PHYSICAL REVIEW VOLUME 106, NUMBER 5 JUNE 1, 1957

Λ^0-Nucleon Interaction*

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(Received January 24, 1957)

A study of the Λ^0-p or Λ^0-n force is of particular interest because the Λ^0-nucleon two-body force need not be related to the potential effective in binding hyperfragments. Thus, an exchange of a single pion between the Λ^0 and the nucleon is forbidden only in the two-body interaction. It is pointed out that the final-state interaction in the production of a Λ^0 and nucleon is the only effective means of studying this force that seems feasible at present. The effect of the Λ^0-nucleon interaction on the spectrum of K mesons produced in nucleon-nucleon and pion-deuteron collisions is investigated. It is pointed out that these experiments can also be used to determine whether the ${}_\Lambda H^2$ is bound.

I. INTRODUCTION

A STUDY of the Λ^0-nucleon force is of particular interest because the two-body interaction is not necessarily intimately related to the interaction of the Λ^0 with two or more nucleons. Thus, the exchange of a single pion between the Λ^0 and nucleon is forbidden only in the two-body problem, but not when more nucleons are present. The virtual process $\Lambda^0 \rightarrow \Lambda^0 + \pi$ is forbidden if isotopic spin is a good quantum number in strong interactions. Figure 1(a) shows the lowest order perturbation theory Feynman diagrams with pion exchanges which contribute to the Λ^0-nucleon force. Figure 1(b) shows a similar diagram when two nucleons are involved. In these figures N indicates either a proton or neutron, since charge independence predicts that the Λ^0-p and Λ^0-n forces are equal. The Feynman diagrams indicate that even the range of the two-body and higher-body forces may be different. The same argument cannot be made for any other known hyperons, since for all of these, single pion exchange is allowed by isotopic spin conservation laws. Furthermore, the nature of the Σ^0-nucleon force can be studied experimentally by scattering the Σ^+ and Σ^- and the use of isotopic spin independence, but no such techniques are available for the Λ^0-nucleon force. It is difficult to study the latter force directly, but some of its properties can be obtained from the final state interaction, which is important close to threshold, in the production of a Λ^0, nucleon, and K meson. Three bodies are required in the final state, as then, for example, the spectrum of produced K mesons can be used to obtain information on the strength of the Λ^0-nucleon force. The effect of such final state interactions have been previously considered for pions.[1] They are investigated here for processes of Λ^0 and nucleon production in nucleon-nucleon and π-deuteron collisions.†

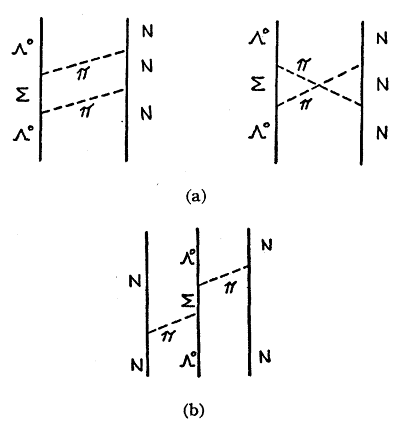

FIG. 1. Feynman diagrams for lowest order pion exchange contributions to Λ^0 nucleon (indicated by the letter N) and Λ^0-two nucleon forces.

II. NUCLEON-NUCLEON COLLISIONS

Consider the collision between two nucleons at an energy just below the production threshold for Σ^0 hyperons, i.e., 1.75 Bev, so that the problem of distinguishing the Λ^0 from the Σ^0 does not arise. If the masses are taken to be 938 Mev for the nucleons, 1.114 Bev for the Λ^0, and 494 Mev for the K meson, then the threshold for production of a Λ^0 and K meson in nucleon-nucleon collisions occurs at 1.58 Bev. At 1.75 Bev, the final state kinetic energy available in the center-of-mass system is 62 Mev, and on the average only one quarter of this is available to the Σ^0-nucleon. It is thus reasonable to assume that only final S-state interactions between the Λ^0 and nucleon will be of importance over most of the spectrum of emitted K meson if the Λ^0-nucleon force is not of the gradient type. This is all the more likely if the range of forces is $\hbar/2\mu c$ (μ=mass of π meson) or less, as would be expected from the exchange of two pions and more, or of K mesons, because for this range, P states of angular momentum should only become important at relative energies of the order of 75 Mev. It is to be expected that the K mesons will be emitted into S or P states of orbital angular momentum, or a mixture thereof, depending on their parity relative to the Λ^0 and on

* Partially supported by the U. S. Atomic Energy Commission.

[1] K. M. Watson and K. A. Brueckner, Phys. Rev. **83**, 1 (1951); A. H. Rosenfeld, Phys. Rev. **96**, 139 (1954).

† *Note added in proof.*—L. B. Okun and M. I. Shmushkevich have considered the effect of the final state interaction in the capture of K^- mesons by deuterium, Soviet Phys. JETP **3**, 792 (1956).

whether parity conjugation[2] is satisfied. Recent experiments[3] of K-meson production in π^{-}-p collisions at 1.3 Bev indicate that even higher orbital angular momenta may be involved, but it is doubtful that this will be true at the energy considered here.

