

Spectral Distribution of the Photomagnetolectric Effect in Semiconductors: Theory

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A solution of the differential equations describing the photomagnetolectric (PME) effect in semiconductors under small injection of arbitrary spatial distribution is derived for the infinite slab. The results are presented in graphical form showing PME response *vs* absorption coefficient with bulk lifetime, surface recombination velocities, and slab thickness as parameters.

I. INTRODUCTION

THE photomagnetolectric (PME) effect which was first observed in cuprous oxide¹ has recently attracted considerable interest.²⁻¹² It is of importance in connection with the measurement of bulk lifetime and surface recombination rates in semiconductors, as a potential tool in the study of surface states, and as a basis for the development of photosensitive devices. The following is an extension of the theory which describes the effect in its dependence on the wavelength of the incident light.

II. SOLUTION OF THE BASIC EQUATIONS

The carrier transport through a semiconductor in the presence of an electric field \mathbf{E} and a steady magnetic field $\mathbf{H} = |H| \mathbf{k}$, and under arbitrary bulk generation, is described by the following equations:

$$(\partial n / \partial t) = -(n - n_0) / \tau_n + q^{-1} \operatorname{div} \mathbf{J}_n + g(x, y, z; t), \quad (1)$$

$$(\partial p / \partial t) = -(p - p_0) / \tau_p - q^{-1} \operatorname{div} \mathbf{J}_p + g(x, y, z; t), \quad (2)$$

$$\mathbf{J}_{\text{tot}} = \mathbf{J}_n + \mathbf{J}_p, \quad (3)$$

$$\mathbf{J}_n = q\mu_n n \mathbf{E} + qD_n \operatorname{grad} n + \tan \theta_n (\mathbf{J}_n \times \mathbf{k}), \quad (4)$$

$$\mathbf{J}_p = q\mu_p p \mathbf{E} - qD_p \operatorname{grad} p + \tan \theta_p (\mathbf{J}_p \times \mathbf{k}), \quad (5)$$

¹ I. K. Kikoin and M. M. Noskov, *Physik. Z. Sowjetunion* **5**, 586 (1934); I. K. Kikoin, *Physik. Z. Sowjetunion* **6**, 478 (1934); G. Groetzinger, *Physik. Z.* **36**, 169 (1935).

² J. Frenkel, *Physik. Z. Sowjetunion* **5**, 597 (1934); **8**, 185 (1935).

³ P. Aigrain and H. Bulliard, *Compt. rend.* **236**, 595, 672 (1953); H. Bulliard, *Ann. phys.* **15**, 52 (1954); P. Aigrain, *Ann. radioélec. Compagn. Gen. de T. S. F.* **9**, 219 (1954).

⁴ H. Bulliard, *Phys. Rev.* **94**, 1564 (1954).

⁵ Moss, Pincherle, and Woodward, *Proc. Phys. Soc. (London)* **66B**, 743 (1953); T. S. Moss, *Physica* **20**, 989 (1954); L. Pincherle, *Proceedings of the Atlantic City Conference on Photoconductivity, November 4, 1954* (John Wiley and Sons, Inc., New York, 1956).

⁶ J. J. Oberly, *Phys. Rev.* **93**, 911 (1954).

⁷ T. M. Buck and W. H. Brattain, *J. Electrochem. Soc.* **102**, 636 (1955).

⁸ G. Grosvalet, *Ann. radioélec. Compagn. Gen. de T. S. F.* **9**, 360 (1954).

⁹ Kurnick, Strauss, and Zitter, *Phys. Rev.* **94**, 1791 (1954); T. S. Moss, *Proc. Phys. Soc. (London)* **66B**, 993 (1953).

¹⁰ W. van Roosbroeck, *Phys. Rev.* **101**, 1713 (1956). For an introduction to the PME effect and additional literature see this comprehensive article.

¹¹ T. M. Buck and F. S. McKim, *Bull. Am. Phys. Soc. Ser. II*, **1**, 294 (1956).

¹² An experimental investigation of the spectral distribution of the PME effect in germanium has been carried out by Brand, Baker, and Mette and is being readied for submittal to *The Physical Review*.

