Angular Momentum Coupling in the Nuclear *p* Shell*

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Certain experimental results involving the nuclei $A=6$–10 are compared with the predictions of the simple spin-orbit coupling model for *p*-shell nuclei in order to investigate the validity of this model and in particular to consider the significance of nucleon reduced widths as determined by deuteron reactions.

I. INTRODUCTION

IN this paper we consider a variety of experimental results[1] involving nuclei in the range $A=6$–10 and the conclusions which may be drawn from them. One of our major interests will be in comparing experimental results with the predictions of the simple single-particle spin-orbit coupling model for the nuclear *p* shell. As emphasized particularly by Inglis,[2] Lane,[3] and Kurath,[4] this model has been remarkably successful in describing many of the features of *p*-shell nuclei; it may well be argued indeed that the model has had far more success than it deserves and by this time one is apt to learn more from its failures than from its further successes. We shall consider specifically a number of (d,p), (d,t), and (d,He^3) reactions, a few γ-ray and particle widths and particle channel spin ratios, one β-decay case, and certain features of the level spectra. We shall not discuss the techniques of calculation, since by this time these are quite well known.

Our other major interest will be in the technique of investigating nuclear wave functions by the use of relative reduced widths as given particularly by deuteron pickup and stripping reactions.[5] We are particularly concerned about this because this technique is a remarkably flexible one which can be used to give a large variety of information about nuclear structure. Its real flexibility is in fact not apparent in the nuclear *p* shell where for the most part a reduced width is simply another parameter to be compared with a prediction. But accurate deuteron reaction data are available in the *p* shell and these, coupled with other available data, can be used to check on the general accuracy of the technique. Here our two major interests are interconnected and we shall find ourselves comparing observed relative deuteron cross sections with the predictions of the simple nuclear model mentioned above.

II. THEORETICAL AND EXPERIMENTAL RESULTS

(a) Notation and General Comments

We write the effective nuclear interaction as the sum of a central internucleon interaction together with a single-particle spin-orbit interaction,

$$H=\sum_{i<j}\left[a_0+a_\sigma\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j+a_\tau\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j+a_{\sigma\tau}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\right]J(r_{ij})+a\sum_i \mathbf{s}_i\cdot\mathbf{l}_i, \quad (1)$$

and without loss of generality we take $a_{\sigma\tau}=7/30$. For the ratio of the usual Slater integral parameters we take $L/K=6$. We write also

$$z'=a_\tau-a_\sigma,\quad z''=a_\tau+a_\sigma,\quad \zeta=|a/K|.$$

The reduced widths from deuteron reactions will always be deduced by using the simple Butler theory.[5] In units of the usual Wigner limit we write the width as $\theta^2=S\theta_0^2$, where θ_0^2 is the single-particle width and S ($n\sum\beta_z^2$ of reference 5) is the factor which is directly comparable with the predictions of a nuclear model. When two channel spins z are available in either a deuteron reaction or in the incident or outgoing side of a resonant reaction, we have for the usual channel spin ratio $x=(\beta_{z_2}/\beta_{z_1})^2$, where, say, $z_2>z_1$.

(b) $A=7$ Spectrum

The difficulty has often been discussed[2] of reconciling the small ratio of the ^{22}P splitting (0.48 Mev) to the ^{22}F splitting (2.85 Mev if we assume the 4.61-Mev and the 7.46-Mev levels to be the doublet members. We consider specifically Li^7; the situation in Be^7 is essentially identical). We point out that, even in the *LS* limit, the ratio of these splittings depends on the central interaction parameters.[6] For if $z'\neq 0$ the central interaction is not diagonal in the space symmetry $[\alpha]$ (we have $\langle {}^{22}P^{[3]}|H_{\mathrm{central}}|{}^{22}P^{[21]}\rangle=-(40)^{\frac{1}{2}}z'K$) and thus even in the *LS* limit the *P*-doublet wave function has an

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[1] For experimental data see the review article of F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[2] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).

