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Hyperfine Structure of Cu^{63} and Cu^{65}

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The hyperfine splittings of the electronic ground states of the stable copper isotopes Cu^{63} and Cu^{65} have been measured by the atomic beam magnetic resonance method. The magnitudes of these splittings are $\Delta\nu(Cu^{63})=11\,733.83\pm0.01$ Mc/sec, $\Delta\nu(Cu^{65})=12\,568.81\pm0.01$ Mc/sec, and their ratio is $\Delta\nu_{63}/\Delta\nu_{65}=0.933567\pm0.000002$. If this ratio is compared with the ratio of the magnetic moments as measured by nuclear magnetic resonance, *viz.* $\mu_{63}/\mu_{65}=0.933424\pm0.000019$, one finds a "hyperfine structure anomaly" of

$$\Delta=[\Delta\nu_{63}\cdot\mu_{65}/\Delta\nu_{65}\cdot\mu_{63}]-1=(0.015\pm0.002)\%.$$

Measurements of the "low-frequency" lines of the two isotopes yield the following values for the Landé g factors of the ground states: $g_J(Cu^{63})=g_J(Cu^{65})=(1.0000\pm0.0005)g_J(Cs^{133})$.

I. INTRODUCTION

A PRECISE determination of the hyperfine structure splittings ($\Delta\nu$) of the ${}^2S_{\frac{1}{2}}$ ground states of Cu^{63} and Cu^{65} by the atomic beam techique has been undertaken with a view towards comparing the ratio of the $\Delta\nu$'s for the two isotopes with the ratio of the magnetic moments as determined by nuclear induction methods. These two ratios are equal if one assumes that the wave function of the electronic state is that corresponding to a point charge at the nucleus and that the nuclear magnetic moment arises from a point dipole. A deviation from equality would imply that these assumptions were not strictly correct and would, with the aid of current theories on the "hyperfine structure anomaly",[1-4] cast some light on the structure of the copper nuclei.

Prior to the present work, hyperfine structure studies of the CuI spectrum had been done by optical methods.[5] From transitions between the $3d^{10}4s\,{}^2S_{\frac{1}{2}}$ ground state and the $3d^{10}4p\,{}^2P$ excited states, the hyperfine structure splittings of the ground state had been deduced to be

$$\Delta\nu(Cu^{63})=0.388\text{ cm}^{-1}=11\,640\text{ Mc/sec},$$
$$\Delta\nu(Cu^{65})=0.416\text{ cm}^{-1}=12\,480\text{ Mc/sec}.$$

These spectroscopic values greatly facilitated the finding of the transitions in the present work.

II. EQUIPMENT

A. Atomic Beam Apparatus

The apparatus has been described previously in connection with work on praseodymium[6] and silver and gold.[7] The operating conditions are very similar to those for silver and gold but for completeness they will be briefly described here.

The deflecting magnets A and B are arranged to detect transitions by the so-called "flop-in" method. That is to say, the gradients as well as the fields are in the same direction with respect to each other so that only transitions which are accompanied by a change in sign of the effective magnetic moment are refocused.

The oven for the generation of the copper beam is a graphite tube of $\frac{3}{8}$-inch outside diameter and $\frac{1}{4}$-inch inside diameter with a vertical slit $\frac{3}{16}$ inch long and 0.027 inch wide. Each charge of copper consists of a slug about $\frac{1}{2}$ inch long and $\frac{1}{4}$ inch in diameter. During most of the final measurements this oven is heated to between 1700°C and 1750°C by an alternating current of 300 amperes at 10 volts.

For detection purposes, the atomic beam is interrupted at 10 cycles per second by a chopper located immediately in front of the oven.

The collimator slit, located between the homogeneous field (C field) and the second deflecting field (B field) is set at anywhere between 0.015 inch and 0.021 inch.

The currents energizing the two deflecting magnets are kept fixed during the entire experiment at the values necessary for refocusing equal and opposite magnetic moments. With the magnet constants given in reference 6, these values are 30 amperes in the A field and 55 amperes in the B field.

