Monte Carlo Calculation of Single Pion Production by Pions*

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A phenomenological calculation of single pion production in a pion-nucleon collision has been made using a $\frac{3}{2}, \frac{3}{2}$ isobaric nucleon model and a Monte Carlo type of calculation on a high-speed digital computer. The results are in good agreement with observations of pion production for incident pion energies of approximately 1.0 and 1.5 Bev.

THE suggestion that the $\frac{3}{2}, \frac{3}{2}$ resonant state of the pion-nucleon system might play an important role in pion production appears to have originated at the Brookhaven Laboratory. Yuan and Lindenbaum,[1] who investigated the energy spectrum of pions arising from high-energy proton bombardment of beryllium, suggested that the observed strong yield of pions at low energies might be attributed to the $\frac{3}{2}, \frac{3}{2}$ resonance at approximately 0.2 Bev. On the basis of arguments of charge independence, Peaslee[2] concluded that the variation of single to double pion production with proton energy, which was observed by Yuan and Lindenbaum, could be well interpreted in terms of the $\frac{3}{2}, \frac{3}{2}$ excited state of either or both nucleons involved in the production reaction. Fowler, Shutt, Thorndike, and Whittemore,[3] who investigated pion production in high-energy n-p collisions, ascribed the observed strong angular correlations of π^+ and π^- mesons with protons and neutrons, respectively, to the excitation of $\frac{3}{2}, \frac{3}{2}$ isobaric nucleon states. Walker, Crussard, and Koshiba,[4] who investigated pion production in high-energy pion-nucleon collisions, also concluded that the observed pion and nucleon angular distributions were consistent with what might be expected from an excited nucleon model.

In the course of an investigation of pion production in heavy nuclei in emulsions, we have also observed that the pion spectrum has a maximum at much lower energies than one finds from Monte Carlo calculations in which an isotropic pion-production model is assumed.[5] Consequently, we have been interested in developing other models of pion production, and in particular the isobar model, which could be applied to Monte Carlo calculations of pion production in nuclei. An analytical evaluation of the effects of nucleon excitation in nucleon-nucleon pion production has recently been made by Yuan and Lindenbaum[6] and Sternheimer and Lindenbaum.[7]

OUTLINE OF THE MODEL

An incident high-energy pion is assumed to collide with a free nucleon at rest. After the collision, it is assumed that a scattered pion, π_1, and a nucleon isobar, N^*, conserve energy and momentum of the initial pion-nucleon system. The mass of the isobar is assumed to be equal to the sum of the masses of the bare nucleon and created pion, π_2, together with the mass-equivalent of the excitation, Q, of the $\frac{3}{2}, \frac{3}{2}$ isobar.

We make the *ad hoc* assumption that the probability for the formation of the isobar with a particular excitation, Q, is proportional to the product of two factors: (a) the observed pion-nucleon total scattering cross section, σ, in the $\frac{3}{2}, \frac{3}{2}$ state at a kinetic energy in the center-of-mass system of the created pion and nucleon equal to Q, and (b) the two-body phase space factor, F, for the scattered pion and isobar, equal to $p_{\pi1}E_{\pi1}E_{N^*}/E$, where $p_{\pi1}$ is the momentum of the scattered pion, $E_{\pi1}$ and E_{N^*} are the total energies of the scattered pion and isobar, respectively, and E is equal to the sum of $E_{\pi1}$ and E_{N^*}, momentum and energy being given in the center-of-mass system of the colliding particles. Assumption (a) implies that, apart from the phase space factor, the formation of the isobar is a process which is quite independent of the scattering of the incident pion. It will be seen later that we also make the same assumption from the converse viewpoint of the scattered pion.

The isobar is then assumed to decay isotropically in its rest system, the available energy, Q, being conserved by the nucleon and created pion. The complete process is thus considered as two successive two-body interactions.