[3] A. M. Lane, Proc. Phys. Soc. (London) **A68**, 197 (1955) and earlier papers referred to therein.

[4] D. Kurath, Phys. Rev. **101**, 216 (1956).

[5] T. Auerbach and J. B. French, Phys. Rev. **98**, 1276 (1955). See also A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953). The treatment of the stripping process itself is due to S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

[6] We owe this comment to Dr. T. Auerbach.

TABLE I. Given in Mev are the energies with respect to the 3/2− ground state of the Li^7 levels arising from the ^{22}P and ^{22}F multiplets. K is given in Mev. Set I has a_0, z', $z''=-0.2$, 0.15, 0.1; set II has a_0, z', $z''=0$, 0.15, 0.1; set III (the usual Rosenfeld mixture[a]) has a_0, z', $z''=0$, 0.1, 0.1; the set used by Inglis[b] and Kurath[c] has a_0, z', $z''=0.12$, 0.12, 0.35.

Set	ζ	K	$E_{1/2}$	$E_{7/2}$	$E_{5/2}$
I	1.7	1.57	0.48	4.2	6.0
II	1.3	1.64	0.48	4.6	6.5
III	1.1	1.20	0.48	4.6	6.1
Expt.			0.48	4.6	7.5 (?)

a L. Rosenfeld, *Nuclear Forces* (North Holland Publishing Company, Amsterdam, 1948), p. 233.
b See reference 2.
c See reference 4.

admixture of $^{22}P^{[21]}$. The corresponding spin-orbit off-diagonal matrix element is opposite in sign for the two J values. Thus by varying z' we can narrow or broaden the P doublet splitting without seriously changing the F splitting. In this way the discrepancy is reduced, but even so a parameter set consistent with the Li^6 spectrum will not actually match the doublet splitting ratio. Table I gives the levels for a few parameter sets (the wave functions for all the sets are closely similar except for the $^{22}P^{[21]}$ components). See Sec. (d) for comments about a different interpretation.

(c) $A=6$ Spectrum

The six levels which develop from the space-symmetric multiplets are known.[7] The essential structure of the spectrum near the LS coupling limit is easily found to depend primarily on the parameter $y=z'/(9a_{\sigma\tau}+3z''-3a_0)$; this parameter must be quite close to 0.05 (say $0.04\leqq y\leqq 0.055$) in order for the first three states to be satisfactorily fitted and for the $(TJ)=(02)$ state to be above these. Combining this with the requirement that the states arising from the P multiplets should be high, we can conclude that we should have $0.1\leqq z'\leqq 0.15$ and on this account it is not worthwhile attempting to improve the Li^7 results by further increasing z'. Of the three sets of parameters in Table I, set II ($y=0.063$) is rather unsatisfactory; the spectra for sets I ($y=0.05$) and III ($y=0.042$) are given in Table II. All in all the Rosenfeld mixture (III) appears to be about the most reasonable for Li^6 and Li^7 and we shall use it from now on.[8]

(d) Particle Widths Connecting $A=6$, 7

Relative cross-section results for $Li^6(d,p)$ to the first two levels of Li^7 are given for $E_d=8$ Mev by Holt and Marsham[9]; corresponding absolute cross sections for $E_d=14$ Mev are given by Levine, Bender, and McGruer[10]; absolute (p,d) cross sections to the first two states of Li^6 are given by Reynolds and Standing[11] ($E_p=17$ Mev) and all the results are discussed thoroughly by the latter authors. They are all in agreement with each other and with the predictions of the simple nuclear model[5] [a minor discrepancy is that a slightly larger value of ζ is needed for $Li^6(d,p)$]. It is quite important that the reduced width ratio determined by Holt and Marsham is identical with that of LBM though the deuteron energies are quite different. The actual value (as deduced by the simple Butler theory) of θ_0^2 is about 0.060.

TABLE II. The calculated Li^6 levels $E(T,J)$ which arise from the space symmetric multiplets are given in Mev for parameter set I (see Table I) with $\zeta=1.25$, $K=0.80$ Mev and for set III with $\zeta=1.35$, $K=1.25$ Mev, and are compared with experiment. All energies are relative to the ground state $(TJ)=(01)$.

