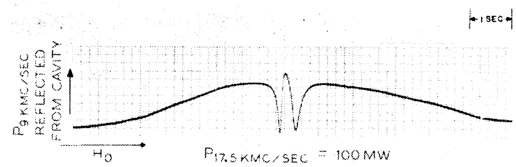

FIG. 3. The central line of the last trace of Fig. 2 is shown on an expanded time scale.

The negative temperature (a term introduced by Purcell and Pound[6] to designate the fact that a higher energy level is more densely populated than the lower one) at complete saturation of the $\Delta S_z=\pm2$ transition will depend essentially upon two parameters: the separations of the energy levels and the ratio of the relaxation times. In a given material the first parameter is fixed. Our attempts were directed toward varying the second parameter in order to obtain lower negative temperatures. A relaxation time ratio of 1:10 between two neighboring transitions was obtained by introducing cerium into the crystal.[5] In order to obtain the full benefit of this large relaxation time ratio for a 9-kMc/sec maser, a dc magnetic field of 2850 oersteds was applied at an angle of 17° from the perpendicular direction of the crystal.[5] Although Eq. (1) refers to the perpendicular direction, the energy levels and transition probabilities are only slightly modified at this small angle. A 90-mg (8% filling factor) lanthanum ethyl sulfate crystal containing $\approx$0.5% Gd^{+++} and $\approx$0.2% Ce^{+++} was used in contact with liquid helium at 1.2°K. A saturating magnetic field at 17.52 kMc/sec was used to induce transitions between the $|-5/2\rangle$ and $|-\frac{1}{2}\rangle$ states as shown in Fig. 1. The signal at 9.06 kMc/sec was applied between the $|-5/2\rangle$ and $|-3/2\rangle$ states. The maser embodies a microwave cavity simultaneously resonant at these two frequencies. The almost critically coupled 9-kMc/sec cavity had a loaded $Q\approx8000$. The 17.5-kMc/sec cavity perversely supporting a spurious mode provided a $Q\approx1000$; this fortunately proved sufficient.

Figure 2 shows the 9-kMc/sec monitoring signal reflected from the cavity as a function of H_0. In the first trace three $\Gamma S_z=\pm1$ transitions are shown, the peaks representing essentially complete reflection as a result of the high magnetic losses associated with the material. The observed resonance line appears broadened since the absorption is not a small perturbation on the cavity as resonance is approached. The succeeding traces show the reflections associated with the $|-5/2\rangle\rightarrow|-3/2\rangle$ transition as the 17.5-kMc/sec power is increased. In the third trace the salt is lossless, corresponding to an essentially infinite spin temperature. The fourth trace shows the onset of negative spin temperatures and the partial overcoming of the losses associated with the empty cavity. In the fifth trace the reflected power exceeds the incident power and oscillations have commenced. Before oscillations commence, a region of amplification must exist. Figure 3 shows the last trace on an expanded time scale.

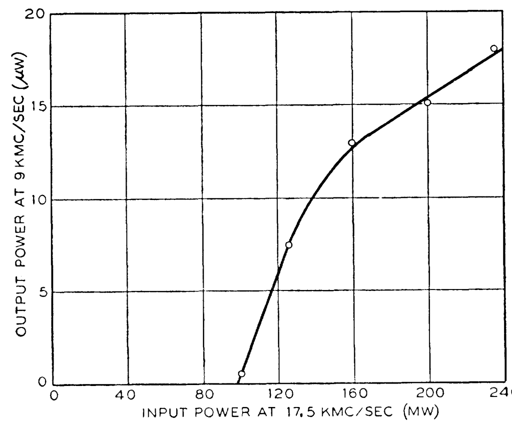

FIG. 4. The 9-kMc/sec output power of the oscillating maser as a function of the saturating power.

At this stage, the 9-kMc/sec monitoring signal was turned off. The dc magnetic field was adjusted to a value resulting in maximum 9-kMc/sec output power from the oscillating maser. The power output was measured with a barretter as a function of the saturating 17.5-kMc/sec power. The results are shown in Fig. 4.

The required saturating power could be materially reduced by the use of a 17.5-kMc/sec cavity having a higher Q. The purpose of this work was merely to show the feasibility of this device.

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Electron Scattering by the Deuteron

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THE object of this note is to point out that the recent Stanford experiments of electron scattering by the deuteron[1] do not necessarily imply any of the far-reaching conclusions drawn in the paper that describes them. The inability to fit the experimental scattering cross sections by those calculated from potentials leading to the correct deuteron binding energy and low-energy neutron-proton scattering phase shift, is the result merely of the special nature of the potentials used.