The C field current is controlled by means of a "water" rheostat which consists of two parallel copper plates partly immersed in a saturated solution of copper sulfate. The depth of immersion of the plates is variable.

For the detection of the copper beam, the electron bombardment ionizer described in reference 7 is used. The ions formed herein are accelerated to about 1150 electron volts, passed through a mass spectrometer magnet of 12.4 cm radius of curvature and finally allowed to strike the first plate of an electron multiplier. The multiplier is made of beryllium-copper, has 16 stages and is operated at 3600 volts. The pulses from the multiplier are amplified, passed through a

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[3] A. Bohr and V. F. Weisskopf, Phys. Rev. **77**, 94 (1950).

[4] A. Bohr, Phys. Rev. **81**, 331 (1951).

[5] References may be found in Landolt-Bornstein, *Zahlenwerte and Functionen* (Springer-Verlag, Berlin, 1952), Vol. 1, Part 5, Sec. 161.

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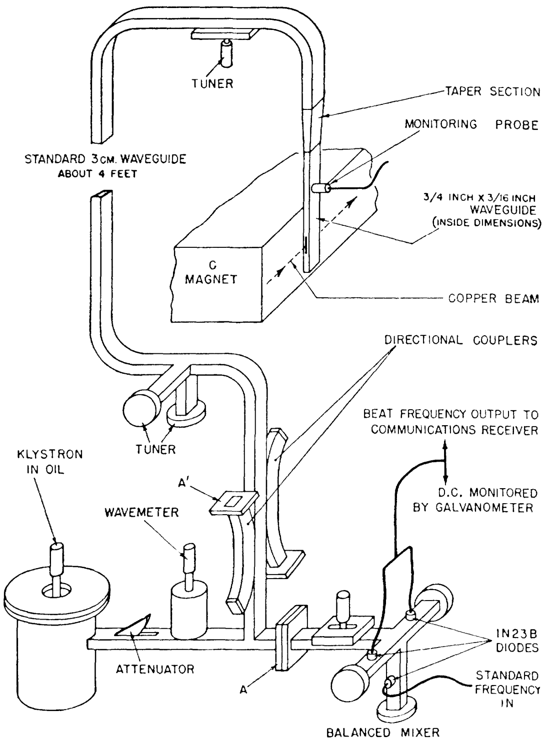

FIG. 1. The microwave system.

scaling circuit and aurally monitored by means of a pair of earphones. At the same time, the pulses are fed into another channel consisting of an amplifier, a demodulator, a 10-cycle narrow-band amplifier, and a phase-sensitive detector. The reference voltage for this phase sensitive detector is derived from a photoelectric cell mounted on top of the beam chopper. The output of the phase sensitive detector is recorded on a 1 milliampere Esterline-Angus recorder.

To keep the total number of background and signal pulses low enough so as not to saturate the narrow band system, the ionizer is operated at very low efficiency. The electron energies are around 45 ev and the total emission current is from 7 to 10 ma.

To test the entire system for the linearity of its response to signals of different strengths, the relative strength of the Cu^{63} and Cu^{65} beams was measured and found to be in the ratio (2.1 ± 0.1) to 1. This agrees satisfactorily with the known relative abundance of 2.23 to 1.

B. Microwave System

As mentioned above, the hyperfine structure splittings of the ground states of the copper isotopes fall in the high end of the 3-cm microwave band, being approximately 11 640 Mc/sec and 12 480 Mc/sec for Cu^{63} and Cu^{65}, respectively. The source of microwave power used in the present experiments for transitions in this region is a Varian Type X-13 reflex klystron. This tube is operated in a bath of mineral oil which is cooled by water flowing in a copper coil immersed in the oil. The power supply is a Browning Model TVN-11 unit slightly modified to supply a maximum beam voltage of 600 volts. The filtering and regulation in this unit is improved to reduce the ripple voltage in both the beam and the reflector supplies to a few millivolts. Aside from oil immersion and the use of a regulated power supply there is no other regulation of the klystron. Coarse frequency adjustments of the klystron are effected by a micrometer screw which varies the size of the cavity. Fine frequency variations such as that required to sweep through a spectral line are effected by adjustment of the reflector voltage either manually or through a low speed motor drive.

