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Diffraction of Thermal Waves in Liquid Helium II*

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Diffraction effects in thermal radiation have been produced in liquid helium II by means of a thermal grating radiating a primary (zero-order) beam accompanied by higher order beams at appropriate Bragg angles. These results at once demonstrate unambiguously the true wave nature of this thermal wave propagation and its conformity to Huygens' principle in the most demanding test to which second sound has been subjected. The general nature of the angular pattern, including the fine structure, conforms to the requirements of diffraction behavior for more customary wave motions. A determination of wave velocity (v_2) for second sound is obtainable from the observed Bragg angles.

I. INTRODUCTION

THE thermal wave propagation in liquid helium II called second sound is quite generally regarded as a true wave phenomenon. Its apparent conformity to the wave equation, rather than to the classical heat flow equation, has been supported by various experiments based upon specific wave properties. Peshkov[1] first demonstrated the progressive change of phase down a traveling thermal-wave system, and later[2] observed second sound using a standing-wave resonance technique. The ability of second sound to proceed independently as self-maintained thermal packets was established by means of the pulse method,[3] which permitted recording photographically the arrival of such pulses followed by their multiple reflections.

The remaining property required for establishing second sound as satisfying unequivocally all demands of the wave equation has been its ability to undergo diffraction in conformity to Huygens' principle. The purpose of the present investigation has been to determine whether such diffraction is observable and, if so, whether the diffraction laws characterizing more familiar forms of wave propagation are obeyed. This is at once the most demanding, yet most conclusive, test to which second sound can be put.

II. METHOD AND EQUIPMENT

(a) Method

The general procedure has been to excite second-sound waves from a "thermal grating" consisting of an array of heater elements possessing a spacial periodicity and undergoing periodic, in-phase heating. Diffraction results would be expected to produce a primary (zero-order) beam normal to the plane of the elements, accompanied by sharply defined secondary beams at appropriate angles. The presence and location of such beams were investigated by means of a thermally-sensitive receiver used to probe the surrounding thermal field.

(b) Equipment

(1) *Thermal Grating*

A "thermal grating" composed of a series of equally-spaced, parallel, line sources constituted the heater array. This grating, shown in Fig. 1, consisted of (28) thin parallel strips (G) of gold evaporated onto a plane Lucite surface (S). When driven in parallel by an alternating voltage applied across the terminating electrodes (E), the resistive elements formed a radiating antenna of constant-phase heating elements.

The spacing interval (δ) between adjacent elements was approximately 0.75 mm so that for the wavelength

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[1] V. Peshkov, J. Phys. (U.S.S.R.) **8**, 381 (1944).
[2] V. Peshkov, J. Phys. (U.S.S.R.) **9**, 150 (1945).
[3] J. Pellam, Phys. Rev. **74**, 841 (1948); **75**, 1183 (1949).

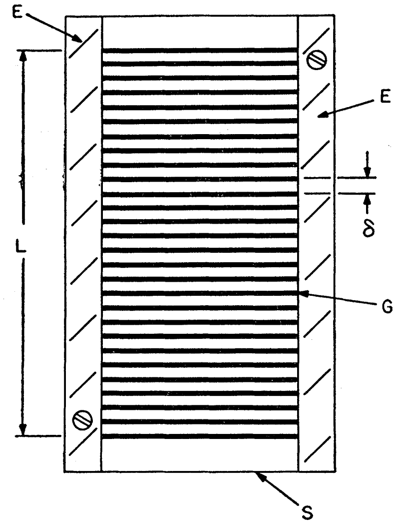

FIG. 1. Thermal grating. (E) Electrodes, (G) grating resistance elements, (L) length of grating array, (S) Lucite block, (δ) grating spacing.

employed predominantly during these experiments ($\lambda \simeq 0.3$ mm) the relative spacing was $\delta/\lambda \simeq 2.5$ Constructive thermal interference might therefore be expected at angles (θ) from the normal given by the "Bragg angles"

$$\theta = \pm \sin(n\lambda/\delta), \quad n = 0, 1, 2, \qquad (1)$$

where the direct beam is represented by $n=0$. Since the effective over-all antenna length (L) was about $L \simeq 70\ \lambda$, the corresponding primary angular beam-width (peak to first minimum) should be roughly $\frac{3}{4}$ of one degree.