Since we are particularly interested in comparing our calculations with the experimental observations of Shutt *et al.*[8] and Walker *et al.*,[9] we shall concern our-

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1 L. C. L. Yuan and S. J. Lindenbaum, Phys. Rev. **93**, 1431 (1954).

2 D. C. Peaslee, Phys. Rev. **94**, 1085 (1954).

3 Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **95**, 1026 (1954).

4 Walker, Crussard, and Koshiba, Phys. Rev. **95**, 852 (1954).

5 Bivins, Metropolis, Storm, Turkevich, Miller, and Friedlander, Bull. Am. Phys. Soc. Ser. II, **2**, 63 (1957).

6 L. C. L. Yuan and S. J. Lindenbaum, Phys. Rev. **103**, 404 (1956).

7 R. M. Sternheimer and S. J. Lindenbaum, Bull. Am. Phys. Soc. Ser. II, **2**, 62 (1957).

8 Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797 (1955).

9 W. D. Walker and J. Crussard, Phys. Rev. **98**, 1416 (1955); Walker, Hushfar, and Shephard, Phys. Rev. **104**, 526 (1956).

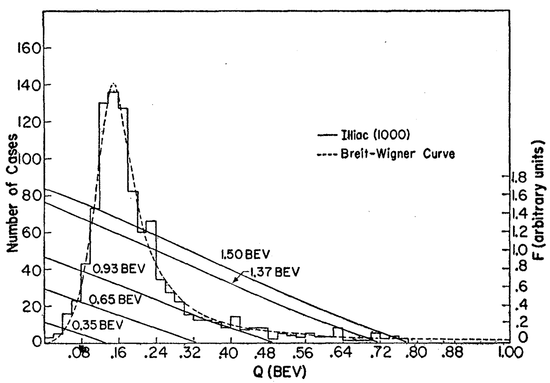

FIG. 1. Histogram of 1000 values of σ used in the Monte Carlo calculation of pion production in pion-nucleon collisions at 1.5 Bev. The broken curve is the calculated Breit-Wigner formula for σ. The solid curves are the calculated phase space factors, F, for the different incident pion energies indicated on the curves.

selves mainly with a description of the process in the plane of the three final particles. The orientation of this plane, about the direction of the scattered pion as axis, is determined by the line of disintegration of the isobar. In assuming that the isobar disintegrates with spherically symmetrical angular distribution in its own system, we have neglected the possibility of pion-pion interaction.

Although a calculation of the momentum and angular distributions in the plane and center-of-mass (c.m.) system of the three final particles yields an adequate test[3] of the isobar model, it is necessary for the purposes of a Monte Carlo calculation in nuclei to obtain the information with respect to the incident pion direction and to the nuclear system. Reference to the incident pion direction has also been made in a number of experimental distributions.[8,9] Thus, in order to convert angles in the c.m. plane into those with respect to the incident pion, we require a function describing the probability for the scattering of one of the particles at a particular angle to the incident pion direction. The particular scattering function employed will be discussed in the next section. The assumption which is implicit in this procedure is that the probability distribution of excitation of isobars remains constant as a function of scattering angle.

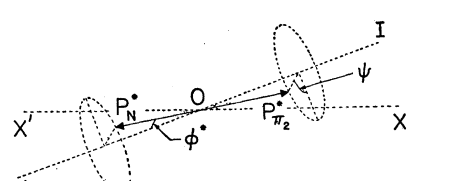

FIG. 2. Disintegration of isobar in center-of-mass system of isobar. ϕ^* is the angle of disintegration with respect to the isobar direction of motion. $I'I$ is the direction of motion of the isobar with respect to the incident pion direction, $X'X$, in the center-of-mass system of the colliding particles.