where $g(x, y, z; t)$ is the rate of (external) pair generation superimposed on the thermal generation and recombination mechanism characterized by τ . The other symbols have the customary meanings. The equations hold under the assumption of Boltzmann statistics and no magnetoresistance. We further assume: (1) local charge neutrality, $p - p_0 = n - n_0$, $\operatorname{grad} p = \operatorname{grad} n$; (2) a slab of semiconductor infinite in the x and z directions; (3) steady state; (4) short- or open-circuit conditions; (5) small Hall angles; (6) small injection levels, so that

$$|p - p_0| = |n - n_0| \ll \frac{1}{2}(p_0 + n_0), \quad (6a)$$

$$|p - p_0| = |n - n_0| \ll (p_0 + bn_0)/(b+1); \quad (6b)$$

(7) constant lifetime; (8) the bulk generation rate is function of y only, $g(x, y, z; t) = g(y)$. One then obtains the following differential equation for the excess carrier density $P = p - p_0$,

$$P'' - L^{-2}P = -D^{-1}g(y), \quad (7)$$

where $L = (D\tau)^{1/2}$ is the diffusion length and $D = D_p b \times (p_0 + n_0)/(p_0 + bn_0)$ is the ambipolar diffusivity. We impose the boundary conditions

$$\mathcal{R}_1 + DP'(0) = s_1 P(0) \quad \text{at } y=0, \quad (8a)$$

and

$$\mathcal{R}_2 - DP'(w) = s_2 P(w) \quad \text{at } y=w, \quad (8b)$$

where $\mathcal{R}_1, \mathcal{R}_2$ are the generation rates on the front and the back surface respectively and s_1, s_2 are the corresponding surface recombination velocities which are assumed constant. If I_1 is the flow density of (unreflected) photons entering the front surface and I_2 is the photon current density leaving the sample on the back surface, the following relationship holds:

$$I_1 = \mathcal{R}_1 + \mathcal{R}_2 + \int_0^w g(y) dy + I_2. \quad (9)$$

A solution of Eq. (7) under the boundary conditions (8) is given by

$$P = \alpha e^{y/L} + \beta e^{-y/L} - (L/D) \int_0^y g(\eta) \sinh[(y-\eta)/L] d\eta, \tag{10}$$

where

$$\alpha = \frac{(\mathfrak{D}+s_1)\mathfrak{R}_2 + (\mathfrak{D}-s_2)\mathfrak{R}_1 \exp(-w/L) + (\mathfrak{D}+s_1)\Gamma}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2) \exp(w/L) - (\mathfrak{D}-s_1)(\mathfrak{D}-s_2) \exp(-w/L)}, \tag{11a}$$

$$\beta = \frac{(\mathfrak{D}-s_1)\mathfrak{R}_2 + (\mathfrak{D}+s_2)\mathfrak{R}_1 \exp(w/L) + (\mathfrak{D}-s_1)\Gamma}{(\mathfrak{D}+s_1)(\mathfrak{D}+s_2) \exp(w/L) - (\mathfrak{D}-s_1)(\mathfrak{D}-s_2) \exp(-w/L)}, \tag{11b}$$

$$\Gamma = \frac{1}{D} \int_0^w g(\eta) \left[D \cosh\left(\frac{w-\eta}{L}\right) + s_2 L \sinh\left(\frac{w-\eta}{L}\right) \right] d\eta, \tag{11c}$$

and $\mathfrak{D} = D/L$. This general solution is valid even if the bulk generation is an arbitrary function of y and may thus be used to investigate photoconductive and PME response when the incident light contains components of different absorption coefficients, in which case the generation function $g(y)$ has the form

$$g(y) = \int_{k_1}^{k_2} i(k) k e^{-ky} dk, \tag{12}$$

where $i(k)$ is the incident photon flow density per unit k (k =absorption coefficient) describing the spectral intensity distribution within a band ranging in absorption coefficient from k_1 to k_2 . If the incident light consists of several lines, each with a photon current density I_i and an absorption coefficient k_i , $g(y)$ is given by

$$g(y) = \sum_i I_i k_i e^{-k_i y}. \tag{13}$$

Similar considerations apply when the sample is illuminated from both sides. If the absorption coefficient is small as compared to the thickness of the slab

so that a noticeable portion of the incident radiation reaches the back surface, internal reflections will occur, leading for a particular wavelength to a generation function:

$$g(y) = I_1 \left[\sum_{i \text{ odd}} (\kappa e^{-kw})^{(i-1)} k e^{-ky} + \sum_{i \text{ even}} (\kappa e^{-kw})^{(i-1)} k e^{-k(w-y)} \right], \tag{14}$$

in which κ denotes the percentage of photons reflected back into the sample when they reach a surface. Since the solution [Eq. (10)] also contains surface generation rates in addition to the bulk generation function, it can be applied to the case where surface levels cause generation independent of bulk absorption at a particular wavelength even if it is not strongly absorbed by the bulk.