The first-order diffraction pattern ($n=1$) would then appear as a set of sharply defined beams at angles $\theta \simeq \pm 24°$ from the normal. The second-order diffraction pattern ($n=2$) would then be expected for the angles $\theta \simeq \pm 55°$. Furthermore, if the usual diffractive principles are operative, a fine structure of minor maxima should be observed accompanying each major beam.

(2) *Receiver System*

The over-all arrangement is given in Fig. 2 where the transmitter grating (T) is shown pivoted at its midpoint about the axis (A). Since the sharp beam requirements of the transmitter system necessitated an extended array of sources ($L \simeq 2.0$ cm), the restrictions of the Dewar system (D) prevented placing the second-sound receiver at a distance large compared to L. Instead, we substituted a plane, phase-sensitive receiver surface (R) located comparatively close to the transmitter array which, as will be shown (Sec. III), was equivalent to examining the radiation pattern at "infinity."

Customary techniques[3] of detecting temperature waves by means of current-carrying carbon surfaces were employed. A rectangular receiver surface supporting uniform current density (between electrodes along opposite walls) measured the instantaneous temperature fluctuation averaged over its area, and thus constituted a phase-sensitive thermal detector. As shown in Fig. 2, this receiver R remained in a fixed horizontal position, while the orientation of the grating was controlled by means of a vertical rod V. The relative angle θ was measured between the vertical (direction of observation) and the normal to T.

(3) *Electronic*

The thermal grating was driven by a 30.4-kilocycle/sec electrical current, thus generating a highly-collimated second-sound beam at 60.8 kilocycles/sec (the usual frequency doubling). Upon arrival at the current-carrying receiver surface, this temperature wave developed a corresponding 60.8-kilocycle/sec electrical voltage, as a result of the (semiconductor) variation of resistance with temperature. This signal was amplified in a narrow-band system, and finally examined (as a difference frequency after mixing) with a very narrow band (4 cps) analyzer, in order to suppress background noise.

III. CONSIDERATIONS OF DESIGN AND OPERATION

(a) Equivalence to Receiver at Infinity

The equivalence between an extended plane receiver surface (R) in proximity to a transmitter T and a point

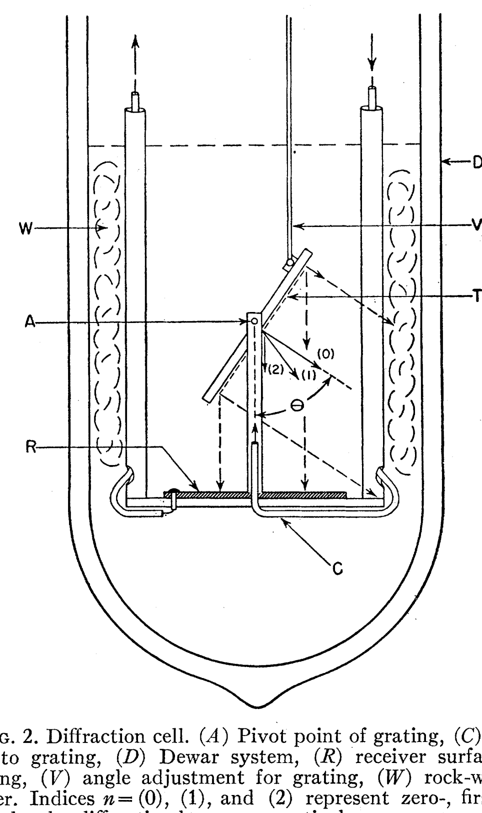

FIG. 2. Diffraction cell. (A) Pivot point of grating, (C) coaxial line to grating, (D) Dewar system, (R) receiver surface, (T) grating, (V) angle adjustment for grating, (W) rock-wool absorber. Indices $n=(0)$, (1), and (2) represent zero-, first-, and second-order diffraction beams, respectively.