Under the assumption that in the initial state the interaction which produces the hyperon is of much shorter range than the nucleon-hyperon force in the final state, the cross section for production in p-p collisions can be written as[1] [$\hbar=c=1$ in Eq. (1) and thereafter]

$$d\sigma=(2\pi)^7 dJ|\psi_f(0)|^2[a+k^2(b+c\cos^2\theta)]/v, \tag{1}$$

where v is the relative velocity of the two protons, $\psi_f(0)$ is the final state Λ^0-nucleon wave function at the origin of coordinates, and a, b, c, are functions of the initial momentum and spin states of the two protons and final spin states of the Λ^0 and nucleon. If the K-meson momentum is specified by $\mathbf{k}$, and its kinetic energy by T, then the phase space dJ for emission into an energy interval dT and a solid angle $d\Omega_K$ is

$$dJ=8\pi(m\mu_K)^{\frac{3}{2}}[T(T_m-1.24T)]^{\frac{1}{2}}dTd\Omega_K. \tag{2}$$

In Eq. (2), T_m is the maximum kinetic energy available in the center-of-mass system, μ_K is the mass of the K meson, and $m=MM_\Lambda/(M+M_\Lambda)$ is the reduced mass of the Λ^0 of mass M_Λ and nucleon of mass M. In both Eqs. (1) and (2) the final state meson, nucleon, and hyperon have been treated nonrelativistically. Should the Λ^0 and nucleon be bound with an energy B, then dJ is

$$dJ=0.805k\mu_K d\Omega_K, \tag{3}$$

and the mesons will be emitted with a fixed energy, T, which is $0.81(B+62)$ Mev if the initial energy is 1.75 Bev.

The conditions of applicability of Eq. (1) have been studied by Watson,[4] and may be summarized as follows:

(a) The relative wavelength, $\lambda\!\!\!^{-}_c$, in the initial state is of short range.

(b) The final state Λ^0 and nucleon have low energy relative to each other in the center-of-mass system.

(c) The final-state interaction has a range $\gg\lambda\!\!\!^{-}_c$ (i.e., $\sigma_{\text{scattering}}\gg\pi\lambda\!\!\!^{-}_c{}^2$).

Conditions (a) and (b) are satisfied for the problem here, and it is expected that (c) holds also.

The value of $|\psi_f(0)|^2$ depends on the interaction potential of the Λ^0 and nucleon, about which very little is known.[5] Should one believe an extrapolation from known bound hyperfragments, then Dalitz has shown[6] that the potential is highly spin-dependent, and that the integral $U\equiv\int Vd^3r$ is equal to -380×10^{-39} Mev-cm^3 (attractive) in the triplet state and $+480\times10^{-39}$ Mev-cm^3 in the singlet state, for a Λ^0 of spin $\frac{1}{2}$ and positive parity. The order of magnitude of the above interaction is such as to almost bind the ${}_\Lambda H^2$ for a range of forces of 1×10^{-13} cm, and is thus not weak. This can be seen most easily for a square well interaction. Then U is equal to $4\pi b^3V/3$, where b is the radius of the potential, and $2mVb^2$ is equal to $\pi^2/4$ for just binding the ${}_\Lambda H^2$. In the triplet state, this occurs when $b=0.94\times10^{-13}$ cm. For a shorter range and a fixed U, the binding is increased.

For a square well interaction between the Λ^0 and nucleon, $|\psi_f(0)|^2$ can be expressed by

$$(2\pi)^3|\psi_f(0)|^2=[\cos^2\kappa b+(p^2/\kappa^2)\sin^2\kappa b]^{-1}, \tag{4a}$$

where κ is equal to $[2m(-V+E)]^{\frac{1}{2}}$, and p is the relative momentum of the Λ^0 and nucleon. For large values of p, this approaches unity, as anticipated. For small values of p, that is $p\ll\kappa$, the effective-range approximation can be used and $|\psi_f(0)|^2$ is proportional to[4]

$$|\psi_f(0)|^2\sim\sin^2\delta_0/p^2=a^2/(1+p^2a^2), \tag{4b}$$

where δ_0 is the S-wave phase shift and a is the scattering length. This expression is a simpler one than that for a square well and does not depend on the shape of the

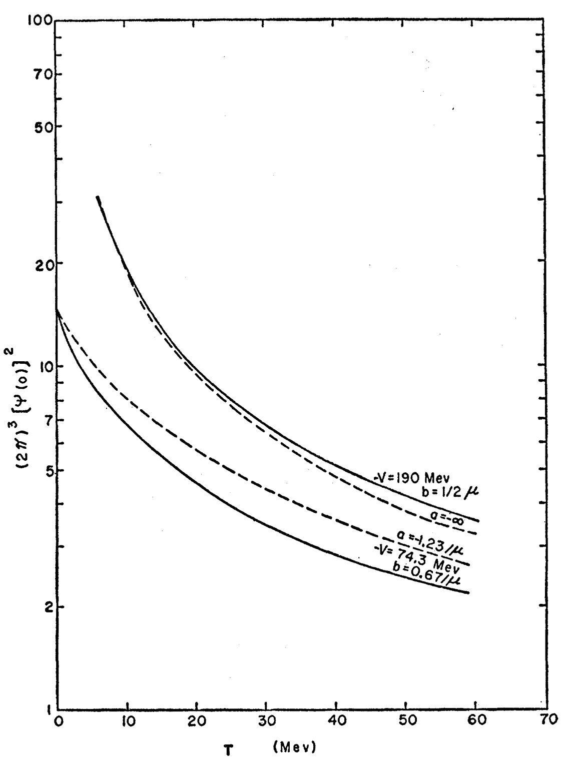

FIG. 2. Comparison of $|\psi_f(0)|^2$ for a square well and for an effective range approximation. The solid curves are for square wells of depth $-V$ and radius b. The dotted curves are the corresponding curves for an effective range a.