FURTHER DETAILS OF THE CALCULATION

One of the first steps in the Monte Carlo calculation is the random selection of a Q value from a probability distribution which is proportional to the total pion scattering cross section, σ, multiplied by the phase space factor, F. We used for the scattering cross section the theoretical Breit-Wigner formula which was fitted to the experimental data by Brueckner[10] and Gell-Mann and Watson.[11] We have allowed the curve to extend to the maximum possible value of Q permitted by the phase space factor. A histogram of the σ values actually selected in a run of 1000 cases for an incident pion-energy of 1.5 Bev is shown in Fig. 1. The smooth curve is the calculated Breit-Wigner formula which is used to compute σ. That the histogram of σ values fits so well merely reflects the randomness of the random number generator.[12] Also shown in Fig. 1 are the phase space factors, F, for the various incident pion energies which were used in the computations.

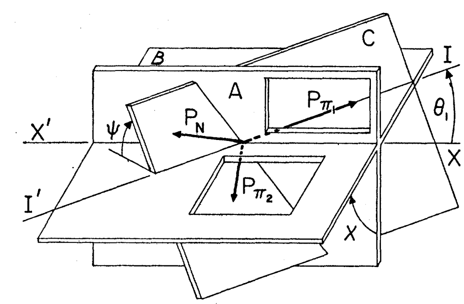

FIG. 3. Pion production in inelastic pion-nucleon collision in center-of-mass system of incident pion and nucleon. Scattered pion, π_1, created pion, π_2, and nucleon, N, conserve momentum in the plane C of the three final particles. Plane A, containing the incident pion direction, $X'X$, is a reference plane in the plane of the paper. Plane B contains the scattered pion direction, $I'I$, at an angle θ_1 to the incident pion direction, and the angle χ is an azimuthal rotation of plane B about the axis $X'X$. Plane C is the disintegration plane of the isobar, and it is rotated about the axis $I'I$ at an angle ψ to plane B.

The effective mass of the isobar is determined by the selection of Q. From the incident pion energy and the masses of the isobar and scattered pion, the momentum of the isobar is evaluated. This then determines the transformation velocity of the isobar in the c.m. system of the isobar and scattered pion.

The isobar is next allowed to explode in its own system and p_{N^*} and $p_{\pi 2^*}$ are calculated from the known Q (see Fig. 2). The probability of the angle of disintegration being between ϕ^* and $\phi^*+d\phi^*$ is assumed to be proportional to the solid angle subtended between these angles. In the Monte Carlo calculation $\cos\phi^*$ is made proportional to a random number.

The final momenta of the nucleon and created pion, p_N and $p_{\pi 2}$, respectively, as well as the relative angles

[10] K. A. Brueckner, Phys. Rev. **86**, 106 (1952).
[11] M. Gell-Mann and K. M. Watson, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1954), Vol. 4.
[12] N. Metropolis *et al.* (to be published).

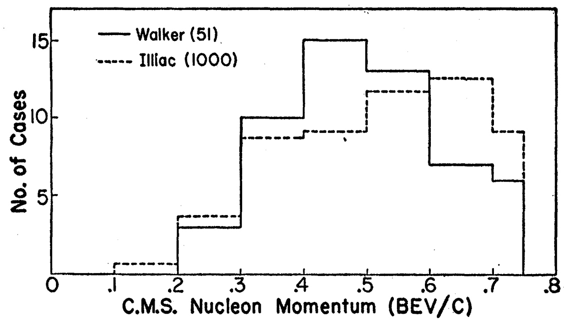

FIG. 4. Distribution of nucleon momenta in c.m. system of colliding particles, for incident pion energy of 1.5 Bev.

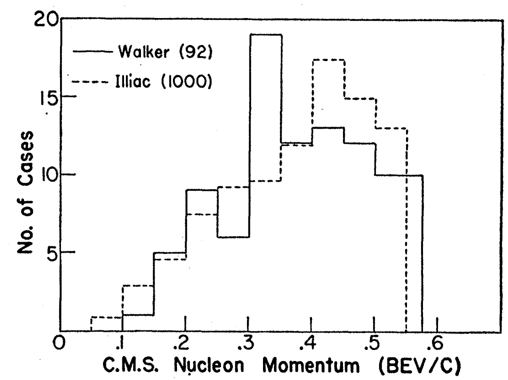

FIG. 6. Distribution of nucleon momenta in c.m. system of colliding particles, for incident pion energy of 0.93 Bev.

between the particles in the "plane," are calculated by using Lorentz transformations into the c.m. system of the isobar and scattered pion (see Fig. 3).

