the experimental points, taken from Fig. 2[1]. Some explanation of this choice of parameters is in order. The maximum value of σ in Eq. (1) cannot be very far from $E=E_a$. At $E=E_a\sim20$ ev, $k_0a_0\sim1$, Eq. (1) makes $\sigma\sim4\pi a_0^2\Gamma_0\Gamma_1/\Gamma^2$; comparing with the experimental value $\sigma=0.045\pi a_0^2$ at maximum, we infer that either $\Gamma_0/\Gamma\ll1$ or $\Gamma_1/\Gamma\ll1$, since $\Gamma_0+\Gamma_1=\Gamma$. If we take $\Gamma_1/\Gamma\ll1$, Eq. (1) has the form

$$\sigma=\frac{C(E-E_1)^{\frac{1}{2}}}{(E-E_a)^2+\frac{1}{4}\Gamma^2},\tag{3}$$

where C (which is adjusted to $\sigma_{\max}$) and $\Gamma\sim\Gamma_0$ are effectively constant in the energy interval under consideration.

The parameters of Fig. 2 correspond to a Γ of 1.0 ev or to a lifetime of about 10^{-15} sec for the He$^-$ compound state. This is not unreasonably short in view of Wu's[9] estimate that the lifetime for autoionization of the $2s^2\,^1S_0$ state of neutral He is 2.5×10^{-15} sec. Our values for γ_0^2 and γ_1^2 also are reasonable from another important standpoint, namely they lie well within the sum rule limit[10] of $3\hbar^2/2ma$, where a is the radius of the compound state. To be precise, a would have to be as large as 100 a_0 for the limit to be exceeded; alter-

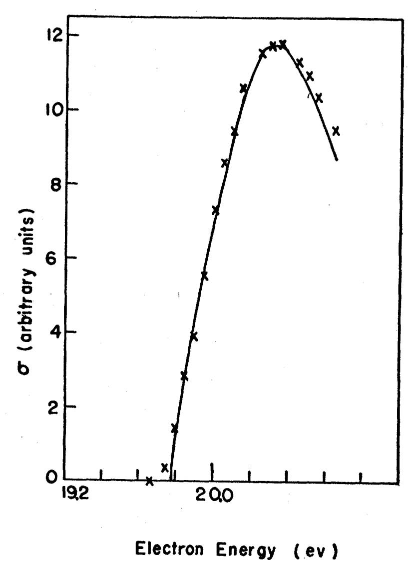

FIG. 2. Total cross section σ for excitation of He metastables, in arbitrary units, as a function of incident electron energy in ev. The solid line is a plot of our best fit obtained with a Breit-Wigner formula. The crosses are experimental points.

[4] Ta-You Wu, Phil Mag. **22**, 837 (1936).
[5] A. V. Phelps, Phys. Rev. **99**, 1307 (1955).
[6] E. P. Wigner, Am. J. Phys. **23**, 371 (1955).
[7] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 558.
[8] We have neglected the slight energy dependence of E_a resulting from its proximity to the $^1S_0$ threshold. Using reasonable values for the reduced width of this channel, the shift can be shown to be small in the interesting energy region. See R. G. Thomas, Phys. Rev. **88**, 1109 (1952).
[9] Ta-You Wu, Phys. Rev. **66**, 291 (1944).
[10] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).

natively, if a is assumed about equal to a_0, $\sum\gamma^2$ is one percent or so of the sum rule limit. For nuclear levels, percentages of this magnitude are interpreted to mean that the single-particle model is beginning to break down but the statistical model is not yet valid.[10,11] The same interpretation for the levels of the three-electron ion He^- is quite believable.