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[2] T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956).

[3] Budde, Chretien, Leitner, Samios, Schwartz, and Steinberger, Phys. Rev. **103**, 1827 (1956).

[4] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

[5] See however, D. B. Lichtenberg and M. Ross, Phys. Rev. **103**, 1131 (1956).

[6] R. H. Dalitz, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956), p. V-40.

potential itself. Equations (4a) and (4b) are compared in Fig. 2 for two ranges and depths of potential of the order of those expected from Dalitz' argument. It is noted that Eq. (4b) is accurate to within 20% over the whole energy region of interest, and is therefore accurate enough for our present purposes. We shall use it henceforth.

The spectrum for mesons emitted into S and P states of orbital angular momentum is shown in Figs. 3 and 4, for various values of the scattering length a. The curves for $a=-\infty$ in both figures correspond to a potential which would just bind the Λ^0-nucleon system. The curve for no final-state interaction ($a=0$) is also shown.

For a binding energy of 0.5 Mev in an S state-interaction, the curve of Fig. 3 for $a=-\infty$ is essentially correct, except in the neighborhood of $T=50$ Mev, where the curve would be modified by the addition of a peak at 50.6 Mev. On the scale indicated, this peak would have a magnitude of 495 units; this should, however, be multiplied by the resolution of the experimental detecting equipment.

In addition to the curves shown in Figs. 3 and 4, the effect of a repulsive potential of the same strength as that attractive one which corresponds to a scattering length of $a=-\infty$ was calculated. The correction for such a repulsive potential to the curve for no final state interaction was found to be negligible (<2%) and the curve was therefore not plotted. Thus, if the Λ^0-nucleon force in an S state is a repulsive one, the method proposed here cannot be used effectively to investigate its characteristics. In the considerations that have led to Figs. 3 and 4, the effects of any interaction between the K meson and Λ^0 or nucleon have been neglected. These, if important, would be expected to chiefly modify the low-energy behavior of the spectrum of emitted K mesons. Furthermore, it has been tacitly assumed that the Λ^0-nucleon potential does not involve a short-range core, which would modify somewhat the results obtained heretofore.

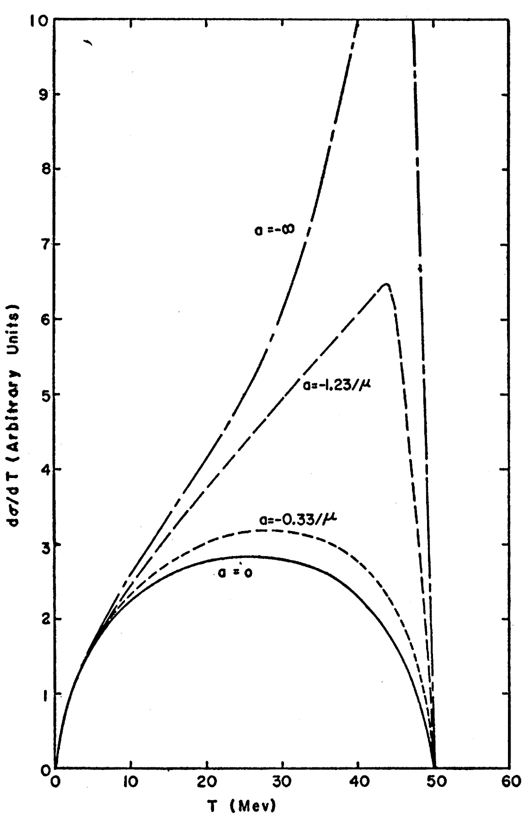

FIG. 3. Effect of final-state interaction on the spectrum of K mesons emitted into S states of angular momentum. All curves have been arbitrarily normalized at a meson energy of 5 Mev, since the magnitude of the matrix element is not known.

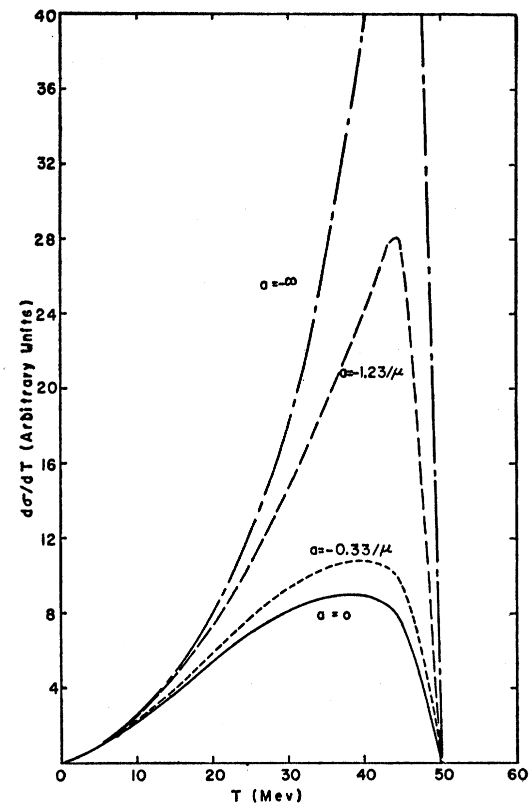

FIG. 4. Effect of final-state interaction on the spectrum of K mesons emitted into P states of angular momentum. All curves have been arbitrarily normalized at a meson energy of 5 Mev, since the magnitude of the matrix element is not known.