Given the axes $X'X$ and $I'I$, the angles ψ and χ, shown in Fig. 3, determine the orientation of the reference plane in space. Although this information is important in analyzing pion production in nuclei, we will not concern ourselves with it because we are considering here only a free nucleon scattering center. The angle ψ, however, is required for determining the directions of the nucleon and created pion relative to the incident pion direction. We have allowed ψ to assume a uniform distribution about the direction of motion of the isobar, i.e., the axis $I'I$, Fig. 3. The problem of choosing angle θ_1 is a difficult one. Probably the most suitable information for this purpose would be a knowledge of the pion-nucleon differential inelastic scattering cross section. The experimental observations closest to this cross section have been obtained by Walker *et al.*,[9] but their distribution is inappropriate because it also contains pions which have come from the disintegration of isobars, i.e., the incident pions have been charge-exchange scattered. Our analysis has shown that a significant fraction, e.g., of the order of 10% at 1 Bev, of the higher energy pions comes from the isobars. However, we cannot readily correct the observed cross sections without having information of the magnitude of inelastic charge-exchange scattering. The differential scattering probability that has been used tentatively in the present analysis is: $P(\theta)=3\cos^4\theta+4\cos^3\theta+1$. This has already been used in the Monte Carlo calculations in nuclear cascades.[5] To a certain extent, the experimental angular distributions of pions and nucleons with respect to the incident pion direction may provide a check on the differential inelastic scattering cross sections in pion-nucleon events.

RESULTS

Histograms of the Monte Carlo calculations are compared with the available experimental results and are shown in Figs. 4–21. In general, the agreement between calculation and experiment is good.

The nucleon momentum distributions for three energies: 1.5 Bev, 1.37 Bev, and 0.93 Bev, are shown in Figs. 4, 5, and 6. It should be pointed out that, according to Walker *et al.*,[9] many of the pion-nucleon collisions observed in their experiments (1.5 and 0.93 Bev) may have occurred on nuclear peripheral nucleons. To some extent this may account for the somewhat poorer

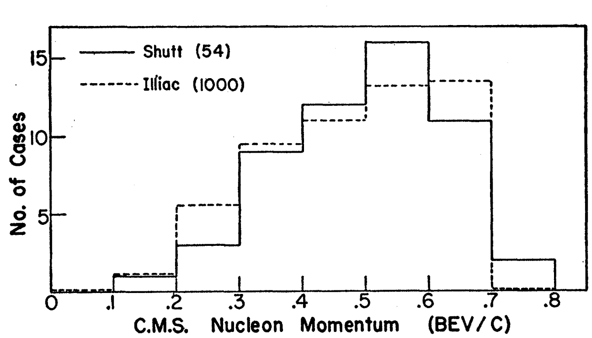

FIG. 5. Distribution of nucleon momenta in c.m. system of colliding particles, for incident pion energy of 1.37 Bev.

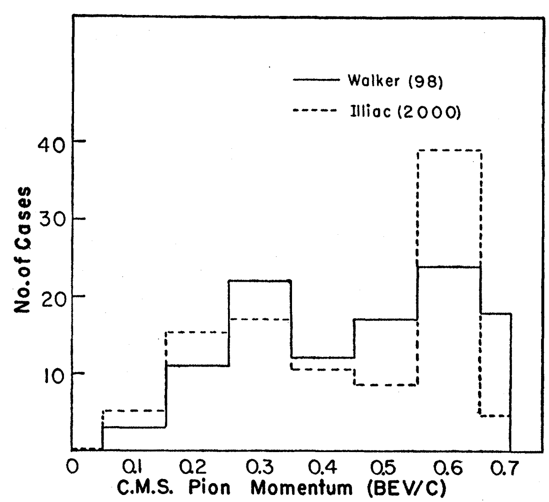

FIG. 7. Distribution of pion momenta in c.m. system of colliding particles, for incident pion energy of 1.5 Bev.