The complete microwave system is shown diagrammatically in Fig. 1. Except for the directional couplers and length of wave guide which extends into the vacuum system and induces transitions in the atoms, all the components shown in the figure are standard 3 cm components. The directional couplers are designed for a frequency of 12 000 Mc/sec. The special length of wave guide which enters the vacuum system and is located in the $\frac{1}{4}$-inch gap of the C field measures $\frac{3}{4}$ inch by $\frac{3}{16}$ inch in inside cross section. Two slots $\frac{1}{16}$ inch wide and $\frac{1}{2}$ inch high are cut in the narrow sides of the wave guide with their centers one-half guide-wavelength (41/64 inch at a frequency of 12 000 Mc/sec) from the short-circuited end. The guide is operated in the TE_{01} mode which provides a maximum of the rf magnetic field one-half guide-wavelength from the end. The rf magnetic lines of forces are perpendicular to the direction of the static magnetic field and hence only $\Delta m_F = \pm 1$ transitions are expected.

To monitor the power in the wave guide, a small probe is inserted into the guide in the center of one of the broad sides about 5/4 guide-wavelength from the center of the slots. The probe is connected to a diode and the rectified current is measured by means of a microammeter.

A tapered section is used to join up the standard 3 cm wave guide outside the vacuum system with the special wave guide inside. A sheet of mica 0.005 inch thick waxed onto the top of the special guide serves as an air barrier between the vacuum and the outside.

The precise measurement of the klystron frequency is effected by comparing it with harmonics of a standard frequency. Two different frequency standards were used in the present experiment corresponding to two series of measurements made about six months apart. In the first series of measurements the frequency standard was a Model 500 Frequency Standard Multiplier of the Polytechnic Research and Development Company. It produced frequencies in the range 263 Mc/sec to 283 Mc/sec by four successive triplings of the output of a very stable oscillator variable over the range 3.25 to 3.50 Mc/sec. The oscillator frequency was continuously monitored by means of a Hewlett-Packard Model 524 B Frequency Counter. The time base of this counter was governed by a 100-kc/sec signal from a primary frequency standard which is accurate to 1 part in 10^9. With this system, the reference frequency in the vicinity of 270 Mc/sec fed into the balanced mixer in Fig. 1 was known to better than 1 part in 10^7. The forty-second to forty-fifth harmonics of the reference frequency were used to beat against the signal to be measured. The beat note, which in the present experiment was made to fall in the range 10 to 15 Mc/sec, was picked up by a Hallicrafters Model SX-62 receiver and measured to within $\frac{1}{2}$ kc/sec by means of a General Radio Type 620 A heterodyne frequency meter. The frequency being measured may therefore be expressed as

$$f_K = n(81 f_F) + f_R, \tag{1}$$

where f_F is the fundamental frequency of the oscillator in the Frequency Standard Multiplier, n is the order of the harmonic (between 42 and 45) of the standard signal fed into the mixer and f_R is the beat frequency as detected in the communications receiver.

In the second series of measurements, the frequency standard was one which provided an output consisting of a mixture of two fixed frequencies, 50 Mc/sec and 250 Mc/sec. These frequencies were obtained by multiplication from a crystal-controlled 1-Mc/sec signal, the frequency of which was known to better than 1 part in 10^8. Beat frequencies between harmonics of this standard and the unknown were measured as before.

The balanced mixer unit may be connected to the point A as shown in Fig. 1 or it may be connected to the point A'. In the latter position the mixer unit takes less power from the klystron and thus permits more power to flow to the transition region. However, this latter position can be used only when the strength of the harmonic being used as reference is sufficiently great to produce a detectable beat with the weaker unknown signal.

Located before and after the wave guide in the C field are two hairpin loops which serve to induce "low-frequency" ($\Delta F = 0$) transitions.