Table I lists the possible transitions for various spin parity assignments of the K meson relative to the Λ^0. The spin of the K meson is assumed to be zero, and that of the Λ^0 $\frac{1}{2}$ or $\frac{3}{2}$. The cross sections for n-p and p-p collisions are expected to be related by charge independence, but this is not considered.

The following conclusions can be drawn from the above curves and table. Proton-proton collisions are considered, unless otherwise stated.

1. If the Λ^0 has a spin of $\frac{1}{2}(\frac{3}{2})$ and positive parity, and the parity of the K meson relative to that of the

TABLE I. Table of allowed angular momenta for production near threshold of Λ^0, nucleon, and K meson in nucleon-nucleon collision. Spectroscopic notation is used to indicate: (initial state relative angular momentum)$\rightarrow$(relative angular momentum of Λ^0 and nucleon in final state). Possible transitions are given for a Λ^0 of spin $\frac{1}{2}$ and $\frac{3}{2}$ and a K meson of spin zero. Only S states of orbital motion are considered for the final-state Λ^0 and nucleon, and the K meson is assumed to be emitted into S or P states of angular momentum.

Λ^0 Spin	Reaction	$pp\rightarrow\Lambda^0pK^+$	$pn\rightarrow\Lambda^0pK^0,\ \Lambda^0nK^+$
$\frac{1}{2}$	Meson in S state K-Λ^0 parity same	${}^1S_0\rightarrow{}^1S_0$	$\left.\begin{matrix}{}^3D_1\\{}^3S_1\end{matrix}\right\}\rightarrow{}^3S_1$, ${}^1S_0\rightarrow{}^1S_0$
	Meson in P state K-Λ^0 parity same	$\left.\begin{matrix}{}^3F_2\\{}^3P_{2,1,0}\end{matrix}\right\}\rightarrow{}^3S_1$, ${}^3P_1\rightarrow{}^1S_0$	$\left.\begin{matrix}{}^3F_2\\{}^3P_{2,1,0}\\{}^1P_1\end{matrix}\right\}\rightarrow{}^3S_1$, $\left.\begin{matrix}{}^3P_1\\{}^1P_1\end{matrix}\right\}\rightarrow{}^1S_0$
	Meson in S state K-Λ^0 parity opposite	${}^3P_1\rightarrow{}^3S_1$, ${}^3P_0\rightarrow{}^1S_1$	${}^3P_1\rightarrow{}^3S_1$, ${}^3P_0\rightarrow{}^1S_0$
	Meson in P state K-Λ^0 parity opposite	$\left.\begin{matrix}{}^1S_0\\{}^1D_2\end{matrix}\right\}\rightarrow{}^3S_1$	$\left.\begin{matrix}{}^1S_0\\{}^1D_2\\{}^3D_{2,1}\end{matrix}\right\}\rightarrow{}^3S_1$, ${}^3D_1\rightarrow{}^1S_0$
$\frac{3}{2}$	Meson in S state K-Λ^0 parity same	${}^1D_2\rightarrow{}^5S_2$	$\left.\begin{matrix}{}^1D_2\\{}^3D_2\end{matrix}\right\}\rightarrow{}^5S_2$, $\left.\begin{matrix}{}^3D_1\\{}^3S_1\end{matrix}\right\}\rightarrow{}^3S_1$
	Meson in P state K-Λ^0 parity same	$\left.\begin{matrix}{}^3F_{3,2}\\{}^3P_{2,1}\end{matrix}\right\}\rightarrow{}^5S_2$, $\left.\begin{matrix}{}^3F_2\\{}^3P_{2,1,0}\end{matrix}\right\}\rightarrow{}^3S_1$	$\left.\begin{matrix}{}^3F_{3,2}\\{}^1F_3\\{}^3P_{2,1}\\{}^1P_1\end{matrix}\right\}\rightarrow{}^5S_2$, $\left.\begin{matrix}{}^3F_2\\{}^3P_{2,1,0}\\{}^1P_1\end{matrix}\right\}\rightarrow{}^3S_1$
	Meson in S state K-Λ^0 parity opposite	$\left.\begin{matrix}{}^3F_2\\{}^3P_2\end{matrix}\right\}\rightarrow{}^5S_2$, ${}^3P_1\rightarrow{}^3S_1$	$\left.\begin{matrix}{}^3F_2\\{}^3P_2\end{matrix}\right\}\rightarrow{}^5S_2$, $\left.\begin{matrix}{}^3P_1\\{}^1P_1\end{matrix}\right\}\rightarrow{}^3S_1$
	Meson in P state K-Λ^0 parity opposite	${}^1D_2\rightarrow{}^5S_2$, $\left.\begin{matrix}{}^1S_0\\{}^1D_2\end{matrix}\right\}\rightarrow{}^3S_1$	$\left.\begin{matrix}{}^1D_2\\{}^3D_{3,2,1}\\{}^3S_1\end{matrix}\right\}\rightarrow{}^5S_2$, $\left.\begin{matrix}{}^1S_0\\{}^1D_2\\{}^3D_{2,1}\end{matrix}\right\}\rightarrow{}^3S_1$

Λ^0 is also positive, then for a local point interaction, it is to be expected that the K meson will be emitted chiefly in an S state of angular momentum. The angular distribution of emitted mesons is spherically symmetric. In such a case one measures the ${}^1S_0({}^5S_2)$ interaction between the Λ^0 and nucleon; if this force is repulsive, the final-state-interaction effect should be negligible and the spectrum should correspond to the curve in Fig. 3 with $a=0$.