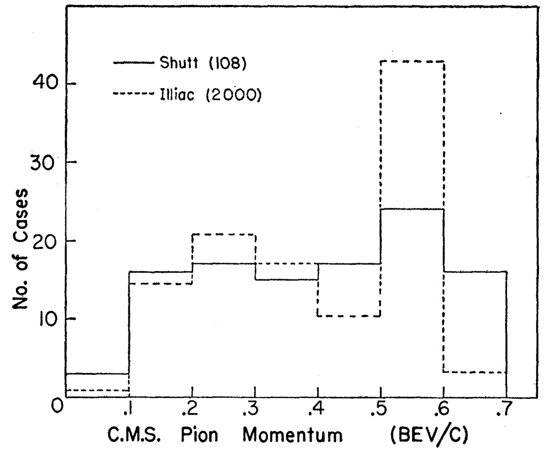

FIG. 8. Distribution of pion momenta in c.m. system of colliding particles, for incident pion energy of 1.37 Bev.

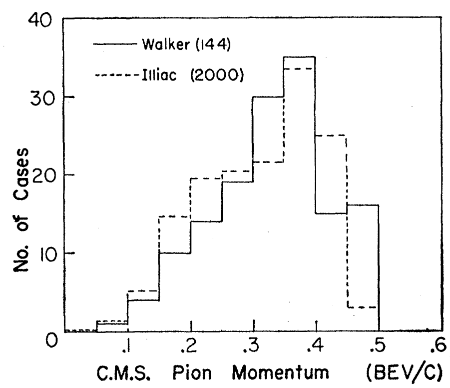

FIG. 9. Distribution of pion momenta in c.m. system of colliding particles, for incident pion energy of 0.93 Bev.

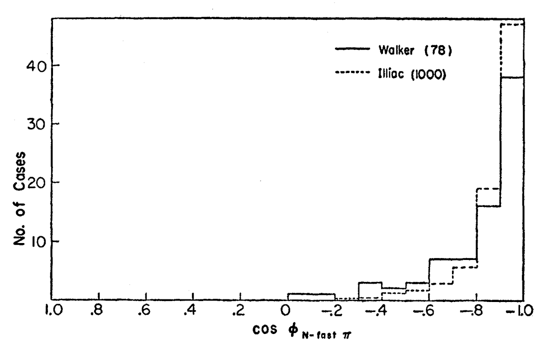

FIG. 10. Angular distribution, in c.m. system of colliding particles, of nucleons relative to the more energetic pions for incident pion energy of 1.5 Bev.

correspondence of the Monte Carlo histograms with experiment in these cases than in the case of the comparison at 1.37 Bev, for which the experiment[8] was performed in cloud chambers using hydrogen gas. It should also be indicated that the experimental histograms are based upon small numbers of cases (numbers are indicated in parentheses in the legends). Generally within statistical error the experimental and calculated histograms agree.

The pion momentum distributions for three energies: 1.5 Bev, 1.37 Bev, and 0.93 Bev, are shown in Figs. 7, 8, and 9. These histograms contain both "fast" and "slow" pions together. The distinction between fast and slow pions was used by experimenters to bring out the qualitative nature of the isobar model. However, as we have already indicated, we find that the fast pion in $\sim 10\%$ of the cases is not the scattered pion. The broad distinction, nevertheless exists, and this is indi-