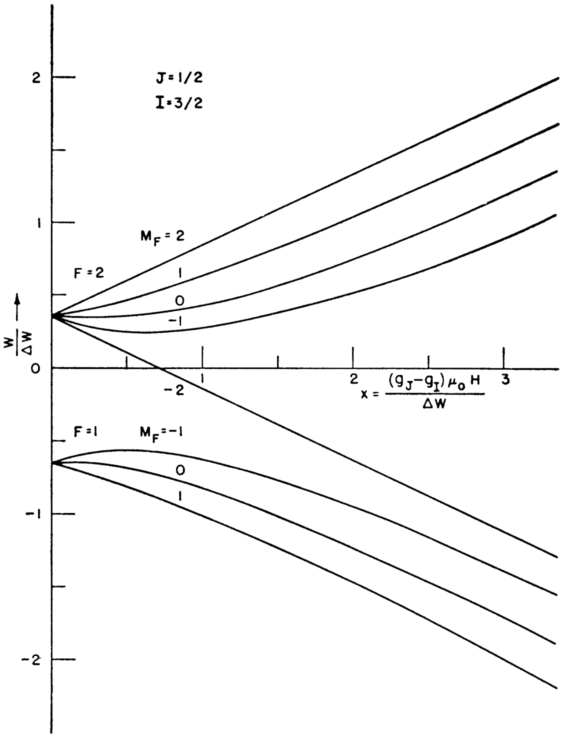

FIG. 2. Energy levels for the case $J = \frac{1}{2}$, $I = \frac{3}{2}$.

III. RESUMÉ OF THEORY

The nuclear spin of both Cu^{63} and Cu^{65} is $\frac{3}{2}$. The two hyperfine levels in the ground state $4\,^2S_{\frac{1}{2}}$ are therefore characterized by the total quantum numbers $F = 2$ and $F = 1$. In a magnetic field, such as the A, B, and C fields, these two levels are split into Zeeman components with energies given by the well-known Breit-Rabi formula[8]

$$W(F,M) = -\frac{\Delta W}{2(2I+1)} - g_I \mu_0 H M \pm \frac{\Delta W}{2}\left(1 + \frac{4M}{2I+1}x + x^2\right)^{\frac{1}{2}},$$

where the ± sign refers to the $F = I \pm \frac{1}{2}$ levels, ΔW is the hyperfine structure separation, $x = (g_J - g_I)\mu_0 H / \Delta W$, μ_0 is the Bohr magneton, and g_J and g_I are negative ratios of the electronic and nuclear magnetic moments in Bohr magnetons to the corresponding angular momenta in $h/2\pi$. A plot of the levels for the case $I = \frac{3}{2}$ is shown in Fig. 2. In weak fields the levels may be represented by a power series expansion of the above formula as follows (for $I = \frac{3}{2}$):

$$\begin{aligned} W(F=2,M_2) &= g_I \mu_0 H M_2 + \tfrac{1}{8}\Delta W \\ &\quad \times (3 + 2M_2 x + 2x^2 - \tfrac{1}{2}M_2^2 x^2 + \cdots), \\ W(F=1,M_1) &= g_I \mu_0 H M_1 + \tfrac{1}{8}\Delta W \\ &\quad \times (-5 - 2M_1 x - 2x^2 - \tfrac{1}{2}M_1^2 x^2 + \cdots). \end{aligned} \tag{2}$$

Transitions between $F = 2$ and $F = 1$ levels, called high-frequency lines, are therefore given by

$$\begin{aligned} \nu_h = \Delta\nu[1 + \tfrac{1}{4}(M_1 + M_2)x + \tfrac{1}{2}x^2 \\ - \tfrac{1}{16}(M_1^2 + M_2^2) + \cdots] + g_I \mu_0 H (M_2 - M_1)/h, \end{aligned} \tag{3}$$

where $\Delta\nu = \Delta W / h$ is the hyperfine structure separation in $\sec^{-1}$. Of the allowed high-frequency transitions satisfying $\Delta M = M_1 - M_2 = 0, \pm 1$, all are observable in the present apparatus except $(F=2, M=-2) \rightarrow (F=1, M=-1)$. This transition is not observable because it is not accompanied by a change in magnetic moment at high fields. The only observable low-frequency line is $(2, -1) \rightarrow (2, -2)$. Its frequency is given to first approximation by

$$\nu_l = \Delta\nu x / 4 \approx 0.6998\, H \text{ Mc/sec}, \tag{4}$$

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[8] G. Breit and I. Rabi, Phys. Rev. 38, 2082 (1931).