In the following, however, we restrict ourselves to the simple special case in which $\mathfrak{R}_1 = \mathfrak{R}_2 = 0$, and $g(y) = I k \exp(-ky)$, where k is the absorption coefficient of the incident monochromatic radiation. Under these conditions, one obtains

$$P = \frac{Iw}{D} \frac{K}{W^2 - K^2} \left[\frac{(K - S_2)(W - S_1)e^{-K} - (K + S_1)(W + S_2)e^W}{(W + S_1)(W + S_2)e^W - (W - S_1)(W - S_2)e^{-W}} e^{-Y} + \frac{(K - S_2)(W + S_1)e^{-K} - (K + S_1)(W - S_2)e^{-W}}{(W + S_1)(W + S_2)e^W - (W - S_1)(W - S_2)e^{-W}} e^Y + e^{-(K/W)Y} \right], \tag{15}$$

with $W = w/L$, $Y = y/L$, $K = kw$, $S_1 = s_1 w/D$, and $S_2 = s_2 w/D$.

For the case where $(1/L) = k$ the solution for P , Eq. (15), assumes the indeterminate form 0/0. Solving the differential equation (7) for $1/L = k$ ($W = K$) then yields

$$P = \frac{Iw}{2D} \left\{ \left[\frac{K + S_2 + (K - 1 - S_2)(K - S_1)}{(K + S_1)(K + S_2)e^K - (K - S_1)(K - S_2)e^{-K}} e^{-K} + \frac{Y}{K} \right] e^{-Y} + \frac{K - S_2 + (K - 1 - S_2)(K + S_1)}{(K + S_1)(K + S_2)e^K - (K - S_1)(K - S_2)e^{-K}} e^{(Y-K)} \right\}. \tag{16}$$

The corresponding change in conductance per unit width, ΔG ; the Dember field, $E_D(y)$; the Dember voltage, V_D ; the short-circuit current density, $J_z^{(sc)}$; the short-circuit current, $I_z^{(sc)}$, per unit width, and

the open-circuit PME field, $E_x^{(oo)}$, are given by

$$J_x^{(so)} = qD\theta(dp/dy), \tag{20}$$

$$\Delta G = q(\mu_n + \mu_p) \int_0^w (p - p_0) dy, \tag{17}$$

$$I_x^{(so)} = \int_0^w J_x^{(so)} dy = -qD\theta[p(0) - p(w)], \tag{21}$$

$$E_D = -q(D_n - D_p)\sigma^{-1}(dp/dy), \tag{18}$$

$$E_x^{(oo)} = -I_x^{(so)}/\sigma w, \tag{22}$$

$$V_D = \int_0^w E_D dy = q(D_n - D_p)\sigma^{-1}[p(0) - p(w)], \tag{19}$$

and may all be calculated by substituting Eq. (15) or (16) respectively. We are particularly interested in photoconductivity, ΔG ,¹³

$$\Delta G = \frac{w^2 q(\mu_n + \mu_p) I}{D} \frac{K}{W^2 - K^2} \frac{1}{W} \left[\frac{(K - S_2)(W - S_1)(1 - e^{-W})e^{-K} + (K + S_1)(W + S_2)(1 - e^W)}{(W + S_1)(W + S_2)e^W - (W - S_1)(W - S_2)e^{-W}} - \frac{(K - S_2)(W + S_1)(1 - e^W)e^{-K} + (K + S_1)(W - S_2)(1 - e^{-W})}{(W + S_1)(W + S_2)e^W - (W - S_1)(W - S_2)e^{-W}} + \frac{W}{K}(1 - e^{-K}) \right] \text{ for } k \neq 1/L, \tag{23}$$

or

$$\Delta G = \frac{w^2 q(\mu_n + \mu_p) I}{2D} \left\{ \left[\frac{K + S_2 + (K - 1 - S_2)(K - S_1)}{(K + S_1)(K + S_2)e^K - (K - S_1)(K - S_2)e^{-K}} \frac{e^{-K}}{K} + \frac{1}{K^2} \right] (1 - e^{-K}) - \frac{1}{K} \frac{e^{-K}}{e^{-K}} - \frac{K - S_2 + (K - 1 - S_2)(K + S_1)}{(K + S_1)(K + S_2)e^K - (K - S_1)(K - S_2)e^{-K}} \frac{e^{-K} - 1}{K} \right\} \text{ for } k = 1/L, \tag{24}$$

and the PME short circuit current, $I_x^{(so)}$,

$$I_x^{(so)} = -q\theta I w \frac{K}{W^2 - K^2} \left[\frac{(K - S_2)(W - S_1)(1 - e^{-W})e^{-K} + (K + S_1)(W + S_2)(1 - e^W)}{(W + S_1)(W + S_2)e^W - (W - S_1)(W - S_2)e^{-W}} + \frac{(K - S_2)(W + S_1)(1 - e^W)e^{-K} + (K + S_1)(W - S_2)(1 - e^{-W})}{(W + S_1)(W + S_2)e^W - (W - S_1)(W - S_2)e^{-W}} + (1 - e^{-K}) \right] \text{ for } k \neq 1/L, \tag{25}$$