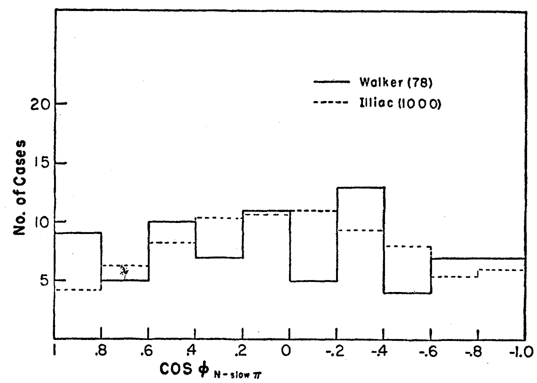

FIG. 11. Angular distribution, in c.m. system of colliding particles, of nucleons with respect to the lower energy pions for incident pion energy of 1.5 Bev.

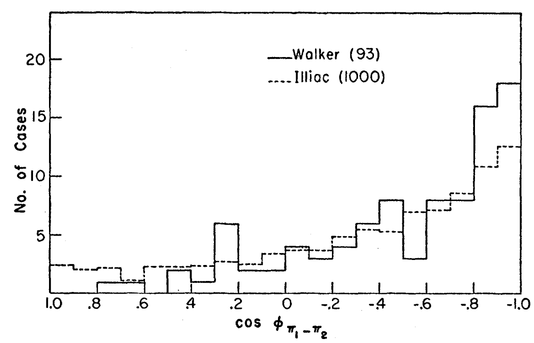

FIG. 12. Angular distribution, in c.m. system of colliding particles, of π_1 relative to π_2 for incident pion energy of 1.5 Bev.

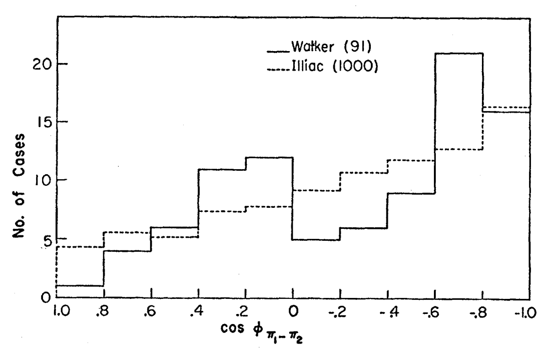

FIG. 13. Angular distribution, in c.m. system of colliding particles, of π_1 relative to π_2 for incident pion energy of 0.93 Bev.

cated by the two peaks in the histograms at approximately 0.2–0.3 Bev/c and 0.5–0.6 Bev/c in Figs. 7 and 8. The poorest correspondence between calculation and experiment occurs at high momentum in the case of the 1.37-Bev incident pion energy. It is possible that the difficulties of high momentum measurement may have increased the 0.6–0.7 Bev/c bin at the expense of the 0.5–0.6 Bev/c.

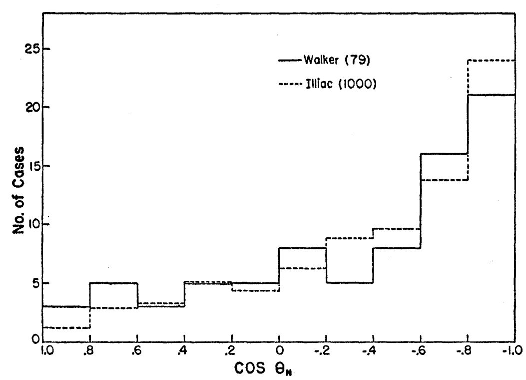

FIG. 14. Angular distribution, in c.m. system of colliding particles, of nucleons relative to the incident pion direction for incident pion energy of 1.5 Bev.