2. If the Λ^0 has a spin of $\frac{1}{2}(\frac{3}{2})$ and positive parity, but the K particle has negative parity with respect to the Λ^0, then the meson is expected to be emitted chiefly into P states of angular momentum. Only the ${}^3S_2({}^5S_2,{}^3S_1)$ interaction between the Λ^0 and p is then measured. The angular distribution of mesons will be $a+b\cos^2\theta$; the ratio of b/a depends on the relative contributions of the ${}^1D_2$ and ${}^1S_0$ states. For a Λ^0 of spin $\frac{1}{2}$, and a contribution from only the ${}^1D_2$ state, the angular distribution is[7] $(1+3\cos^2\theta)$. The spectrum will be of the type given in Fig. 4 (proportional to k^2dJ).

3. Finally, if parity conjugation is a good quantum number, a combination of (1) and (2) will hold. (Λ^0's of negative parity then also exist, but the interaction of these with nucleons will probably be strong in P states and therefore need not be considered.) The angular distribution will not, in general, be spherically symmetric, and the spectrum will be of the form $(A+Bk^2)dJ$, and thus will be a mixture of Figs. 3 and 4.

[7] K. M. Watson and C. Richman, Phys. Rev. 83, 1256 (1951).

4. Should 1 (or 2) hold, then for a Λ^0 of spin $\frac{1}{2}$, information on the ${}^3S_1({}^1S_0)$ interaction can be obtained in n-p collisions.

5. Finally, it may be pointed out that by similar arguments to those presented in this section, it should be possible to obtain information on the spin and parity assignments of bound hyperfragments from studies of such reactions as $p+\mathrm{He}^4\rightarrow{}_\Lambda\mathrm{He}^5+K$ near threshold.

III. π-d COLLISIONS

Information on the Λ^0-nucleon interaction can also be obtained by investigating the collisions of pions with deuterons. Available data indicate that the relative cross sections for production of the Λ^0 in nucleon-nucleon[8] and π-nucleon[9] encounters are of the same order of magnitude. However, one would expect the final-state interaction to be partially washed out by the motion of the struck nucleon in the deuteron. The reactions which can be initiated by charged pions are[10]

$$\pi^++d\rightarrow K^++\Lambda^0+p, \tag{5a}$$

$$\pi^-+d\rightarrow K^0+\Lambda^0+n. \tag{5b}$$

Reaction (a) can occur only when the π^+ strikes the neutron and reaction (b) only when the π^- collides with

[8] M. M. Block *et al.*, Phys. Rev. 103, 1484 (1956).

[9] Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. 98, 121 (1955).

[10] If the ${}_\Lambda\mathrm{H}^2$ and ${}_\Lambda n^2$ are bound, additional reactions to the bound states can occur. The modifications of the relations given below can be obtained from reference 12, and are similar to those in Sec. I. They will not be treated here.

the proton of the deuterium target. Reaction (b) can be directly related to the production cross section in π^--proton collisions by means of the impulse approximation.[11] This approximation has been used to relate π-d collisions to π-nucleon collisions by Fernbach, Green, and Watson[12] and others. The derivation will therefore not be given here. It should be noted, however, that the nature of the approximation differs from that used in the phenomenological approach of Sec. II. Thus, the final state interaction is not approximated by $|\psi_f(0)|^2$, but instead, it is assumed that the pion-nucleon production matrix element is not dependent on the momentum of the struck nucleon. A discussion of this approximation is given by Fernbach *et al.*[12] For an incident π meson of energy ω_0 and momentum $\mathbf{k}_0$, the matrix element for the reaction (5b), for example, can be written as

$$h_{fi}=\langle t|r_p|t\rangle I_1{}^t+\langle s|r_p|t\rangle I_1{}^s, \qquad (6)$$

with

$$I_1{}^t=\int\psi_f{}^{*t}(\mathbf{r})\exp[\tfrac{1}{2}i(\mathbf{k}_0-\mathbf{k})\cdot\mathbf{r}]\phi_d(\mathbf{r})d^3r, \qquad (7)$$

where all symbols not explicitly defined here are the same as those used by Fernbach *et al.*[12] In Eq. (7) $\psi_f{}^t$ is the final triplet-state Λ^0-nucleon wave function. A similar expression holds for $I_1{}^s$. The cross section for production is

$$\frac{d^2\sigma}{d\Omega_K dT}=\left(\frac{d\sigma_p}{d\Omega_K}\right)h_1{}^t+\frac{(2\pi)^4}{v_\pi}J_0L_p{}^s(h_1{}^s-h_1{}^t), \qquad (8)$$

with

$$h_1{}^t=\frac{k\omega}{J_0}\int d^3p\,\delta(E_f-E_0)|I_1{}^t|^2. \qquad (9)$$

A similar expression holds for $h_1{}^s$.