or

$$I_x^{(so)} = -\frac{q\theta I w}{2} \left\{ \frac{K + S_2 + (K - 1 - S_2)(K - S_1)}{(K + S_1)(K + S_2)e^K - (K - S_1)(K - S_2)e^{-K}} e^{-K} - \left[\frac{K + S_2 + (K - 1 - S_2)(K - S_1)}{(K + S_1)(K + S_2)e^K - (K - S_1)(K - S_2)e^{-K}} e^{-K} + 1 \right] e^{-K} + \frac{K - S_2 + (K - 1 - S_2)(K + S_1)}{(K + S_1)(K + S_2)e^K - (K - S_1)(K - S_2)e^{-K}} (e^{-K} - 1) \right\} \text{ for } k = 1/L. \tag{26}$$

The ratio of the PME short-circuit current over the change in conductance, is thus independent of incident light intensity.

These equations reduce to simpler expressions in the following special cases:

(1) Slab thick compared to L : $W \gg 1$; $e^W \gg 1$; $e^{-W} \ll 1$.

(2) Slab thin compared to L : $W \ll 1$; $e^W \simeq 1 + W$; $e^{-W} \simeq 1 - W$.

(3) Strong absorption in the thickness of slab: $1/k \ll w$; $K \gg 1$; $e^{-K} \ll 1$. This case provides a result that agrees with the literature.

(4) Weak absorption in the thickness of slab: $(1/k) \gg w$; $K \ll 1$; $e^{-K} \simeq 1 - K$. Because of internal re-

flections in this case, however, the more general formula, Eq. (14), should be used.

(5) Absorption large in a diffusion length: $1/k \ll L$; $K \gg W$.

(6) Absorption small in a diffusion length: $1/k \gg L$; $K \ll W$.

(7) Front and/or back surface recombination dominant over bulk recombination: $L \gg D/s_1$ and/or D/s_2 ; $W \ll S_1$ and/or S_2 .

(8) Bulk recombination dominant over front and/or back surface recombination: $L \ll D/s_1$ and/or D/s_2 ; $W \gg S_1$ and/or S_2 .

¹³ For $s_1 = s_2$ this expression is equivalent to the one derived by H. B. DeVore, Phys. Rev. **102**, 86 (1956).

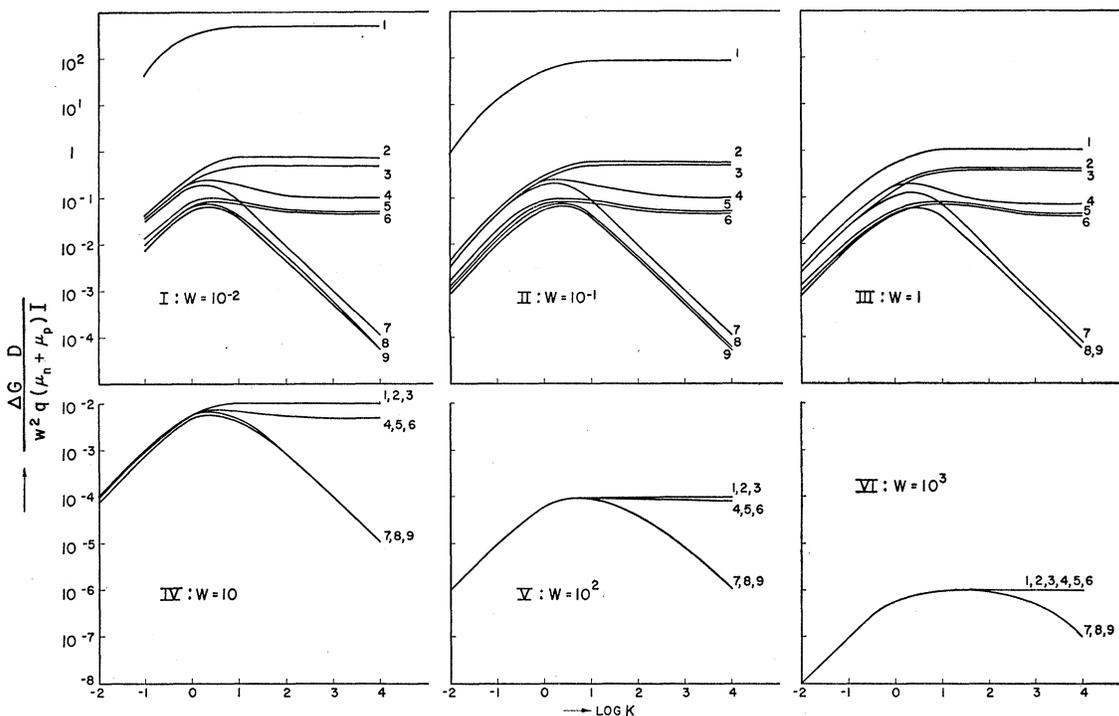


FIG. 1. Photoconductance as a function of absorption coefficient for various values of lifetime and surface recombination velocity.