Let us assume that the tensor force can be neglected for the present purpose. (According to reference 1 this is a justified assumption.) Then we may easily write down explicitly a family of central neutron-proton triplet potentials which exactly produce the deuteron binding energy E_B (and no other bound state) as well as the low-energy behavior of the triplet s-phase shift:

$$k\cot\delta = -\alpha^{-1} + \tfrac{1}{2} r_0 k^2, \tag{1}$$

where the scattering length α is given by

$$\alpha^{-1} = R^{-1} - \tfrac{1}{2} r_0 R^{-2}, \tag{2}$$

and the "size of the deuteron" is

$$R = K^{-1} = \hbar (2\mu E_B)^{-\frac{1}{2}}.$$

The simplest family of potentials is that for which the effective-range approximation (1) [as well as (2)] is exactly correct for all energies. They are[2]

$$V_c(r) = -4K \frac{d}{dr}\left[\sinh\phi r \frac{g_c(K,r)}{g_c(K+\phi,r)-g_c(K-\phi,r)}\right], \tag{3}$$

where

$$g_c(k,r) = k^{-1}(e^{-kr} + c\sinh kr),$$

and

$$\phi = r_0^{-1}[1 + (1-2r_0\alpha^{-1})^{\frac{1}{2}}] = 0.944\times 10^{13}\ \mathrm{cm}^{-1} > K.$$

The parameter c may take on any positive value.

The (normalized) deuteron wave function for the potential (3) is given by $\psi(\mathbf{r}) = r^{-1}u_c(r)$,

$$u_c(r) = 2\left(\frac{cK}{\phi^2 - K^2}\right)^{\frac{1}{2}} \frac{\sinh\phi r}{g_c(K+\phi,r)-g_c(K-\phi,r)}, \tag{4}$$

$\mathbf{r}$ being the coordinate of one particle relative to the other.

If the square of the wave function is interpreted as a static charge density at the distance $r' = r/2$ from the deuteron center of mass, then the form factor for the scattering of electrons by this static charge in the first Born approximation is

$$F_c(q) = \int_0^\infty dr u_c^2(r) \frac{\sin(qr/2)}{qr/2},$$

where (in the absence of recoil)

$$q = 2\hbar^{-1}|\mathbf{p}|\sin(\tfrac{1}{2}\theta),$$

$\mathbf{p}$ being the electron momentum and θ the scattering angle.

Since the denominator in (4) is

$$g_c(K+\phi,r) - g_c(K-\phi,r) = (K\sinh\phi r + \phi\cosh\phi r)e^{-Kr} + c(\phi\cosh\phi r \sinh Kr - K\sinh\phi r\cosh Kr),$$

it is clear that as $c\to\infty$ the entire contribution to the integral in $F_c(q)$ comes from $r\sim 0$. Therefore

$$\lim_{c\to\infty} F_c(q) = \lim_{c\to\infty}\int_0^\infty dr u_c^2(r) = 1$$

because of the normalization of $u_c(r)$.

On the other hand, consider the limit as $c\to 0$. Then all contributions come from $r\sim\infty$, and therefore

$$\lim_{c\to 0} F_c(q) = \lim_{c\to 0} K(\phi^2 - K^2)c\int_a^\infty dr \frac{\sin(qr/2)}{qr/2} \times[(\phi+K)e^{-Kr} + \tfrac{1}{2}c(\phi-K)e^{Kr}]^{-2}$$

$$= \lim_{\eta\to\infty} 2K\int_a^\infty dr \frac{\sin(qr/2)}{qr/2}[e^{Kr-\eta} + e^{\eta-Kr}]^{-2},$$

where

$$\eta = -\tfrac{1}{2}\log\left(\frac{c}{2}\cdot\frac{\phi-K}{\phi+K}\right).$$

If we set $x = Kr - \eta$, this becomes

$$\lim_{c\to 0} F_c(q) = 2\lim_{\eta\to\infty}\int_{aK-\eta}^\infty dx \frac{\sin[q(x+\eta)/2K]}{q(x+\eta)/2K}(e^x + e^{-x})^{-2}$$

$$= \lim_{\eta\to\infty}\frac{\sin[q\eta/2K]}{[q\eta/2K]}$$

$$= \begin{cases} 0 & \text{if } q>0 \\ 1 & \text{if } q=0, \end{cases}$$

because

$$\int_{-\infty}^\infty dx(e^x + e^{-x})^{-2} = \tfrac{1}{2}.$$

Consequently, by changing the parameter c, the form factor can be made to vary between the most extreme possible values. In other words, the deuteron binding energy and the effective range do not, by themselves, allow us to draw any conclusions whatever concerning the spread of the deuteron wave function. As another illustration of this fact, we find by arguments similar to the above that[3]

$$\lim_{c\to\infty}\langle r^2\rangle_c = 0, \quad \lim_{c\to 0}\langle r^2\rangle_c = \infty.$$

This directly shows the independence of the average neutron-proton distance in the deuteron on the one hand, and the deuteron binding energy and neutron-proton s-wave scattering phase shift on the other.

In conclusion it may be noted that, although one may fit the experimental form factor by proper choice of c with one of the above potentials, this does not seem to be a very reliable way of fixing c. There is too much uncertainty owing to the additional effect of the finite proton size.

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