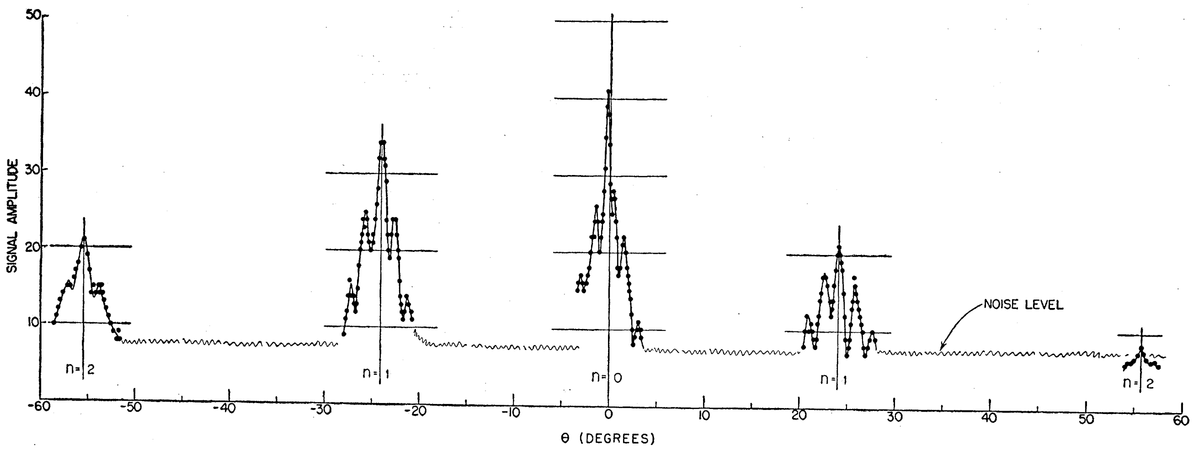

FIG. 3. Observed diffraction pattern for temperature waves. Received signal amplitude (arbitrary units) *versus* angle (θ) from grating normal. Central beam ($n=0$) occurs at $\theta \simeq 0°$, the first secondary set ($n=1$) at $\theta=\pm 24.1°$, and the second ($n=2$) at $\theta=\pm 55.5°$ in accordance with Eq. (1). Distortion of central-beam peak probably results from standing waves. Note the diffraction pattern asymmetry discussed in text.

receiver at infinity may be recognized in various ways. Qualitatively, a set of cylindrical wavelets originating from a plane array of line sources should present a single phase front (for enough sources) at each of the specific angles representing constructive interference. Since a phase-sensitive receiving surface is involved, the electrical signal generated falls to zero with deviation from these optimum orientations by angle $\simeq(\lambda/L)$.

Conversely, the same result may be achieved by employing the *reciprocity* theorem. Since the receiving surface (R) was appreciably longer than the thermal grating (T), a uniform-phase signal emanating *from R* would present at T the same effective plane-wave front as a point signal originating at infinity. Accordingly, if one treats T temporarily as a receiver, the resultant arriving signal (S) would be (in terms of quantities already defined)

$$S=\sin\left(N\frac{\pi\delta}{\lambda}\sin\theta\right)\bigg/\sin\left(\frac{\pi\delta}{\lambda}\sin\theta\right),$$

which is the same expression as for the signal at infinity due to the thermal grating of N elements as a generator.

(b) Spacial Separation of Beams

Substitution of an extended receiver surface adjacent to the transmitter array is necessary for achieving high resolution without going to megacycle frequencies; that is, for a transmitter length ($L=2.0$ cm) adequate to produce a sharp beam, the space constraints of the Dewar system effectively preclude detection at "large" distances, such that $r\gg L$. On the other hand, examination by a small receiver of the thermal wave field adjacent to the transmitting array could achieve at best a regional mapping of the beam "cross sections." But by providing an extended, plane receiver surface exposed to all radiation within a beam (and acting as a Huygens surface relative to infinity) the actual angular sharpness of the beams could be observed at short range.