For the deuteron wave function $\phi_d=N(e^{-\alpha r}-e^{-\beta r})/r$, with $\alpha=45.5$ Mev and $\beta=7\alpha$ is used. If all final-state interactions are neglected, then

$$\psi_f{}^0=e^{i\mathbf{p}\cdot\mathbf{r}}/(2\pi)^{\frac{3}{2}}, \qquad (10)$$

and

$$h_1{}^t=8\sqrt{2}N^2m^{\frac{3}{2}}\frac{k\omega}{J_0}\left\{\frac{T_p{}^{\frac{1}{2}}}{[\alpha^2+(p-y)^2][\alpha^2+(p+y)^2]}+\frac{T_p{}^{\frac{1}{2}}}{[\beta^2+(p-y)^2][\beta^2+(p+y)^2]}+\frac{1}{2^{\frac{3}{2}}ym^{\frac{1}{2}}(\beta^2-\alpha^2)}\times\ln\left[\frac{\alpha^2+(p-y)^2}{\alpha^2+(p+y)^2}\cdot\frac{\beta^2+(p+y)^2}{\beta^2+(p-y)^2}\right]\right\}, \qquad (11)$$

where $y=|\mathbf{q}_0-\mathbf{q}|/2$ and $T_p=p^2/2m=T_m-T-2y^2/(M+M_\Lambda)$.

[11] G. F. Chew, Phys. Rev. **80**, 196 (1950).
[12] Fernbach, Green, and Watson, Phys. Rev. **84**, 1084 (1951).

As in Sec. II, only the S-state interaction between the Λ^0 and nucleon will be taken into account. In evaluating h_1, the S-wave part of the plane wave in Eq. (10) is then replaced by

$$\begin{aligned}\psi_f(\mathbf{r})&=(1/2\pi)^{\frac{3}{2}}\sin(pr+\delta_0)e^{-i\delta_0}/pr, \quad &r\geqslant b\\ &=A\sin(\kappa r)/r, &r\leqslant b\end{aligned} \qquad (12)$$

where a square well interaction has been chosen for illustrative purposes. An approximate evaluation of h_1 for this condition is given in the Appendix.

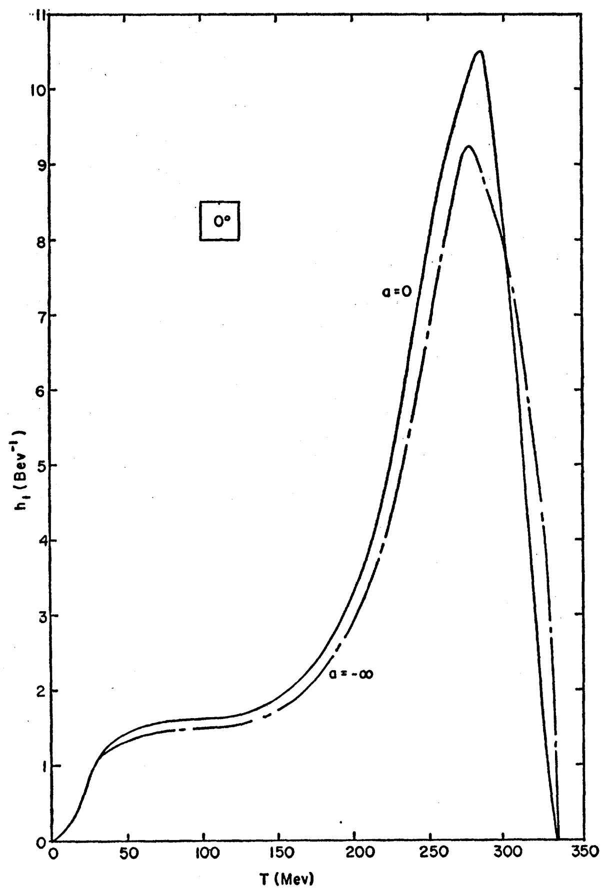

FIG. 5. Plot of h_1 as a function of K-meson energy, T, for forward emission and incident pion kinetic energy of 900 Mev. The curve for $a=0$ corresponds to no final-state interaction; that for $a=-\infty$ corresponds to a final-state square well interaction of depth 190 Mev and radius 1/2 μ.

As in the nucleon-nucleon production process, it is desirable to choose as high an energy as possible without having appreciable Σ^0 production. For an average nucleon kinetic energy of 15 Mev in the deuteron, the threshold for Σ^0 production occurs approximately at 760 Mev. This energy almost corresponds to the production threshold of the Λ^0 from a nucleon at rest in the deuterium target. Thus $d\sigma_p/d\Omega_K$ is equal to zero in the laboratory frame except in the forward direction; the cross section in deuterium is therefore expected to

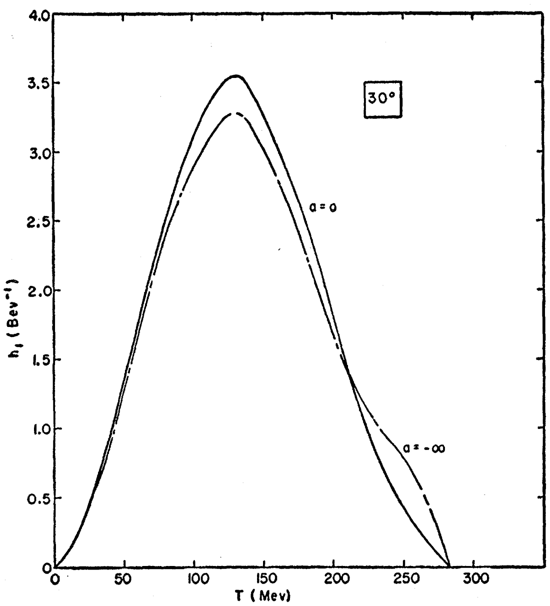

FIG. 6. Same as Fig. 5, but for mesons emitted in the laboratory system at an angle of 30° to the incident pion beam.

