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Pion Production by Inelastic Scattering of Electrons in Hydrogen*

G. B. YODH† AND W. K. H. PANOFSKY
Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California
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Measurements have been continued on the ratio of single-pion production yield in hydrogen by electrons to the yield by the corresponding photons. At an incident electron energy of 600 Mev, pions were detected at energies of 60, 147, and 170 Mev, emitted at 75° to the beam; and of 70 Mev at 135° to the beam. The results are expressed in terms of "the equivalent number of photons" N_e:

θ_{lab}	T_π	N_e
75°	60 Mev	0.0202±0.0007
75°	147 Mev	0.0177±0.0007
75°	170 Mev	0.0145±0.0020
135°	70 Mev	0.0178±0.0007

The results are compared with theoretical calculations and found to be in good agreement.

I. INTRODUCTION

THE study of positive-pion production by electrons inelastically scattered from hydrogen ($e+p\rightarrow e'+n+\pi^+$) has been extended.[1,2] A measurement of the ratio of the pion-production yield by 600-Mev-electrons to the yield by the corresponding photon bremsstrahlung was carried out. Measurements were made of pions of kinetic energies 60, 147, and 170 Mev at a laboratory angle $\theta=75°$, and of pions of kinetic energy 70 Mev at a laboratory angle $\theta=135°$. The results are expressed in terms of "the equivalent radiation length"[2] X_e defined so that the pion yield due to direct electron production is equal to the pion yield due to photons produced by electrons in a radiator of thickness X_e radiation lengths.

The equivalent radiation length X_e is very nearly equal to a quantity N_e as used previously[2] and as used by Dalitz and Yennie[3] and Curtis.[4] The "equivalent number of virtual photons" N_e relates the cross section σ_e for electron production of mesons to the cross section $\sigma(k)$ of production of mesons of a specific energy by the relation

$$\sigma_e=N_e\int_{\Delta k}\sigma(k)\frac{dk}{k}, \tag{1}$$

where Δk defines the energy band of photons useful in producing pions of energies as limited by the energy acceptance band of the detector.

The quantities X_e and N_e are related by the equation[2]

$$N_e=X_e(kN_k)(1-aZ^2), \tag{2}$$

where $N_k dk$ is the number of photons produced in the photon energy band k to $k+dk$ per unit radiation length thickness of a thin radiator; $(1-aZ^2)$ is the Coulomb correction to the Bethe-Heitler formula as measured by Brown[5] and calculated by Bethe and Maximon[6] and Olsen.[7]

The results quoted in the previous paper[2] (hereafter referred to as I) do not include certain corrections. These have now been incorporated, and the old and newer results have been analyzed jointly. As discussed in I, this experiment requires precise knowledge of the location of the liquid hydrogen volume along the beam direction. For this reason primarily, a new vacuum-insulated liquid hydrogen target was constructed which enabled us to locate the leading edge of the hydrogen volume to within 0.020 in. The arrangement is shown in Fig. 1. The design of the target follows in general the design of the Chicago target,[8] as to the use of a separate hydrogen reservoir and level-indicating condensers. The target cell was a cylinder 6 in. long and 3 in. in diameter with $\frac{1}{16}$-in. brass wall; the front and back windows through which the electron beam passed was 0.002-in. beryllium-copper. The outer vacuum envelope had 0.002-in. stainless steel windows in the path of the beam; the outgoing pions pass through $\frac{3}{16}$ in. of copper of the outer envelope in addition to 0.020-in. copper of a radiation shield and the hydrogen cell wall.

II. EXPERIMENTAL ARRANGEMENT

The experimental arrangement is basically the same as described in I. The geometries of the setup at 75° and 135° are shown in Fig. 2.

Improvement was made in the method of determining the pion-detection profile[2] $f(x)$. A thin copper target was exactly located on a slide, positioned with a graduated screw. By means of a transit, the front edge

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† Now at Tata Institute for Fundamental Research, Bombay, India.
1 Panofsky, Newton, and Yodh, Phys. Rev. **98**, 751 (1955).
2 Panofsky, Woodward, and Yodh, Phys. Rev. **102**, 1392 (1956).
3 R. H. Dalitz and D. R. Yennie (to be published).
4 R. Curtis (private communication).
5 K. L. Brown, Phys. Rev. **103**, 243 (1956).
6 H. A. Bethe and L. C. Maximon, Phys. Rev. **87**, 156 (1952), and **93**, 768 (1954).
7 H. Olsen, Phys. Rev. **99**, 1335 (1955).
8 Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953).