Set	$E(03)$	$E(10)$	$E(02)$	$E(12)$	$E(01)$
I	2.19	3.58	3.7	5.5	4.6
III	2.19	3.58	4.7	6.1	6.2
Expt.	2.19	3.57	4.5	5.3	5.4

We must note however the possibility of a discrepancy concerning the 5/2− level at 7.46 Mev. As long as we are close to the LS limit we find for the neutron decay of this level $S\simeq\frac{2}{3}(^{24}P)^2$, where (^{24}P) is the amplitude of this multiplet in the 5/2− state. Then the intermediate coupling calculation above gives $S\simeq 0.02$ for the lowest 5/2− (predominantly ^{22}F) and $S\simeq\frac{2}{3}$ for the next 5/2− which would be predominantly ^{24}P and should lie roughly 2 Mev higher. The (d,p) cross sections to the 7.46-Mev level has not been measured, but the elastic scattering experiment[12] gives a reduced neutron width about $\frac{1}{4}$ of the Wigner limit in striking disagreement with prediction. Besides this a discrepancy in the widths for the $\alpha+t$ breakup of the 4.61-[10] and 7.46-Mev[12] levels also suggests that these levels should not be regarded as members of the same doublet. For the 4.61-Mev level the width (not reduced) is given as 300±100 kev; for the 7.46-Mev level the corresponding width is 114 kev (smaller despite the much larger kinetic energy). These facts are both consistent with the view of Meshkov and Ufford[8] that the 7.46-Mev level is in fact the second 5/2− state which would be largely ^{24}P. On the other hand, Lane[8] has explained the neutron reduced width by using a much larger spin-orbit parameter than found above, thereby departing considerably from the LS limit.

7 See in particular K. Allen, *Proceedings of 1954 Glasgow Conference on Nuclear and Meson Physics* (Pergamon Press, London, 1955).

8 For other calculations concerning these nuclei see G. E. Tauber and T. Y. Wu, Phys. Rev. **93**, 295 (1954); A. M. Lane, Proc. Phys. Soc. (London) **A68**, 189 (1955); A. Feingold, Phys. Rev. **101**, 258 (1956); S. Meshkov and C. W. Ufford, Phys. Rev. **101**, 734 (1956); M. Morita and T. Tamura, Progr. Theoret. Phys. Japan **12**, 653 (1954).

9 J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 1032 (1953).

10 Levine, Bender, and McGruer, Phys. Rev. **97**, 1249 (1955). We shall refer to this paper as LBM.

11 J. B. Reynolds and K. G. Standing, Phys. Rev. **101**, 158 (1956).

12 Johnson, Willard, and Bair, Phys. Rev. **96**, 985 (1954).

(e) $Li^7(d,t)$ and $Li^7(d,He^3)$ Reactions

Absolute cross sections are given by LBM[10] for these reactions leading to the first two states of Li^6 and of He^6. Thus they are of particular interest because four separate states of the $A=6$ polyad are involved. However the ambiguities and difficulties encountered in the usual Butler theory for (d,p) reactions may well be expected to be more severe when dealing with these cases. For example, it is not at all established that the use of a simple transform for finding a deuteron in a triton will be satisfactory. On the other hand, these reactions would be extremely useful if it should turn out eventually that they may safely be used in the analysis of nuclear wave functions.

For the present we use the simple theory and deduce from experiment a reduced width θ^2. We use Irving's[13] three-particle wave function and the usual Hulthén deuteron wave function. The major formal change[14] is that the usual transform function is now replaced by

$$P(K)=\int d\tau \exp\{i\mathbf{K}\cdot[\mathbf{r}_3-\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)]\} \times\phi_t(1,2,3)\phi_d(1,2)=P(0)I(K), \quad (2)$$

where the integration is over the internal three-particle coordinates and, in an obvious notation, $\mathbf{K}=\frac{2}{3}\mathbf{k}_0-\mathbf{k}_1$. For Irving's wave function we find $P^2(0)=113$ while for $K\leqq 0.7$, $I(K)\simeq[1+2.4K^2]^{-1}$ (the unit of length in each case is 10^{-13} cm).