Curve No.	1	2	3	4	5	6	7	8	9
$\text{Log}_{10} S_1$	-3	-3	-3	1	1	1	5	5	5
$\text{Log}_{10} S_2$	-3	1	5	-3	1	5	-3	1	5

(9) Front surface recombination dominant over back surface recombination or vice versa: $S_1 \gg S_2$ or $S_2 \gg S_1$.

(10) In addition there are the cases: $K \gg S_1$; $K \ll S_1$; $K \gg S_2$; $K \ll S_2$.

Substantial simplifications, however, usually only result from a combination of two or more of conditions 1 through 10. Therefore, the specialized equations have not been given for any individual case.

III. DISCUSSION

To illustrate the results of the theory the photoconductance, the PME short-circuit current, and the ratio of the two functions has been plotted in Figs. 1 through 3 as a function of absorption coefficient for various combinations of lifetime and surface recombination velocities. Although most of the curves are self-explanatory, some of the important features shall be pointed out in the following by calculating several special cases. The portions of the curves with $K < 1$ are not discussed because in this range the more accurate formula, Eq. (14), should be used (requiring the knowledge of κ).

A. Photoconductance (Fig. 1)

We consider the case of strong absorption, $K \gg 1$, and small front surface recombination velocity, $K \gg S_1$,

$W \gg S_1$. The photoconductance is then given by the expression

$$\frac{\Delta G D}{w^2 q (\mu_n + \mu_p) I} \approx \frac{1}{W^2} + \frac{1}{W^2 - K^2} \frac{K^2}{W^2} \times \frac{2S_2}{(W + S_2)e^W - (W - S_2)e^{-W}}. \quad (27)$$

The second term which is negative contributes significantly only if $K \gg W$ and $W < 1$, in which case we obtain

$$\frac{\Delta G D}{w^2 q (\mu_n + \mu_p) I} \approx \frac{1}{W^2} - \frac{1}{W^2} \frac{S_2}{W^2 + S_2}. \quad (28)$$

The photoconductive response is thus independent of absorption coefficient and reaches its maximum value (corresponding to a given bulk lifetime) for vanishing back surface recombination. This case is illustrated by curves I, 1, 2, 3; II, 1, 2, 3; and III, 1, 2, 3. For $K \ll W$ or $W > 1$, the influence of the back surface disappears and one obtains

$$\Delta G D / [w^2 q (\mu_n + \mu_p) I] \approx 1/W^2, \quad (29)$$

exemplified by curves IV, 1, 2, 3; V, 1, 2, 3; and VI, 1, 2, 3. The photoconductive response is diminished by decreasing lifetime,