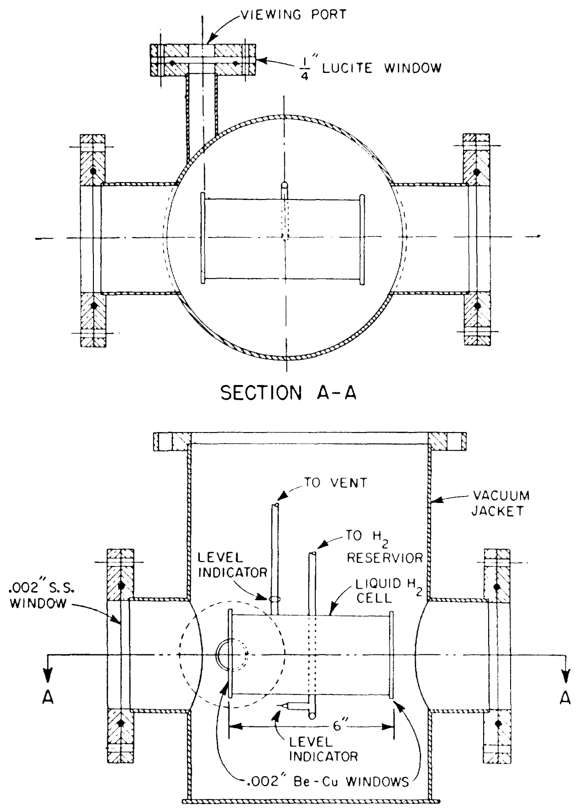

FIG. 1. The target chamber of the liquid hydrogen vessel. Note the viewing port used for locating the front edge of the target accurately, the thin windows, and the capacitor-type hydrogen-level indicators.

of the hydrogen target was lined up with the front edge of the thin target located at the start of the pion detection profile $f(x)$. Thus all the errors in the knowledge of the position of the target were reduced to negligible values.

One secondary electron monitor was placed ahead of the hydrogen target and a second one behind. This eliminated any systematic errors due to beam modification when the radiator was inserted.

A close check was maintained of the beam centering and size during the runs. This was particularly important for the backward (135°) measurement.

III. PROCEDURE

The experimental procedure is the same as in I. The pion yield was measured with the radiator in and then out. The neutron and gamma-ray contamination of the incident beam was measured by deflecting the electron beam so as to miss the hydrogen target altogether. In previous calculations of X_e, no account was taken of the thick-target and degradation corrections; these are now included.

Let us use the same notation as in I: Let A be the yield of pions produced by the electron-induced pion reaction plus that produced by the photons radiated in the material in the electron beam of radiation length X_j other than the additional radiator. Let B be the yield as in A but including the yield due to the photopions produced by the x-rays from an additional radiator X_R radiation lengths thick. Let counts be taken as in Table I. Then

$$\begin{aligned} A&=C^{-+}-C^{--}-(C_d^{-+}-C_d^{--})X_jf_2+X_ef_4,\\ B&=C^{++}-C^{+-}-(C_d^{++}-C_d^{+-})X_Rf_1+X_jf_2+X_ef_3f_4, \end{aligned} \tag{3}$$

where f_1 is the reduction in thickness of radiator due to thick-target corrections, f_2 is the reduction in thickness of the material other than the radiator due to thick-target corrections, f_3 is the reduction in X_e due to degeneration of primary electron energy by radiation straggling in the radiator, and f_4 is the reduction in X_e due to degeneration of primary electron energy from radiation straggling in the extraneous material. This gives

$$X_e=X_R\frac{f_1}{f_4(R-f_3)}-X_j\left(\frac{R-1}{R-f_3}\right)\frac{f_2}{f_4}, \tag{4}$$

where R equals B/A.