The results which follow from the LBM data are shown in Table III. The reduced widths for the first two states are larger by a factor 2 than the (p,d) values. This could be remedied by abandoning Irving's transform and substituting another which is larger in the region of interest ($K\leqq 0.7$) but has about the same variation with K in this region (in order that the excellent agreement with the angular distributions be not destroyed). Since $\int P^2(\mathbf{K})d\mathbf{K}$ is bounded, the transform must then fall off much more quickly at large K than Irving's transform. We have checked that this can be consistently arranged.

Much more troublesome is the rapid monotonic decrease in $\theta_0{}^2$ as we go to higher levels. It is difficult to believe that this variation is a true one. It cannot be conveniently ascribed to a rapid decrease in the transform function with increasing K for then the angular distributions would not be fitted. Possibly the S values for the two $T=1$ states are seriously in error but in view of the many successes of the intermediate coupling theory for $A=6$ this seems improbable. It seems more likely that the $\theta_0{}^2$ variation represents a basic defect in the way the simple theory treats the dynamics of the reaction[15] (note that the triton or He^3 kinetic energies vary from 15 Mev for the ground state to 7 Mev for the highest state considered).

Our final conclusion is that, until more information is available concerning (d,t) and (d,He^3) reactions we cannot safely use them for measuring relative reduced widths. In this connection more cases where the same pair of levels are connected, say, by (p,d) and also by (d,t) reactions would be particularly valuable.

TABLE III. The reduced widths θ^2 as deduced from the $Li^7(d,t)$ and $Li^7(d,He^3)$ data of LBM[a] are given and divided by the calculated relative reduced widths S to give $\theta_0{}^2$. The S values for the extreme cases ($\zeta(Li^6)=\zeta(Li^7)=0, \infty$) are also listed. K is given in units of 10^{13} cm^{-1}.

(T,J)	K	θ^2	$S(LS)$	$S(jj)$	$S(\text{int})$	$\theta_0{}^2$
01	0.30	0.11	5/6	9/20	0.82	0.13
03	0.39	0.061	7/15	21/20	0.59	0.10
10	0.44	0.055	5/6	5/4	0.84	0.065
12	0.50	0.017	2/3	1/4	0.63	0.027

[a] See reference 11.

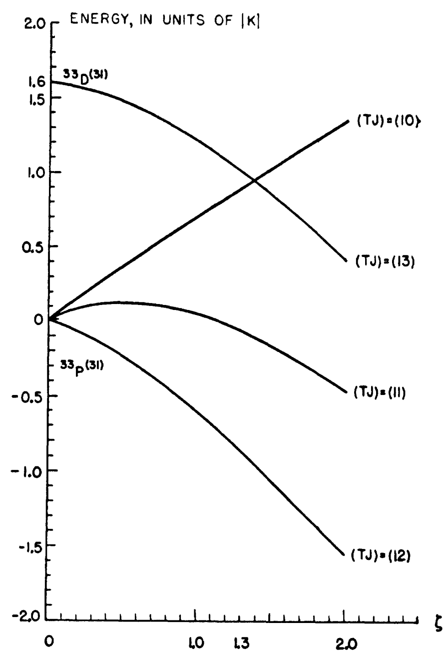

FIG. 1. The level structure for the first four levels in Li^8. The LS separation between the two multiplets for an arbitrary exchange mixture is $2(3a_{\sigma\tau}+z''-a_0)=2z'/3y$.

(f) Li^8 Spectrum and the $Li^7(d,p)Li^8$ Reaction

Figure 1 shows the level structure calculated near LS coupling and with the Rosenfeld interaction for the first four levels of Li^8. Three odd-parity levels only have been observed and these can be safely identified as $J=2$, 1, 3 as indicated. We find the proper relative spacing for the three levels at $\zeta\simeq 1.3$ with $K=1.25$ Mev but this could be changed somewhat by altering the parameters of the central interaction (Kurath's inter-

[13] J. Irving, Phil. Mag. 42, 338 (1951).