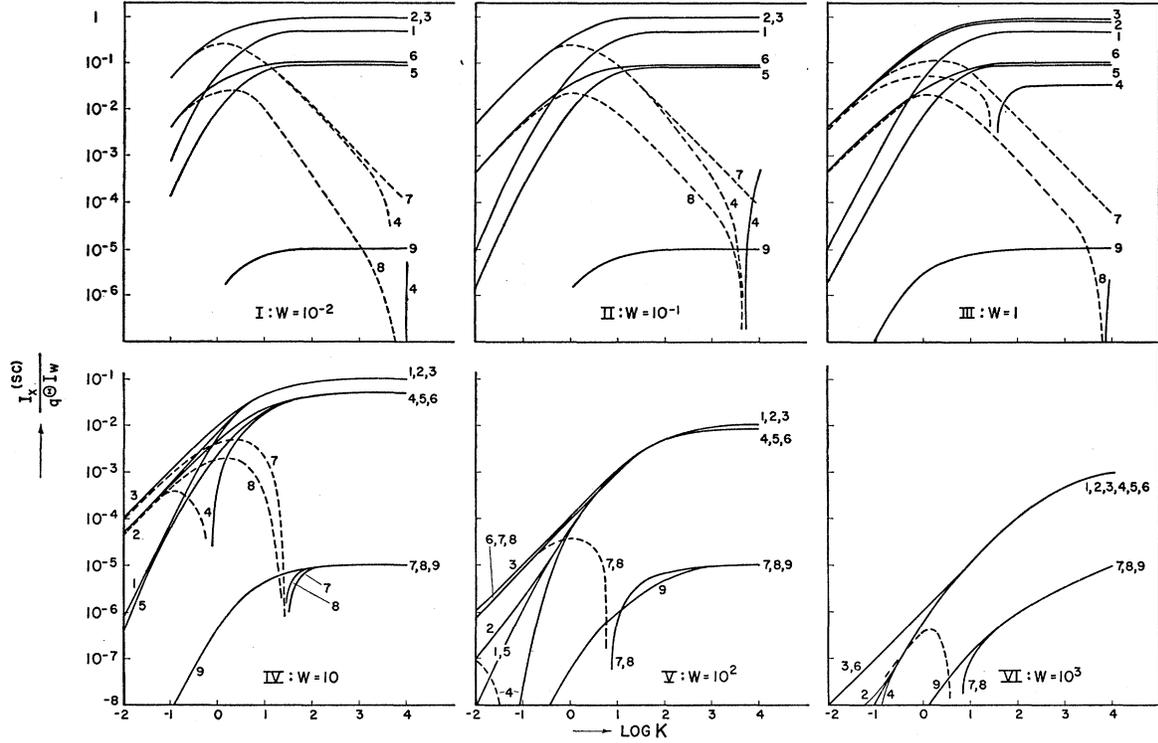


FIG. 2. PME short-circuit current as a function of absorption coefficient for various values of lifetime and surface recombination velocity (—>0, ---<0).

Curve No.	1	2	3	4	5	6	7	8	9
$\text{Log}_{10} S_1$	-3	-3	-3	1	1	1	5	5	5
$\text{Log}_{10} S_2$	-3	1	5	-3	1	5	-3	1	5

When both surface recombination velocities are negligible, $W, K \gg S_1, S_2$, and the absorption is strong, $K \gg 1$ (curves II, 1; III, 1; IV, 1; V, 1; VI, 1), Eq. (29) holds quite generally.

When the front surface recombination is dominant, however, $S_1 \gg K, W$, one finds under most conditions that the photoconductance for large absorption varies close to $1/K$ as in all the curves 7, 8, 9 in Fig. 1.

The curves 4, 5, 6 in Fig. 1 constitute intermediate cases all of which exhibit the well-known maximum of photoconductance at an absorption coefficient, k , between $1/w$ and $10/w$, (w =thickness of slab).

The maximum photoconductive response is evidently reached by strong absorption in thin slabs with high bulk lifetime and negligible surface recombination.

B. PME Response (Fig. 2)

The photomagnetolectric response is shown in Fig. 2. For the case of strong absorption, $K \gg 1$, negligible front surface recombination, $K, W \gg S_1$ and long bulk lifetime, $W \ll 1$, one finds

$$\frac{I_x^{(sc)}}{q\theta I w} \approx \frac{S_2}{W^2 + S_2} \frac{1}{K} \quad (30)$$

This situation holds for curves I, 1; II, 1. If in addition the back surface recombination is dominant over the bulk recombination, $W^2 \ll S_2$, one finds the maximum value of the PME response to be

$$-I_x^{(sc)}/(q\theta I w) \approx 1. \quad (31)$$

This case prevails in curves I, 2, 3; II, 2, 3.

On the other hand, if both surface recombination velocities are dominant over the bulk lifetime, $S_1 \gg W, S_2 \gg W$, one finds

$$-I_x^{(sc)}/(q\theta I w) \approx 1/(1+S_1) \quad \text{for } S_2 \gg S_1 \quad (32)$$

(curves I, 2, 3, 6; II, 2, 3, 6), or

$$-I_x^{(sc)}/(q\theta I w) \approx 1/(2+S) \quad \text{for } S_1 = S_2 = S \quad (33)$$

(curves I, 5, 9; II, 5, 9), or

$$\frac{I_x^{(sc)}}{q\theta I w} \approx \frac{S_2}{S_1(1+S_2)} \frac{1}{K(1+S_2)} \quad \text{for } S_1 > S_2 \quad (34)$$

(curves I, 4, 7, 8; II, 4, 7, 8). In the last case, Eq. (34), the function is negative for small values of K , its magnitude decreases like $1/K$ and reverses its sign at