IV. CORRECTIONS

The thick-target corrections made according to formulas derived by Wilson[9] give

$$f_1\cong 1-\frac{X_R}{2}\left[\int_{E_0-k}^{E_0}\frac{dk}{k}+\Sigma_\pi(k)+\Sigma_c(k)\right]_{\text{radiator}}; \tag{5}$$

$$f_2\cong 1-\Sigma_j\frac{X_j}{2}\left[\int_{E_0-k}^{E_0}\frac{dk}{k}+\Sigma_\pi(k)+\Sigma_c(k)\right]_{\text{extraneous material}}; \tag{6}$$

k is the energy of a photon producing a given pion calculated from the kinematics of the reaction $\gamma+p\rightarrow\pi^++n$; $[\Sigma_\pi(k)]^{-1}$ and $[\Sigma_c(k)]^{-1}$ are the mean free

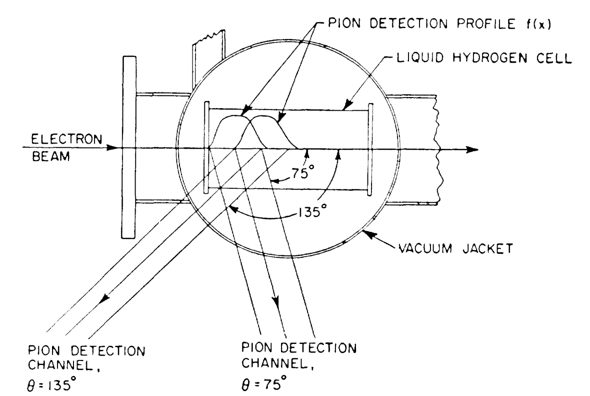

FIG. 2. Target geometries for $\theta=75°$ and $\theta=135°$. Note the positions of the pion detection profiles relative to the front edge of the target and the material traversed by pions during exit from the target.

[9] R. Wilson, Proc. Phys. Soc. (London) A66, 638 (1953).

paths for pair production and Compton scattering, respectively, measured in units of radiation length.

The radiation length, pair-production mean free path, and Compton mean free path were used as tabulated by Bethe and Ashkin.[10]

In order to calculate the correction factors f_3 and f_4, it was necessary to use the theoretically-calculated values of X_e for incident electrons of different energies. Approximate values of $X_e(E,k)$ for a given photon of energy k and for different incident electron energies E were used as calculated from the formulas of Blair and others.[11] The correction factor f_3 is then given by

$$f_3=\int_{E_0}^{E_0-k} X_e(E,k)W(E_0,E)dE \Big/ X_e(E_0,k), \quad (7)$$

where $W(E_0,E)$, the probability that an electron of initial energy E_0 emerges from the radiator with a degraded energy E, is calculated by using the radiation-straggling formula[12]:

$$W(E_0,E)=\Gamma(b_l,y_0)/\Gamma(b_l); \quad (8)$$

where $\Gamma(b_l,y_0)$ is the incomplete gamma function; y_0 equals $\log(E_0/k)$; and b_l is essentially the thickness in radiation lengths of the material in question. A similar definition applies for f_4 where $W(E_0,E)$ is the probability for energy degradation in the extraneous material. The reduction factor f_4 is found to be almost equal to unity.

V. PION ENERGY

The mean pion energy for a given magnet setting was determined by measuring a yield curve of pions as a function of the beam energy. The derivative of this excitation curve was obtained and the first moment calculated to define the mean energy of the incident beam. From this, the pion energy was calculated by the kinematics of the reaction $\gamma+p\rightarrow\pi^++n$. Figure 3 shows the yield curves for 70-Mev pions at 135° and for 147-Mev pions at 75°. The energies of the pions were found to be $T_\pi=(60\pm10)$, (147 ± 15), and (170 ± 15) Mev for $\theta=75°$; and (70 ± 10) Mev for $\theta=135°$.