The results quoted in the preceding paragraph support our belief that a one-level approximation to the $^3S_1$ cross section is meaningful and useful. We note that we could have chosen $\Gamma_0/\Gamma \ll 1$ in Eq. (1), in which event Eq. (3) is replaced by

$$\sigma = \frac{C'(E-E_1)^{\frac{1}{2}}}{(E-E_a)^2+(2m/\hbar^2)(E-E_1)\gamma_1{}^4}, \tag{4}$$

where the quantities other than E are energy-independent. Recognizing that the denominator in Eq. (4), like the denominator in Eq. (3), is a quadratic function of E, a little manipulation shows that Eq. (4) yields the same family of curves as does Eq. (3). Hence with $\Gamma_0/\Gamma \ll 1$ the solid line of Fig. 2 still is our best fit to the data, but it now corresponds to $E_1=19.79$ ev as before, $E_a=20.43$ ev, $\gamma_0{}^2=4.1\times10^{-11}$ ev cm, $\gamma_1{}^2=1.4$, $\times10^{-8}$ ev cm. The lifetime of the compound state is the same as previously. The lack of structure in the elastic cross section,[12] not easily explicable if Γ_0/Γ is of the order of unity, favors the supposition that in fact $\Gamma_0/\Gamma \ll 1$ rather than $\Gamma_1/\Gamma \ll 1$.

We remark that although in either case, $\Gamma_1/\Gamma \ll 1$ or $\Gamma_0/\Gamma \ll 1$, we find E_a very close to Wu's[4] value for the energy of the He^- $1s2s^2$ level, we do not cite this agreement as evidence favoring our one-level analysis of the data. Insisting that Wu's He^- compound state is responsible for the observed $^3S_1$ excitation is logical only if it can account for all scattering data in this energy region. Otherwise, since the level spacing probably is close, it is more reasonable to postulate additional compound states nearby, any one of which may lie closer to 19.82 ev than the He^- $1s2s^2$ level and therefore be the compound state producing the observed $^3S_1$ resonance. A $1s2s^2$ compound state of He^-, because it should have a large reduced width for breakup into He $1s2s$ levels, is consistent with our $\Gamma_0/\Gamma_1 \ll 1$ fit and with Phelp's datum[5]: a de-excitation cross section more than 75% of the theoretical limit at 0.04 ev. However, on the assumption that the $2\,^1S_0$ excitation also proceeds mainly via this same compound state, Phelp's result—even if only approximately correct—implies the combined $^3S_1$ and $^1S_0$ excitation cross section must decrease past 0.04 ev above the $^1S_0$ threshold instead of rising as Schulz and Fox's observations do. In sum, while the possibility cannot be dismissed, there is no compelling reason to identify Wu's level (whose energy is estimated[4] only to ± 0.5 ev) with the level responsible for the observed structure in the $^3S_1$ cross section.

The energy dependence implied by Eq. (3) in the immediate vicinity of threshold is the same as that predicted from the more general many-level theory.[2] The success of our fit to Fig. 2[1] with Eq. (3) shows that there is no reason to doubt the $(\Delta E)^{\frac{1}{2}}$ law right at threshold, but also demonstrates (as Wigner[2] cautions) that the energy range in which the $(\Delta E)^{\frac{1}{2}}$ law is valid can become unobservably small when there is a resonance close to threshold. Because the "true" cross section for monoenergetic electrons is not known in the tailing region of Fig. 2[1], and because Schulz and Fox's energy scale need not be exact,[1] we regard the threshold energy E_1 as a parameter in Eq. (3) and choose it to get the best fit to the experimental points of Fig. 2[1] in the energy range above about $E=19.85$ ev. Especially in the energy range 20.3–20.6 ev, where the higher compound states which may be needed to account for the second peak can make important contributions to the $2\,^3S_1$ excitation cross section, our one-level approximation may not be justified. Thus we place no emphasis on the particular values of E_a, $\gamma_0{}^2$, and $\gamma_1{}^2$ we have found, despite the fact that a good fit to Fig. 2[1] allows very little latitude in these parameters. The value $E=19.79$ ev for E_1 does seem significant, however; we have no definite proof but our numerical work indicates that varying E_1 in a many-level fit to the data would produce much the same effect, at energies below 19.85 ev, as varying E_1 in the simpler expression (3).