$$K = S_1/S_2 \quad (35)$$

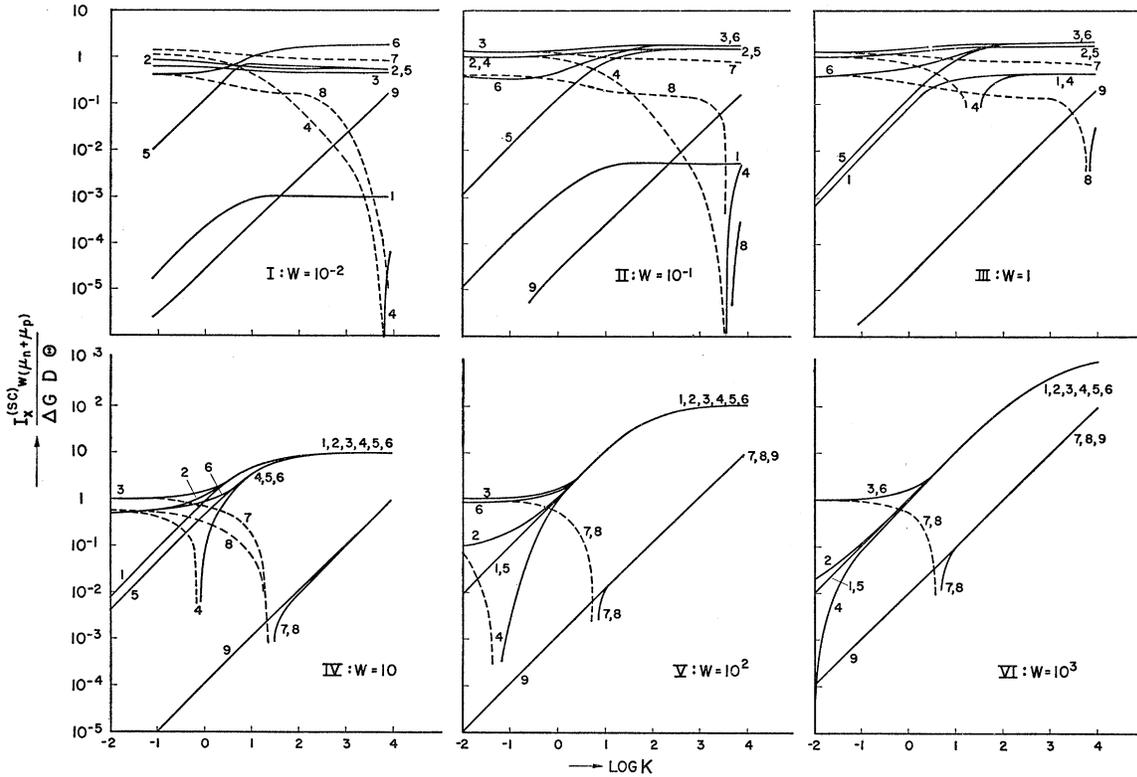


FIG. 3. Ratio of PME short-circuit current over photoconductance as a function of absorption coefficient for various values of lifetime and surface recombination velocity (— >0 , --- <0).

Curve No.	1	2	3	4	5	6	7	8	9
$\text{Log}_{10} S_1$	-3	-3	-3	1	1	1	5	5	5
$\text{Log}_{10} S_2$	-3	1	5	-3	1	5	-3	1	5

(curves I, 4, 8; II, 4, 8). The limiting value at large K is recombination one finds

$$\lim_{K \rightarrow \infty} \left(-\frac{I_x^{(sc)}}{q\theta I w} \right) = \frac{S_2}{S_1(1+S_2)} \quad (36)$$

In general, the sign reversal for long bulk lifetimes, $W \ll 1$, occurs where K is a solution of the transcendental equation

$$S_1 K e^{-K} + (S_1 + S_2 + W^2) e^{-K} + K S_2 = S_1 + S_2 + W^2, \quad (37)$$

reducing to

$$K = (S_1 + S_2 + W^2) / S_2 \quad (38)$$

for large K .

In the case of short bulk lifetimes, $W \gg 1$, and strong absorption, $K \gg W$, one obtains

$$-I_x^{(sc)} / (q\theta I w) \simeq 1 / (W + S_1), \quad (39)$$

thus independent of the back surface recombination. In addition, for dominant bulk recombination, $W \gg S_1$, one has

$$-I_x^{(sc)} / (q\theta I w) \simeq 1 / W \quad (40)$$

(curves V, 1-6; VI, 1-6), and for dominant front surface

$$-I_x^{(sc)} / (q\theta I w) \simeq 1 / S_1 \quad (41)$$

(curves V, 7-9; VI, 7-9).

The sign reversal in this case ($W \gg 1$) occurs where K is a solution of

$$(K + W) e^{-K} / (W + S_2) \simeq -(K - W) / (K + S_1), \quad (42)$$

which for $K \ll W$ reduces to

$$K \simeq \ln[(W + S_1) / (W + S_2)]. \quad (43)$$

For K to be positive, S_1 has to be larger than S_2 . In the case where the front surface recombination is dominant, $S_1 \gg W$, and the back surface recombination negligible, $S_2 \ll W$, we obtain

$$K \simeq \ln(S_1 / W) \quad (44)$$

(curves V, 7, 8; VI, 7, 8).