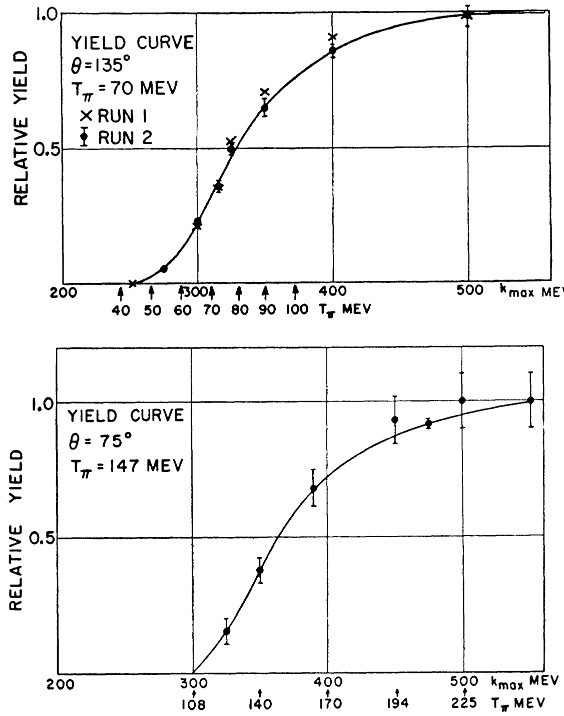

FIG. 3. Pion yield curves as a function of the maximum energy of the photon beam. The mean pion energy and its energy spread were calculated from the derivative of these curves.

TABLE I. Definition of symbols used to designate counts under various conditions.

Type count	Additional radiator	Hydrogen target	Beam
C^{++}	in	full	undeflected
C^{-+}	out	full	undeflected
C^{+-}	in	empty	undeflected
C^{--}	out	empty	undeflected
C_d^{++}	in	full	deflected
C_d^{-+}	out	full	deflected
C_d^{+-}	in	empty	deflected
C_d^{--}	out	empty	deflected

VI. RESULTS AND DISCUSSION

The results, indicating all the corrections, are given in Table II, in which all runs are included. The weighted averages of all the measurements of X_e and N_e for a given pion energy and angle are given in the last column. For the 60-Mev case, a correction has to be made for double-pion production, as described previously,[2] and the value of X_e^{single} is shown It is seen that the final accuracy achieved is about 4% in all except the 170-Mev case. The correction factors discussed above are tabulated also. Dalitz and Yennie[3] have calculated theoretical values of N_e using the exact kinematics pertaining to this experiment under various physical assumptions; N_e values were calculated for various multipole orders of the absorbed photons. In the theoretical curves in Fig. 4, the curve labeled "phenomenological mixture" represents a weighted average of these values corresponding to the analysis of photoproduction of pions made by Watson.[13] The contribution from longitudinal matrix elements is neglected. The curve labeled "Chew Low" is based on the static model

[10] H. A. Bethe and J. Ashkin in *Experimental Nuclear Physics*, edited by E. Segrè (John Wiley and Sons, Inc., New York, 1953), Vol. I, p. 264 ff.

[11] J. S. Blair, Phys. Rev. **75**, 907 (1948) and mimeographed notes (unpublished); Thie, Mullin, and Guth, Phys. Rev. **87**, 962 (1952).

[12] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), second edition, p. 224, Eq. (15).

[13] K. M. Watson (private communication), and Watson, Keck, Tollestrup, and Walker, Phys. Rev. **101**, 1159 (1956).

TABLE II. Summary of experimental results. Here $R=B/A$; X_R is the thickness of additional radiator in radiation lengths; X_j is the thickness in radiation lengths of material other than the additional radiator; f_1 and f_2 are thick-target correction factors for X_R and X_j, respectively; f_3 is the correction factor for energy degradation of the electron beam in X_R. The "equivalent number of photons" N_e is related to X_e by the equation $N_e=X_e(kN_k)(1-aZ^2)$; the notation is discussed in the text.