[14] See, for example, H. C. Newns, Proc. Phys. Soc. (London) A65, 916 (1952).

[15] In this connection see A. Werner, Nuclear Phys. 1, 9 (1956).

action[4] favors a value $\zeta \simeq 2.0$). In this connection it would be very useful to identify the $J=0$ state arising from the ^{33}P multiplet.

LBM[10] give absolute cross sections for $Li^7(d,p)$ leading to the first two levels and these yield $\theta^2=0.054$, 0.028, respectively. Theoretical S values are given in Table IV; for $\zeta=0$, 1, 2, we have $S^*/S=1.6$, 2, 2.3, respectively compared with the experimental value 1.9. This too favors the small spin-orbit parameter though for any value $\zeta \leqq 2$ the agreement is adequate. For the single-particle width we have $0.050 \leqq \theta_0{}^2 \leqq 0.060$.

(g) 440-kev Li^7+p Reactions

The 17.63-Mev resonant state in Be^8 which is reached in this reaction is the analog of the 0.97-Mev level of Li^8 and thus with the same wave functions used above we can examine some features of the Li^7+p reaction.

The ground-state γ-ray angular distribution is experimentally isotropic to within $\sim 6\%$; the theoretical angular distribution is $1-\{(5-x)/(5+7x)\}\cos^2\theta$ where x= channel spin ratio. For $\zeta=0$, 1, 2 the calculated values of x are found to be 5, 100, 14, respectively, and this strongly favors a coupling scheme either very close to (LS) or else quite close to $\zeta=2$. For $\zeta=2$ we would have $d\sigma/d\omega=1+0.09\cos^2\theta$. The channel spin value also affects the elastic scattering angular distribution but from the published discussions, it is not clear how accurately x is determined by the distribution. A value $x=5$ is satisfactory.

The experimental γ-ray width[16] for the ground-state transition has the relatively small value (for a 17.6-Mev γ ray) of about 17 ev. The transition here may reasonably be termed unfavored since, near LS coupling, there is no transition between the dominant multiplets of each wave function ($^{33}P^{[31]}$ and $^{11}S^{[4]}$), the major contributions arising from $^{31}P^{[31]} \to {}^{11}S^{[4]}$ and $^{33}P^{[31]} \to {}^{13}P^{[31]}$. It follows easily then that near LS coupling the calculated $M1$ width will vary as ζ^2. The radiation width for the 17.6-Mev γ ray is calculated to be 0, 3.6, 12.1 ev for $\zeta=0$, 1, 2, respectively, thus suggesting a slightly larger value than $\zeta=2$.[17]

The proton width in Li^7+p is about 12 kev which corresponds to $\theta^2 \simeq 0.13$, in strong disagreement with the value 0.028 for the corresponding bound state reduced neutron width found by $Li^7(d,p)$. The two cases however are not dynamically identical and the significance of this disagreement is not clear.

(h) $Be^9(d,p)Be^{10}$ Reaction

There is fair agreement between prediction and experiment for the first two levels of Be^{10}. Predicted S values (using $\zeta=1.5$ for Be^9, $\zeta=5$ for Be^{10}) are, for the ground and first excited states, 2.2 and 0.2, respectively. For $E_d=4$ Mev[18] the reduced widths θ^2 are 0.050, 0.010 and for $E_d=14$ Mev,[19] 0.093, 0.012; for $E_d=8$ Mev relative reduced widths are given by Green and Middleton[20] as 22 and 8, respectively. Then from the ground-state cross section we have roughly $\theta_0{}^2 \simeq 0.025$ ($E_d=4$ Mev), $\theta_0{}^2 \simeq 0.04$($E_d=14$ Mev) and we note the apparent increase with increasing deuteron energy.