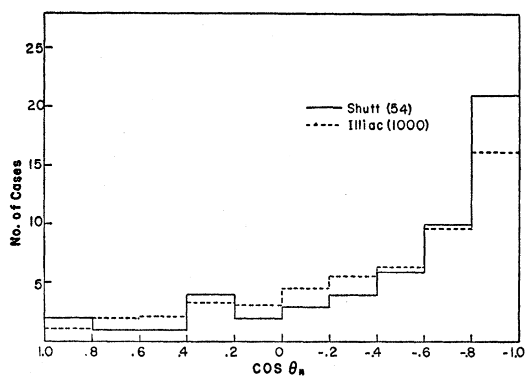

FIG. 15. Angular distribution, in c.m. system of colliding particles, of nucleons relative to the incident pion direction for incident pion energy of 1.37 Bev.

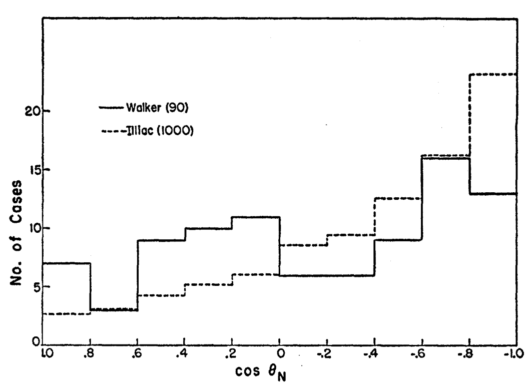

FIG. 16. Angular distribution, in c.m. system of colliding particles, of nucleons relative to the incident pion direction for incident pion energy of 0.93 Bev.

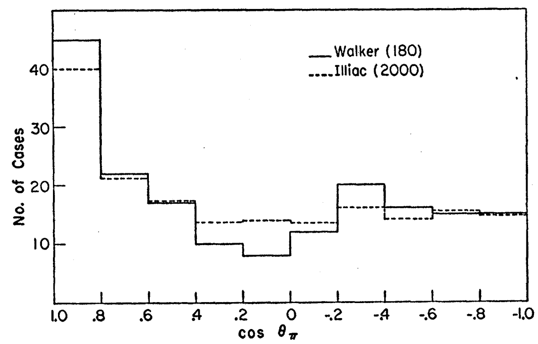

FIG. 17. Angular distribution, in c.m. system of colliding particles, of both pions with respect to the incident pion direction for incident pion energy of 1.5 Bev.

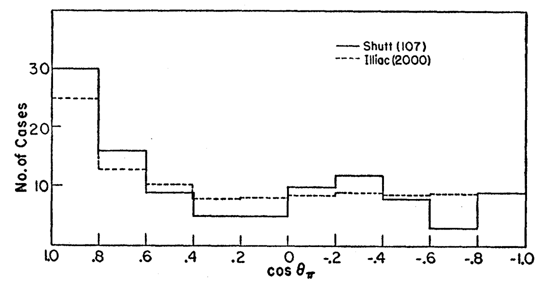

FIG. 18. Angular distribution, in c.m. system of colliding particles, of both pions with respect to the incident pion direction for incident pion energy of 1.37 Bev.

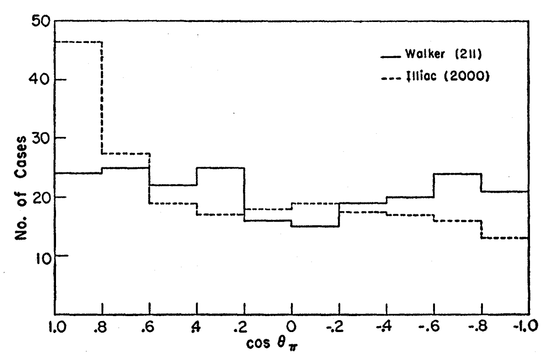

FIG. 19. Angular distribution, in c.m. system of colliding particles, of both pions with respect to the incident pion direction for incident pion energy of 0.93 Bev.

Although it cannot be decided experimentally whether the fast pion is always the scattered pion, the angular distributions of the fast and slow pions are features which can be definitely checked with calculation. These angular distributions with respect to the nucleon are shown in Figs. 10 and 11 for an incident pion energy of 1.5 Bev. The angular distributions of the two pions with respect to one another are shown in Figs. 12 and 13 for energies of 1.5 and 0.93 Bev. The agreement is generally close.