It will be recognized from the geometry of Fig. 2, where the beam boundaries are indicated by dashed lines, that the direct (zero-order) beam would fall obliquely across the right-hand portion of the receiver surface (R) at the same time that the first secondary beam (first-order diffraction) at $\theta\simeq 24°$ would be observable by normal incidence. The practical unimportance of this circumstance follows immediately upon noting that the resulting zones of positive and negative temperature fluctuations from such oblique waves mutually cancel, resulting in no effective signal. Any objection was precluded, however, by designing the system for detection of the second-order diffraction beam at sufficient angle from the normal (55°) that the principal beam misses the receiving surface altogether (the separation between the midpoints of T and R of 2.5 cm permits this clearance). This is the situation represented in Fig. 2 where the orientation of T precludes the arrival of zero-order thermal radiation upon R. Accordingly, the second-order pattern has been separated from the direct beam in space as well as in phase, leaving no ambiguity in the observation that thermal waves leave the generator at specific oblique angles.

(c) Elimination of Interference

Interference from stray signals multiply reflected between Dewar and equipment surfaces was eliminated by placing rock-wool (W) at various positions (see Fig. 2), to intercept and absorb any extraneous second-sound beams.

IV. RESULTS

The results are summarized in Fig. 3 where the observed second-sound signal (arbitrary units) is plotted as a function of angle θ, for an ambient temperature of 1.29°K and frequency of 60.8 kilocycles/sec. The effective angular dependence of second sound radiated from the thermal antenna is in general over-all agreement with expectations based on usual diffraction behavior.

The radiation pattern shows a sharp central (zero-order) peak emitted normally to the plane of the array ($\theta=0$), a pair of secondary (first-order diffraction) beams at $\theta\simeq\pm 24°$, and another set of secondary (second order diffraction) beams at $\theta\simeq\pm 55°$. Finer structure in the radiation pattern is represented by the array of minor peaks accompanying each major peak.

The various orders of maxima fall at positions in agreement with Eq. (1) and provide, in fact, an independent determination of the velocity (v_2) of second sound. In this manner a velocity of (19.0) m/sec was observed at the temperature (1.29°K) employed for the measurements, in reasonable agreement with earlier values by other methods. An interesting aspect of this application is that the (slight) contraction of the thermal grating upon cooling from room temperature introduces no error. Since the contraction of the Lucite base produces the same proportional change in the lever arm ($L/2$) as in the grating-spacing δ, the value of λ obtained by using room-temperature dimensions requires no correction for grating shrinkage.

The observed beam-widths show order-of-magnitude agreement with the angular spread computed for diffractive behavior. For example, the angular half-width of the main peak for the central beam ($n=0$) measured from the mid-point to the first minimum is about 1.0°, compared to a computed value (for $a/\lambda\simeq 70$) of 0.8°. Somewhat greater comparative sharpness results for the higher order beams. Thus the ($n=1$) peaks also have half-breadths of 1.0° compared to an expected 0.9°; and for the ($n=2$) peak the observed value has increased only to 1.2° compared to an expected 1.5° (resulting from the reduced effective grating-length subtended at this oblique angle).

The distortion observable in the main peak of the zero-order pattern results from standing waves set up between the transmitting and receiving surfaces, and is therefore limited to conditions of nearly exact parallelism. A condition of general asymmetry in the over-all diffraction also persists, which evidently indicates nonuniformity of beam intensity or nonuniform sensitivity of receiver surface, or both. Particularly when enhanced by the "unbalancing" effects of wave absorption (the coefficient of temperature attenuation for second sound is[4] $\alpha=0.17$ cm^{-1} at this temperature and frequency), such variations can be expected to produce asymmetry of this nature. Actually the peak at $\theta\simeq+55°$ was below the noise level (see Fig. 3) for the power used in the remainder of the pattern. The peak was measured by increasing the generator power and normalizing these data to conform to the previous power level.

V. CONCLUSIONS

The capability of liquid helium II to respond to the excitation of a thermal diffraction-grating to produce highly-collimated second-sound beams of several orders of diffraction provides the final evidence of the unambiguous wave nature of this propagation. We regard this as the most demanding and conclusive test to which thermal waves in liquid helium II have been subjected.

[4] W. B. Hanson and J. R. Pellam, Phys. Rev. 95, 321 (1954).