fall off rapidly with increasing angle. Perhaps a more appropriate choice of initial energy is 900 Mev, which corresponds to the threshold for Σ^0 production, if the struck nucleon is at rest in the deuteron. For Λ^0 production, the maximum angle in the laboratory system

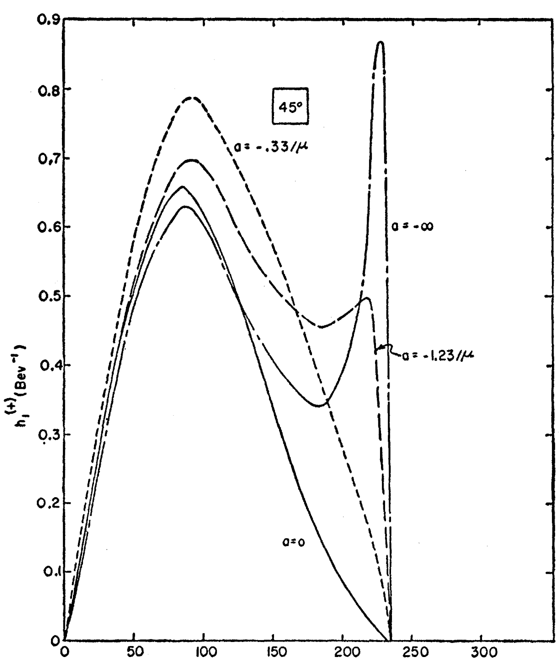

FIG. 7. Same as Fig. 5, but for mesons emitted in the laboratory system at an angle of 45° to the incident pion beam. In addition the curve for $a=-0.33\,\mu$ corresponds to a square well of depth 74 Mev, radius $2/3\,\mu$; that for $a=-1.23/\mu$ to a square well of depth 81 Mev, radius $1/2\,\mu$.

at which a K meson will then appear for a struck nucleon at rest is 53°. The cross section is expected to fall rapidly beyond this angle; the impulse approximation predicts zero cross section.

Plots of h_1 with and without final-state interaction in Λ^0-nucleon S states are shown in Figs. 5, 6, and 7 for laboratory angles of 0°, 30°, and 45°, respectively. At 0° and 30°, the difference found between no interaction and an attractive interaction that corresponds to a scattering length, a, of $-\infty$ (well depth 190 Mev, well radius $1/2\,\mu$) is too small to be measurable. It is only at 45° and at the high-energy end of the spectrum that the difference between a relatively strong interaction and no interaction becomes significant. For this reason, curves for less attractive potentials are only shown at this angle. At all three angles the modification of the K-meson spectrum, due to a potential of the same

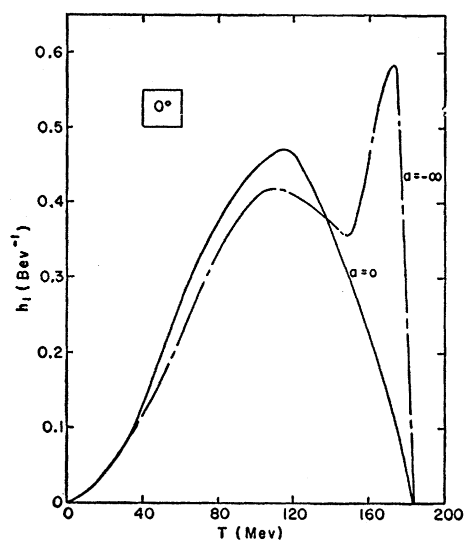

FIG. 8. Same as Fig. 5 but for an incident pion kinetic energy of 760 Mev.

strength as that corresponding to $a=-\infty$ ($V=+190$ Mev and $b=1/2\,\mu$), was found to closely parallel that for no interaction, as in nucleon-nucleon collisions. Similar results to those obtained at 45° were found at 0° for an incident meson kinetic energy of 760 Mev. At this energy, h_1 is plotted at 0° as a function of a K-meson kinetic energy in Fig. 8.

Since both the cross section for production of strange particles in π-p collisions at 900 Mev and the spin-flip probability in this process are not known, it is not possible to evaluate the cross section for production in π-d collisions. However, it is expected that the effects shown in Figs. 7 and 8 will be present. Thus if the potential is attractive in the triplet S state and repulsive in the singlet S state (assuming Λ^0 has spin$=\frac{1}{2}$), then h_1^t is approximately equal to h_1^s over most of the K-meson spectrum, except near the high-energy end, where h_1^t is much larger than h_1^s. The production cross section [see Eq. (8)] will therefore be approximately proportional to h_1^t over the whole spectrum.

Unlike proton-proton collisions, the final-state parallel-spin and antiparallel-spin interaction cannot be measured separately. Since both of these spin states are allowed, in general, only the effective average of the two is measured (i.e., only the attractive one, if one of them is repulsive). In addition, the effect of the final-state interaction is decreased by the motion of the struck nucleon in the target. It therefore appears that π-p collisions are not as useful as nucleon-nucleon collisions in determining properties of the Λ^0-nucleon force.