For the higher levels things are unclear; one of the three levels near 6 Mev could possibly have $l=1$ and correspond to the second 2+ level as predicted by Inglis[2] and Kurath[4] (there is a rather strong prediction that the first four p^n levels should be $J=0$, 2, 2, 3). As discussed also by Green and Middleton,[20] a major difficulty with the 7.37-Mev level whose spin is given as 3 is that its (d,p) reduced width is $\theta^2=0.013$ (at $E_d=14$ Mev) which should correspond to $S \sim 0.3$. The calculated width is very small; for $\zeta(Be^9)=\zeta(Be^{10})=0$, we have $S=0$; for $\zeta(Be^9)=1.5$, $\zeta(Be^{10})=4$, 5 we have $S=0.012$, 0.011 and we have verified that this value cannot be essentially increased while using any reasonable ζ value for Be^9. The neutron width as measured by elastic scattering[21] gives $\theta^2=0.017$ which agrees with the (d,p) value.

(i) $E2$ Lifetime in B^{10}

The mean γ-ray lifetime of the first excited state of B^{10} is given by Thirion and Telegdi[22] as $(7\pm 2)\times 10^{-10}$ sec. The $E2$ lifetime has been calculated for several ζ values. Specifically we find (for an rms radius 3×10^{-13} cm) $\tau=2.9$, (∞), 8.9, (∞), 6.6, (∞) and 23×10^{-10} sec for $\zeta=0$, 1.4, 3.8, and ∞, respectively.[23] In listing the τ values, the symbol (∞) indicates that the $E2$ matrix element changes sign between the two adjacent values and thus must vanish at least once between them

TABLE IV. The relative reduced widths calculated for the three known Li^8 levels for different values of the Li^8 spin-orbit parameter. For $\zeta=1$, 2 the Li^7 spin-orbit parameter is 1.1 but for the LS limit both values are taken zero.

(T,J)	$\zeta=0$	$\zeta=1$	$\zeta=2$
12	10/9	1.09	0.98
11	2/3	0.54	0.42
13	2/5	0.33	0.33

[16] This case has been considered in more detail by D. Kurath, Bull. Am. Phys. Soc. Ser. II, 1, 180 (1956).

[17] The calculated values depend somewhat on the central force exchange mixture. Near the LS limit we have for the m-reduced matrix element of the magnetic moment operator $\langle \| \mu/\mu_0 \| \rangle = \zeta(3)^{-\frac{1}{2}}\{(2g_p-2g_n-1)G_1{}^{-1}+G_2{}^{-1}\}$, where $G_1=9a_0+(4-4s)a_\sigma-10a_\tau+(9-12s)a_{\sigma\tau}$; $G_2=(4s-13)a_\sigma+a_\tau+(12-4s)a_{\sigma\tau}$; $s=L/K$.

[18] Fulbright, Bruner, Bromley, and Goldman, Phys. Rev. 88, 700 (1952).

[19] K. B. Rhodes and J. N. McGruer, Phys. Rev. 92, 1328 (1953); and J. N. McGruer (private communication).

[20] T. S. Green and R. Middleton, Proc. Phys. Soc. (London) A69, 28 (1956).

[21] Bockelman, Miller, Adair, and Barschall, Phys. Rev. 84, 69 (1951). See also Willard, Bair, and Kington, Phys. Rev. 98, 669 (1955).

[22] J. Thirion and V. L. Telegdi, Phys. Rev. 92, 1253 (1953). A later measurement by J. S. Severiens and S. S. Hanna [Phys. Rev. 100, 1254(A) (1955)] gives $9.5\pm 2\times 10^{-10}$ sec.

[23] We are indebted to Dr. D. Kurath, who has also calculated this lifetime, for pointing out a numerical error in our original calculation.

(corresponding to infinite lifetime). With this erratic variation of the lifetime with ζ there is no problem in fitting the experimental result[24] but the significance is not clear.