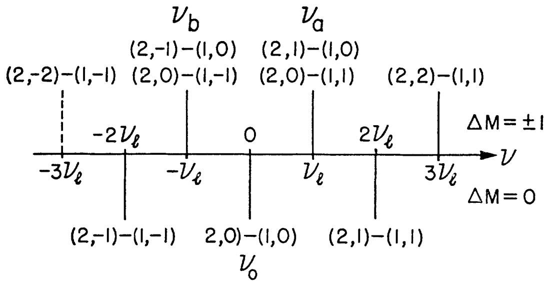

FIG. 3. Spectrum of high-frequency lines at weak fields.

where H is in gauss and where we have set $g_J=2$ and neglected g_I. To the same approximation, Eq. (3) may be written

$$\nu_h=\Delta\nu+(M_1+M_2)\nu_l. \tag{5}$$

At magnetic fields of the order of 1 gauss, the error in this approximation is of the order of 1 kilocycle per second, which is about a hundred times less than the line width in the present measurements. The spectrum given by Eq. (5) is shown diagrammatically in Fig. 3. From Eq. (5) the following relations may be noted between the various Zeeman components:

$$\Delta\nu=\nu_0=\tfrac{1}{2}(\nu_a+\nu_b), \tag{6a}$$

$$\nu_l=\tfrac{1}{2}(\nu_a-\nu_b), \tag{6b}$$

where ν_0, ν_a, ν_b are the components as designated in Fig. 3. To a first-order approximation, the hyperfine separation is simply equal to ν_0 or to the average of ν_a and ν_b. The low-frequency line is just one-half the difference between ν_a and ν_b. This latter relation, along with Eq. (4), permits us to determine the magnetic field strength at the exact position of the wave guide whereas this would not be true if the low frequency line were measured directly with a hairpin loop. This loop must of necessity be located before or after the wave guide and the field at either of these positions may not be the same as that at the wave guide.

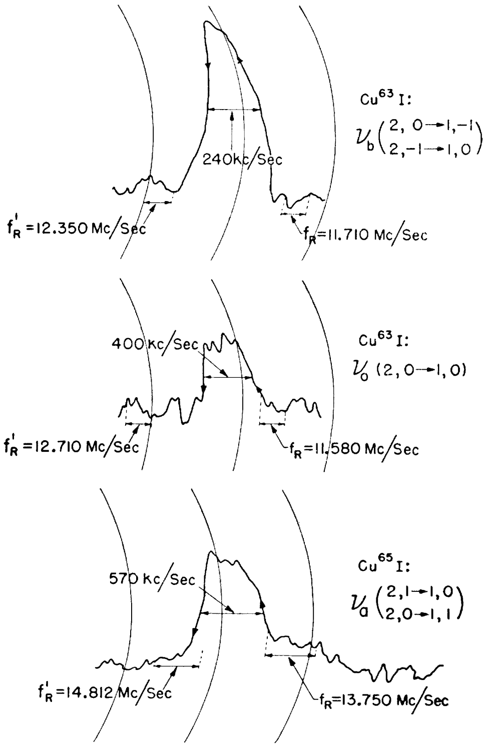

FIG. 4. Typical lines observed in the first series of measurements. f_R and f_R' denote beat frequencies as detected in the radio receiver. The actual klystron frequency corresponding to any beat frequency is the sum of the beat frequency and the appropriate multiple of 250 Mc/sec. The arrows on the curves indicate the direction of sweep.