To get some feeling for the permissible variation in E_1, in Fig. 3(a) we compare the experimental points with the best fit we could get using E_1 equal to 19.85 ev. The $E_1=19.85$ ev curve is a slightly yet discernibly inferior fit; larger variations in E_1 soon make the fit much worse. Apparently, provided our success in fitting the data with a one-level formula is neither fortuitous nor trivial, we may assert that Schulz and Fox's energy scale is in error by not more than about 0.05 ev. That the fit is not wholly trivial can be seen from Fig. 3(b), where we compare the experimental points of Fig. 2[1] with the best fit obtainable from the four-parameter family

$$\sigma = C(E-E_1)^{\frac{1}{2}}(E^2+aE+b), \tag{5}$$

wherein C, E_1, a, and b are energy-independent adjustable constants. Though Eq. (5) is not a bad fit, it is not as good as Eq. (3), mainly because Eq. (5) cannot duplicate the observed linear dependence on $E-E_1$ over as large a portion of the energy range below 20.1 ev as does Eq. (3).

CONCLUDING REMARKS

Schulz and Fox's data suggest a legitimate challenge to the utility of the threshold theory in atomic or

[11] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Press, Cambridge, 1953), p. 312.

[12] H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Clarendon Press, Oxford, 1952) Chapt. I–IV, especially p. 10.

molecular excitation by electrons, namely: that its energy range of validity generally lies so close to threshold as to be practically unobservable, although the fundamental assumptions of the theory are justified. The theory itself in its present form[2] gives no quantitative information concerning its range of validity. On this basis we urge more careful studies than heretofore[12] of electron excitation cross sections in a variety of atoms and molecules, to determine whether a range of validity less than 0.1 ev is typical or atypical. Moreover such studies would clarify the significance of the linear dependence on ΔE observed in the energy range 19.8–20.1 ev. A purely accidental linear behavior over so extended an interval between the cross section peak and the tailing [presumably $(\Delta E)^{\frac{1}{2}}$] region is improbable. Yet our one-level formula implies such linearity only when the ratio of Γ to $E_a - E_1$ is close to two. At the moment we see no reason why $\Gamma/(E_a - E_1)$ need be two when a one-level approximation is applicable, nor why a many-level formula need produce a linear curve past the $(\Delta E)^{\frac{1}{2}}$ threshold region.

In conclusion, we recognize and have ourselves mentioned theoretical objections to the use of a one-level approximation in this He excitation problem. We are much concerned therefore with the possibility of experimental confirmation of our analysis. The only obvious direct test is a high-resolution measurement of the shape of the ${}^3S_1$ excitation curve near threshold. As we have explained, the shape of our "best fit" curve near threshold probably is the least equivocal of our results. Thus we feel that if the experimental curve near threshold does not follow the solid curve of Fig. 2 our one-level approximation is contradicted, while conversely a successful prediction of the detailed shape of the experimental curve near threshold surely would be strong evidence in favor of our approach.

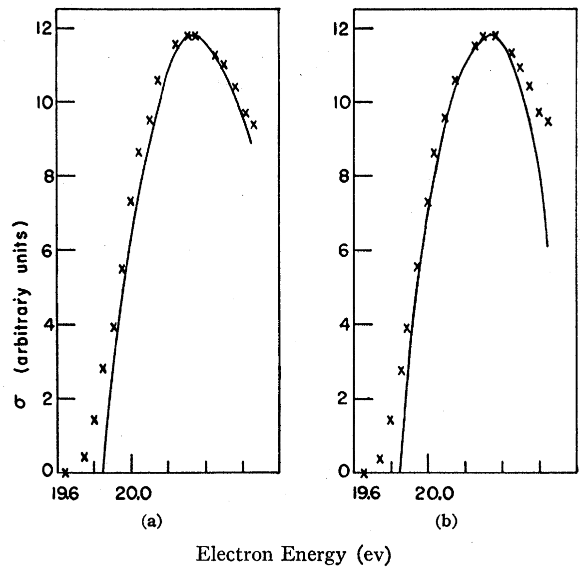

FIG. 3. Total cross section σ for excitation of He metastables, in arbitrary units, as a function of incident electron energy in ev. The solid line in (a) is obtained from the Breit-Wigner formula, using a threshold of 19.85 ev. The solid line in (b) is our best fit obtained using Eq. 5. The crosses are experimental points.

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