T_π	θ_{lab}	k	$E=E_0-k$	f_1	f_2	f_3	Run	R	X_R	X_j	X_e	Mean values
60 Mev	75°	230 Mev	370 Mev	0.979	0.992	0.996	I-1	2.100	0.0358	0.01092	0.02096 ± 0.00112	$\overline{X}_e=0.01988\pm0.00069$
							I-2	1.936	0.0358	0.1638	0.02110 ± 0.00167	$\overline{X}_e^{\text{single}}=0.02091\pm0.00069$
							I-3	2.383	0.03593	0.01092	0.01456 ± 0.00332	$\overline{N}_e^{\text{single}}=0.02022\pm0.00067$
							I-4	2.367	0.03593	0.00705	0.01868 ± 0.00133	
							I-5	2.307	0.03593	0.00748	0.01943 ± 0.00199	
147 Mev	75°	360 Mev	240 Mev	0.972	0.988	0.975	II-1	1.875	0.0358	0.01659	0.02275 ± 0.00214	$\overline{X}_e=0.02000\pm0.00081$
							II-2	1.955	0.0358	0.01659	0.01955 ± 0.00087	$\overline{N}_e=0.01766\pm0.00072$
170 Mev	75°	400 Mev	200 Mev	0.969	0.996	0.971	III-1	2.3192	0.0352	0.00890	0.01662 ± 0.00231	$\overline{X}_e=0.01663\pm0.0023$; $\overline{N}_e=0.01445\pm0.0020$
70 Mev	135°	300 Mev	300 Mev	0.975	0.988	0.975	IV-1	1.8106	0.0358	0.02096	0.02170 ± 0.00140	$\overline{X}_e=0.02000\pm0.00082$
							IV-2	1.8611	0.0358	0.02096	0.01929 ± 0.00101	$\overline{N}_e=0.01775\pm0.00073$

of Chew and Low[14] for the pion-nucleon interaction. Both of these calculations are sensitive to the behavior of the interaction "off the energy shell"—i.e., for momentum transfers in excess of the energy transfer. Hence the theoretical values can be depressed somewhat by increasing the size of the region of interaction. The experimental measurements do not permit us to distinguish these possibilities to a significant extent. Large longitudinal contributions are however ruled out. This is a consequence of the fact that the largest fraction of the contribution to electron production originates from small electron-scattering angles where the theoretical predictions become insensitive to detailed physical assumptions pertaining to the meson processes. Therefore we can conclude only that the results are in excellent agreement with the calculations; this is more significant regarding the electrodynamic than the meson-theoretical assumptions of the calculations.

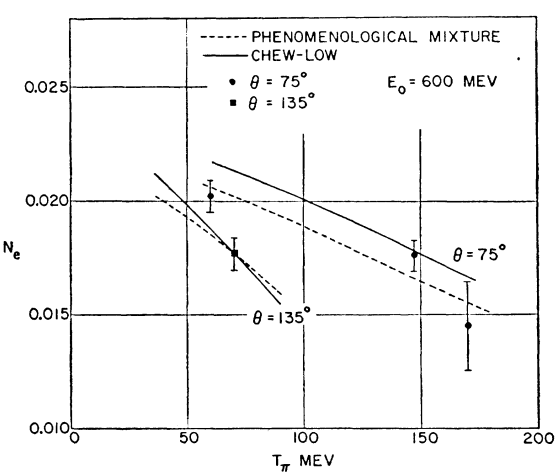

FIG. 4. Plot of theoretical values of N_e as a function of pion energy for $\theta=75°$ and $\theta=135°$. Two sets of curves are given: (1) based on the Chew-Low model for the pion-nucleon interaction; and (2) based on the phenomenological analysis of photoproduction according to Watson and others. The experimental results are also plotted with the experimental errors.

[14] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570, 1579 (1956).

A point of interest relates to the fact that the "Watson phenomenological mixture" for our lowest-energy measurement ($T_\pi=70$ Mev, $\theta=75°$) corresponds primarily to electric-dipole absorption of the incident photon. The theoretical magnetic-dipole value ($N_e=0.0238$) is five standard deviations away from the experimental value. The agreement with the N_e value for π^+ production near threshold (S-wave production) for electric-dipole absorption of the photon is, of course, a direct consequence of the pseudoscalar nature of the charged pion, without any assumption of a more detailed nature.

The complementary experiment—observing the inelastically-scattered electrons, while integrating over all pions produced in the process—is being carried out at this laboratory. This experiment should be more sensitive to the physical assumptions such as the physical extent in space of the matrix elements, since it is possible to focus attention on large electron-scattering angles.

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