For a more precise value of $\Delta\nu$ than that given by the first-order approximation of Eq. (6a) or when the high frequency lines are measured at fields of more than 1 or 2 gauss, a second-order correction is necessary. With this correction $\Delta\nu$ is given by

$$\begin{aligned}\Delta\nu&=\tfrac{1}{2}(\nu_a+\nu_b)\left[1-(1-\tfrac{1}{8})x^2\right]\\&=\tfrac{1}{2}(\nu_a+\nu_b)\left[1-14\left(\frac{\nu_a-\nu_b}{\nu_a+\nu_b}\right)^2\right],\end{aligned} \tag{7}$$

and $\Delta\nu=\nu_0(1-\tfrac{1}{2}x^2)$. In the present experiment, the second-order correction is of the order of -2 kc/sec.

IV. PROCEDURES AND RESULTS

With the mass spectrometer tuned to Cu^{63}, the more abundant isotope, the klystron frequency was set near the value of $\Delta\nu$ as known from optical spectroscopy. The current in the C magnet was initially adjusted to give "zero-field transitions" (nonadiabatic transitions) and then varied slowly by means of the motor-driven water rheostat so as to give a monotonically increasing magnetic field. The rate of change of this field as determined by the low-frequency line was about 1.5 gauss per minute. After a sweep from zero field to about 50 gauss, which corresponds to a frequency variation in ν_l of about 30 Mc/sec, the klystron was reset at 30 Mc/sec away and the sweep repeated.

The first high-frequency line found was the $(2,0)\to(1,-1)$, $(2,-1)\to(1,0)$ line of Cu^{63} as identified by its dependence on magnetic field at very low fields. Subsequently, with the C field fixed, the klystron reflector voltage was swept over a range of 10 volts to observe all the components of the high-frequency group.

In the first series of measurements the widths of the observed lines ranged from 300 kc/sec to 900 kc/sec. Typical lines are shown in Fig. 4. The line width is several times greater than that observed in previous experiments with the same apparatus. The exact cause of this large width was not obvious. Inhomogeneity of the C field and spectral impurity of the klystron signal were suspected. Since it was felt that it would take considerable time to correct either of these faults and since operating conditions were otherwise very good at the time, it was decided to take a complete set of measurements before attempting to make the lines narrower.

As mentioned before, the wave-guide orientation was such as to induce $\Delta M=\pm 1$ transitions only. At the maximum microwave power attainable, however the $\Delta M=0$ lines of Cu^{63} also appeared, though much weaker. The probable explanation is that the slots opened for the passage of the beam in the narrow sides of the wave guide interrupt the currents in the wave guide and cause a considerable distortion of the field from the TE_{01} mode. The central line ν_0 (Fig. 3) is, to a first-order approximation, independent of the field and hence its width should be relatively independent of field inhomogeneities. Yet its observed width of 500 kc/sec is of the same order as that of the field dependent lines. This suggested that the inhomogeneity of the C field was not the principal cause of the abnormal width. The low intensity of this line together with its large width precludes its use in the present experiment in the final evaluation of the $\Delta\nu$'s. It is listed in Table II, however, for comparison purposes.

The lines ν_a and ν_b are the ones which have been used in the evaluation of $\Delta\nu$. The exact value of $\Delta\nu$ was taken to be the mean value of ν_a and ν_b with a small second-order correction. Since all the measurements were made at C fields of about 2 gauss, the second-order corrections were of the order of 1 or 2 kc/sec. Because of the breadth of the lines, a large number of measurements were made. To allow for the drift of the C field with time, ν_a and ν_b were measured alternately. Each line was actually swept through forwards and backwards to average out the effect of the finite time constant (usually, 4 seconds) of the detection and recording system. The line position was taken to be midway between two half-power points. Its exact frequency was determined by interpolation between two frequency markers located at the beginning and at the end of each sweep as shown in Fig. 4. These frequency markers are, of course, the beat note between the klystron frequency and a known harmonic of a standard frequency. Unfortunately, the klystron signal had a frequency spread of a few hundred kilocycles per second because of some 60-cycle modulation. The center of this spread however was not difficult to locate. The reproducibility of measurements was found to be very high—of the order of 5 kc/sec.