IV. CONCLUSIONS

It has been shown that the spectrum of K mesons produced close to threshold in nucleon-nucleon and π-deuteron collisions can be used to obtain direct experimental information on the Λ^0-nucleon interaction. These experiments are important, since the Λ^0-nucleon force need not be intimately related to the force which binds the Λ^0 in hyperfragments.

The measured spectrum of K mesons emitted in nucleon-nucleon encounters can be directly related to the scattering length of the S-state Λ^0-nucleon potential, if the latter is attractive. For a repulsive interaction the spectrum of emitted K mesons is expected to be almost the same as that for no interaction. Measurements of the K-meson spectrum can also be employed to determine whether the ${}_{\Lambda}H^2$ hyperfragment is bound.

The energy distribution of the K-mesons produced is less sensitive to the Λ^0-nucleon force in π-d collisions than in nucleon-nucleon collisions. In π-d encounters, measurable deviations of the spectrum due to the Λ^0-nucleon force occur only at laboratory angles close to the maximum possible for production from a nucleon at rest in the deuteron. The effect of the final-state interaction is reduced by the motion of the nucleons in the target. Furthermore, only an effective average of the Λ^0-nucleon force in the two possible spin states can be measured, while in nucleon-nucleon collisions the forces in these two states can be obtained separately.

The author wishes to thank Dr. Hiroomi Umezawa for a stimulating discussion and Mr. Paul Stevens for aid in numerical computations.

APPENDIX

The derivation of the correction to I_1 [defined by Eq. (7)] resulting from S-state interactions is considered below. The correction, ΔI_1, is

$$\Delta I_1=\frac{4\pi}{(2\pi)^{\frac{3}{2}}y}\int_0^\infty [\phi_f^*(r)-\phi_0^*(r)]\sin yr\phi_d(r)r\,dr, \quad \text{(a)}$$

where $y=|\mathbf{q}_0-\mathbf{q}|/2$, ϕ_0 is the S-wave term in the expansion of $\exp(i\mathbf{p}\cdot\mathbf{r})$,

$$\phi_0(r)=\sin pr/pr, \quad \text{(b)}$$

and ϕ_f is the final S-state radial wave function, which for a square well and a phase shift δ_0 can be written as

$$\begin{aligned}\phi_f(r)&=e^{-i\delta_0}\sin(pr+\delta_0)/pr, & r\geqslant b\\ &=A\sin(\kappa r)/r, & r\leqslant b.\end{aligned} \quad \text{(c)}$$

The integral $\int_0^\infty\phi_0^*\sin yr\phi_d r\,dr$ is elementary, and gives

$$\int_0^\infty \phi_0^*\sin yr\phi_d r\,dr =\frac{N}{4p}\ln\left[\frac{\alpha^2+(p+y)^2}{\alpha^2+(p-y)^2}\cdot\frac{\beta^2+(p-y)^2}{\beta^2+(p+y)^2}\right]. \quad \text{(d)}$$

The remaining term is approximated as follows:

$$\int\phi_f^*\sin yr\phi_d r\,dr\approx\frac{e^{+i\delta_0}\sin\delta_0}{p}\int_0^\infty(\sin pr\cot\delta_0 +\cos pr)\sin yr\phi_d dr+N\int_0^b\left[\frac{A\sin\kappa r}{r} +\left(\frac{1}{a}-\frac{1}{r}\right)\frac{\sin\delta_0 e^{+i\delta_0}}{p}\right]\sin yr(\beta-\alpha)r\,dr. \quad \text{(e)}$$

In the second integral of Eq. (e), both ϕ_d and some terms in ϕ_f have been approximated by their leading terms in an expansion in r. This is felt to be reasonable, since as $r\to b$, $\sin yr$ oscillates more and more rapidly and the expression in square brackets approaches zero. The resultant expression is

$$\begin{aligned}\int\phi_f^*\sin yr\phi_d r\,dr\approx&\frac{N}{p}\sin\delta_0 e^{+i\delta_0}\\ &\times\Bigg\{(\beta-\alpha)\left[\frac{y(1-b/a)\cos yb}{(\kappa^2-y^2)}-\kappa(1-b/a)\frac{\cot\kappa b\sin yb}{(\kappa^2-y^2)}\right.\\ &\left.-\frac{1}{y}(1-\cos yb)\right]-\frac{e^{-\alpha b}}{a}\frac{(\alpha\sin yb+y\cos yb)}{(\alpha^2+y^2)}\\ &+\frac{e^{-\beta b}}{a}\frac{(\beta\sin yb+y\cos yb)}{(\beta^2+y^2)}+\frac{y}{a(\alpha^2+y^2)}-\frac{y}{a(\beta^2+y^2)}\\ &+\frac{1}{2}\tan^{-1}\left(\frac{2\alpha y}{\alpha^2+p^2-y^2}\right)-\frac{1}{2}\tan^{-1}\left(\frac{2\beta y}{\beta^2+p^2-y^2}\right)\\ &+\frac{1}{4ap}\ln\left[\frac{\alpha^2+(p-y)^2}{\alpha^2+(p+y)^2}\cdot\frac{\beta^2+(p+y)^2}{\beta^2+(p-y)^2}\right]\Bigg\}.\end{aligned}$$