The comparison between calculation and experiment shown in the histograms thus far has been in the c.m. system of the colliding pion and nucleon and the plane

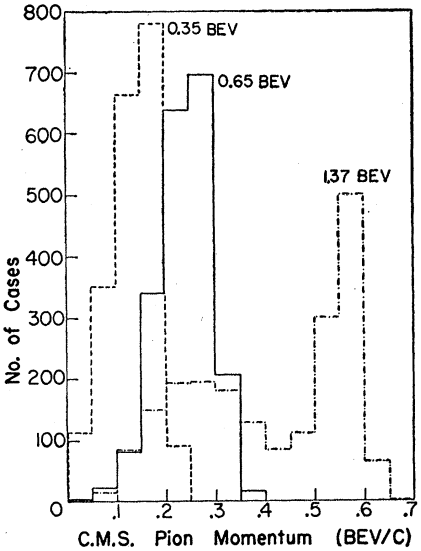

FIG. 20. Distribution of pion momenta, in c.m. system of colliding particles, for different incident pion energies.

containing the three final particles. We have already pointed out that a comparison on such a basis should prove a sufficient test of the isobar model. The agreement between the two sets of data is satisfactory, and we therefore believe that the isobar model approximately describes single pion production in the energy region studied.

The angular distributions of the final particles with respect to the incident pion can be calculated in the present model only by inserting in the computation a rather inaccurately-known differential pion-nucleon inelastic scattering cross section. For the formula already given in the previous section, the angular distributions of the nucleons relative to the incident pion direction are given for three energies: 1.5, 1.37, and 0.93 Bev, in Figs. 14, 15, and 16. It is seen that there is reasonable agreement between the two sets of histograms, especially at the higher energies. The poorer agreement at the lowest energy of 0.93 Bev may possibly be attributable to a less-forward peaking of the differential scattering cross section at this energy than is assumed in the formula used. Walker *et al.*[9] give some experimental indication of this in their result.

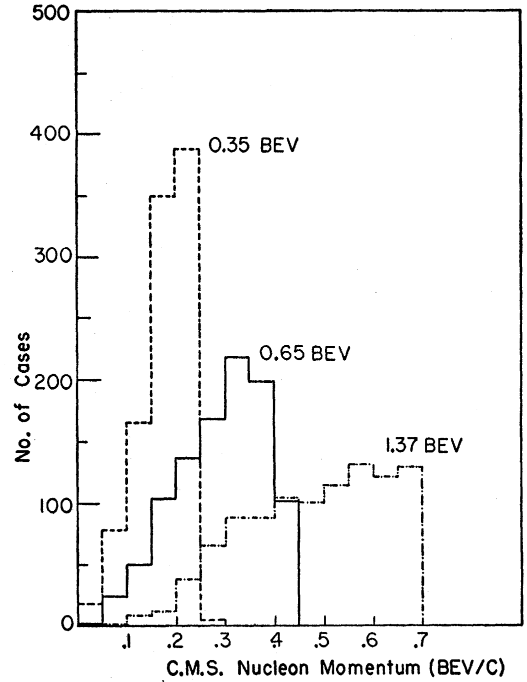

FIG. 21. Distribution of nucleon momenta, in c.m. system of colliding particles, for different incident pion energies.

The angular distributions of both pions with respect to the incident pion direction are shown in Figs. 17, 18, and 19. These histograms, while showing general agreement between calculation and experiment, exhibit a divergence at 0.93 Bev which is similar to that observed in the nucleon case of Fig. 16, above.

Finally, in order to indicate what variations might be expected at comparatively low energies from the isobar model, we have calculated the distributions of pion and nucleon momenta shown in Figs. 20 and 21 for a number of incident pion energies.

We wish to acknowledge the generous allotment of computing time on Illiac by the staff of the Digital Computer Laboratory.