(j) C^{10} β Decay

This case is interesting because, apart from the forbidden ground state transition, there are two pure Gamow-Teller (G.T.) transitions and one pure Fermi transition. Thus if we are willing to calculate the nuclear matrix elements we may determine from the energies and partial lifetimes the absolute strengths of the separate interactions. The calculated Fermi matrix element, as is well known, is $|\int 1|^2=2$. For the G.T. matrix element for the first excited state we find $|\int \boldsymbol{\sigma}|^2=5.9$, 0.77, 0.69, 0.88, 10/3 for $\zeta=0, 4, 5, 7, \infty$. The surprisingly large difference between the values for $\zeta=7$ and $\zeta=\infty$ is indicative of the fact that the composition of the calculated wave functions changes quite markedly between these limits; for $\zeta\leqq 7$ the predominant symmetry is [42] while for $\zeta=\infty$ the dominant symmetry is [321].

There are experimental uncertainties in the determinations of both the available energy and the partial lifetimes, but at first we choose the values favored by Ajzenberg and Lauritsen.[1] Then, following Gerhart,[25] we write

$$A=ft\left\{\left|\int 1\right|^2+R\left|\int \boldsymbol{\sigma}\right|^2\right\}, \qquad (3)$$

where $R=(g_{GT}/g_F)^2$. The Fermi branch gives $A=5300$ sec indicating, by its satisfactory agreement with the value deduced more accurately by Gerhart from his O^{14} data, that the adopted energy and partial lifetime are not unsatisfactory. With Gerhart's A (6500 sec) we now determine R as a function of the intermediate coupling parameter; we find $R=1.0$, 7.8, 8.7, 6.7, 1.8 for $\zeta=0, 4, 5, 7, \infty$ and these values may be compared with the value $R=1.37_{-0.30}^{+0.40}$ found by Gerhart by combining the O^{14} and the isolated neutron data (the latter value of R is found by Lane[8] to be quite satisfactory for several simpler p-shell nuclei and is in essential agreement with the values deduced indirectly by several authors).

It is quite clear that, except for coupling schemes close to the LS or jj extremes, we do not have a satisfactory result. We have verified that this conclusion is unaltered if we vary the energy and lifetimes within the assigned experimental errors and conclude from it that the intermediate-coupling wave functions are unsatisfactory. We have on this account not calculated the matrix element for the second Gamow-Teller transition.

[24] This is contrary to the statement concerning this decay by D. H. Wilkinson, Phil. Mag. **1**, 127 (1956), footnote on p. 149.

[25] J. B. Gerhart, Phys. Rev. **95**, 288 (1954). See also A. Winther and O. Kofoed-Hansen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, 14 (1953).

III. CONCLUSIONS

We now attempt to draw some conclusions. To begin with we note that, with one exception, we have been unable to unearth any essential troubles with the predictions of the simple spin-orbit nuclear model for $A=6, 7, 8$. For this one discrepancy (the nature of the 7.46-Mev level in Li^7), of particular value would be a measure of the (d,p) width of this level and further studies in general of the Li^7 states. With regard to Li^8 a determination of the position of the 0^+ level would, as pointed out above, be helpful in fixing the parameters of the effective nucleon-nucleon interaction. Besides this, a measure of the reduced widths for further levels of Li^8 would be instructive; for it seems likely that of all the p-shell nuclei which may be studied by (d,p) reactions [as opposed to the experimentally more difficult (p,d) and (n,d) and apparently untrustworthy (d,t) and (d,He^3)] the Li^8 nucleus is that for which most definite predictions can be made.[26] We make no comments about the $A=9$ polyad which continues in comparative experimental obscurity and we refer only to earlier calculations.[27] For $A=10$ there are quite strong indications that the simple nuclear model is unsatisfactory.

From one set of experimental results we have concluded that the simplest treatment of the (d,t) and (d,He^3) reactions is probably unsatisfactory for determining relative reduced widths. But further experiments measuring the absolute cross sections for these reactions are needed, particularly in cases where they can be compared with the simpler (p,d) or (n,d) reactions (or their inverses).