3. Ratio of PME Response Over Photoconductance

The ratio of PME response over photoconductance is often measured because of its independence from the incident light intensity. Its behavior as a function of

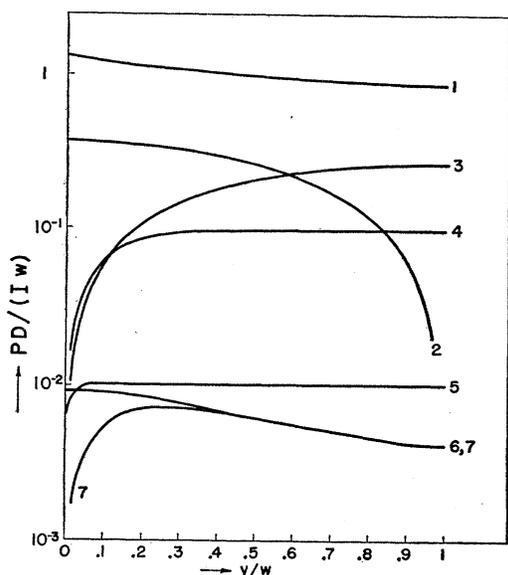


FIG. 4. Reduced carrier density, $PD/(Iw)$, vs reduced distance, y/w , through the slab.

Curve No.	$\text{Log}_{10} W$	$\text{Log}_{10} K$	$\text{Log}_{10} S_1$	$\text{Log}_{10} S_2$
1	0	2	-3	-3
2	-1	0	-3	5
3	-1	0	5	-3
4	-1	1	5	-3
5	-1	2	5	-3
6	1	0	-3	-3
7	1	0	5	-3

absorption coefficient is plotted in Fig. 3. The ratio changes the sign at the same K as the PME response, of course, and its general characteristics become apparent by a comparison with the corresponding curves of Figs. 1 and 2. The plot shows under what conditions the effect is strongly dependent on absorption coefficient in which case extreme caution is advised when such measurements are used to determine recombination parameters (unless monochromatic light of a well-defined absorption coefficient is used). The graphs also show under what circumstances the effect is particularly sensitive to bulk lifetime or one of the surface recombination velocities leading to guidelines for the proper experimental conditions, depending on which quantity is to be measured. In certain cases it appears feasible to measure two or all three of the recombination parameters by fitting an experimental curve to the theoretical ones. Measuring the sample with the light first incident on one surface and then on the other may also prove helpful in many cases.

Figure 4 which is practically self-explanatory has been included to show the carrier density profiles across the slab for several conditions.

It is a pleasure to acknowledge the help of Mrs. H. S. Perlman in carrying out all the numerical calculations and to thank F. A. Brand and H. Mette, all of these Laboratories, for interesting discussions on the experimental aspects of the problem.

Effect of Isotopes on Low-Temperature Thermal Conductivity*

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An analysis of the experimental results of the low-temperature thermal conductivity of single crystals of Si, Ge, and KCl indicates that their maximum conductivities are limited by isotope scattering of phonons rather than by umklapp processes. The isotope scattering in solid helium and sapphire is sufficiently low so that nearly pure umklapp scattering can be observed. The measured thermal conductivities of SiO_2 , KBr, and diamond appear to still be limited by impurities. However, even in pure crystals of these three materials the isotope scattering precludes the appearance of pure umklapp scattering.

IN simple insulating crystals the thermal energy is transported by the lattice vibrations.¹ The vibrations of an infinite, perfectly periodic lattice can be described in terms of phonons.^{2,3} Any disturbance in the perfect periodicity of the lattice which produces scattering of the phonons produces a thermal resistance.

* Part of this work was done while the author was at Cornell University.

¹ P. Debye, *Vorträge Über die Kinetische Theorie der Materie und der Elektrizität* (B. G. Teubner, Berlin, 1914), pp. 17-60.

² R. E. Peierls, *Ann. Physik* **3**, 1055 (1929).

³ J. Frenkel, *Wave Mechanics—Elementary Theory* (Oxford University Press, London, 1932), p. 266.

A monatomic crystal of an element which possesses more than a single, naturally occurring isotope will not have a perfectly periodic lattice. Atoms of the several isotopes will be randomly distributed on the lattice sites. This random variation in the atomic mass will produce a thermal resistivity. This isotope effect was pointed out by Pomeranchuk⁴ in 1942, and a calculation of the magnitude of the thermal resistivity caused by isolated lattice points of differing mass was given by Klemens⁵ in 1955. The isotope resistivity will be most

⁴ I. Pomeranchuk, *J. Phys. (U.S.S.R.)* **6**, 237 (1942).

⁵ P. G. Klemens, *Proc. Phys. Soc. (London)* **A68**, 1113 (1955).