Concerning the use of (d,p) reduced widths we feel, as a result of studying the cases above as well as heavier nuclei,[28] that the (d,p) width, as deduced simply by using Butler's theory, is indeed a quantitatively useful parameter not only for comparing various levels of the same nucleus but for more general applications. In particular, as long as we exclude cases where the kinetic energies are low, we find no experimental evidence for the extremely large fluctuations of the single-particle (d,p) widths which some authors seem to expect. One of the most interesting questions (first discussed by Thomas[29]) then involves the relationship between a (d,p) width and the corresponding resonant reaction width [and thereby the significance of the absolute (d,p) width]. So far as we can find, the situation here is still unclear and further cases should be studied. Finally we emphasize that the most valuable experiments which could be done in order to improve our

[26] For a β-decay calculation see B. G. Jancovici, Nuovo cimento **1**, 840 (1955).

[27] French, Halbert, and Pandya, Phys. Rev. **99**, 1387 (1955). See also Kurath, reference 4.

[28] In particular for the situation at $A\sim 40$ see J. B. French and B. J. Raz, Phys. Rev. **104**, 1411 (1956).

[29] R. G. Thomas, Phys. Rev. **91**, 453(A) (1953).

understanding of (d,p) widths involve the measurement, over a wide energy range, of (d,p) cross sections as a function of energy.

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Inelastic Proton Scattering from Gold at 6 Mev*

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A thin gold target has been bombarded with 6.0-Mev protons. Inelastically scattered protons, corresponding to Coulomb excitation of the 545-kev, 279-kev, and 268-kev levels in Au^{197}, have been observed in a magnetic spectrograph. The angular distribution of the inelastic groups seems to follow the semi-classical theory.

I. INTRODUCTION

THE detection of the inelastically scattered particles in Coulomb excitation has several advantages compared with detection of the subsequently emitted gamma rays or conversion electrons:

1. Direct information about the excitation energy is obtained independent of the possibility of cascade transitions in the decay of the excited levels.
2. The cross section for excitation of a level can be obtained in a very simple way from the ratio of the intensity of the corresponding inelastic group to that of the elastic group, if one assumes simple Rutherford scattering for the elastic group.
3. The determination of the cross sections does not depend on a knowledge of conversion coefficients and branching ratios.

One of the drawbacks of the method is that the purity requirements of the target material and its support are extreme. Every contamination will give rise to an elastic peak with cross section 10^2 to 10^4 times larger than that of the inelastic groups to be observed. Often these contaminant peaks can be identified by energy or angle shifts, but for heavy masses the differences in shifts become small. In any case, their presence obscures regions of the spectrum.

Another disadvantage compared to gamma-ray measurement is imposed by the necessity of using targets thin to the emitted particles, with consequent loss of intensity.

In view of these considerations, it appeared worthwhile to extend into the region of the heavy elements the inelastic scattering studies undertaken with the broad-range spectrograph[1] associated with the MIT-ONR electrostatic generator.

Preliminary runs with 7.45-Mev protons on a thin gold target showed that it was possible to detect the two well-known states in gold at 279 kev and 545 kev.[2,3] The barrier penetration at 7.5-Mev bombarding energy is low, and the inelastic groups must be due at least in part to Coulomb excitation. Interesting information can be deduced from the cross sections[4] if only Coulomb excitation occurs. To insure that this was the case, it was decided to continue the measurements with 6.00-Mev protons, even though the background is a little higher than at 7.45 Mev. At 6 Mev, the cross section for formation of a compound nucleus[5] is about 0.2 mb compared with a Coulomb excitation cross section of about 0.8 mb.

II. FORMULAS

The elastic scattering of 6-Mev protons from gold is expected to follow the Rutherford scattering law:

$$\frac{d\sigma(\theta)_R}{d\omega}=\frac{e^4Z_1{}^2Z_2{}^2}{16E^2}\left(\frac{1}{\sin^4(\frac{1}{2}\theta)}\right)=1.295Z_1{}^2\left(\frac{Z_2}{E}\right)^2\left(\frac{1}{\sin^4(\frac{1}{2}\theta)}\right)\text{ mb/sterad,}\quad (1)$$

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