In Tables I(a) and I(b) are listed the data of the first series of measurements. The values of $\Delta\nu$ obtained from these data are shown in Table II. Close agreement between the measurements of different days is noted. The weighted mean values of the $\Delta\nu$'s from this set of measurements are

$$\Delta\nu(\mathrm{Cu}^{63})=11\,733.841\pm 0.006\ \mathrm{Mc/sec},$$
$$\Delta\nu(\mathrm{Cu}^{65})=12\,568.822\pm 0.005\ \mathrm{Mc/sec},$$

where the probable errors are from internal consistency considerations alone. There may be some systematic error which is greater than the 5 or 6 kc/sec quoted, such as the exact centers of the frequency markers not corresponding to the frequencies as read on the frequency meters.

About six months after the above observations, a second set of measurements was made with a purified klystron signal and a new frequency standard. The latter has already been described above. The klystron signal was purified through the simple expedient of

TABLE I. Data of the first series of measurements. All frequencies in Mc/sec.

Time in minutes[a]	ν_a	ν_b	$(\nu_a+\nu_b)/2$	$(\nu_a-\nu_b)/2$	Second-order correction from $(\nu_a-\nu_b)/2$
	(a) Measurements of ν_a and ν_b of Cu^{63} (first series)				
not recorded		11 732.114	11 733.840	1.726	−0.002
	11 735.566				
	11 735.529		11 733.856	1.674	−0.002
		11 732.182	11 733.832	1.650	−0.002
	11 735.471		11 733.839	1.633	−0.002
		11 732.206			
		11 732.217	11 733.837	1.620	−0.002
	11 735.457				
	Average		11 733.841 ±	0.008	
42	11 734.934				
54		11 732.721	11 733.831	1.110	−0.001
74		11 732.717	11 733.835	1.117	−0.001
90	11 734.961				
96	11 734.956		11 733.845	1.111	−0.001
109		11 732.744	11 733.841	1.097	−0.001
118	11 734.924		(11 733.849)	(1.075)	−0.001
127		11 732.804	(11 733.858)	(1.054)	−0.001
135	11 734.902		11 733.852	(1.050)	−0.001
144		11 732.800			
	Average		11 733.843 ±	0.009	
	(b) Measurements of ν_a and ν_b of Cu^{65} (first series)				
not recorded	12 570.92		12 568.79	2.13	−0.002
		12 566.66	12 568.82	2.16	−0.002
	12 570.98		12 568.82	2.16	−0.002
		12 566.66			
	Average		12 568.80 ±	0.014	
		12 566.985	12 568.827	1.842	−0.002
	12 570.669$_7$				
	12 570.714$_7$		12 568.824	1.890	−0.002
		12 566.934$_8$	12 568.830	1.895	−0.002
	12 570.724$_6$		12 568.829	1.895	−0.002
		12 566.934			
	Average		12 568.827 ±	0.003	

[a] Zero minute is about ½ hour after the C field is set. The mean values of ν_a, ν_b and the differences are computed by interpolation from a plot of ν_a and ν_b against a time axis. The bracketed figures are given statistical weight ½ because of appreciable field drift at these points.

TABLE II. $\Delta\nu$ evaluated from first series of measurements. The third column indicates the lines that are used to determine $\Delta\nu$. Second-order correction is applied only to results with statistical error less than 0.010 Mc/sec. The measurements of ν_0 are not included in the final evaluation of $\Delta\nu$ for reasons mentioned in the text. The first-listed measurement of Cu^{65} is given less weight than the second because only three frequency markers were used instead of four for two successive tracings of ν_a and ν_b.

Isotope	Line observed	$\Delta\nu$(Mc/sec)	Weighted mean (Mc/sec)
Cu^{63}	ν_0	11 733.8±0.1	
	ν_0	11 733.83±0.05	
	ν_a, ν_b	11 733.839±0.008	11 733.841±0.006
	ν_a, ν_b	11 733.842±0.009	
Cu^{65}	ν_a, ν_b	12 568.810±0.014	12 568.822±0.005
	ν_a, ν_b	12 568.825±0.003	

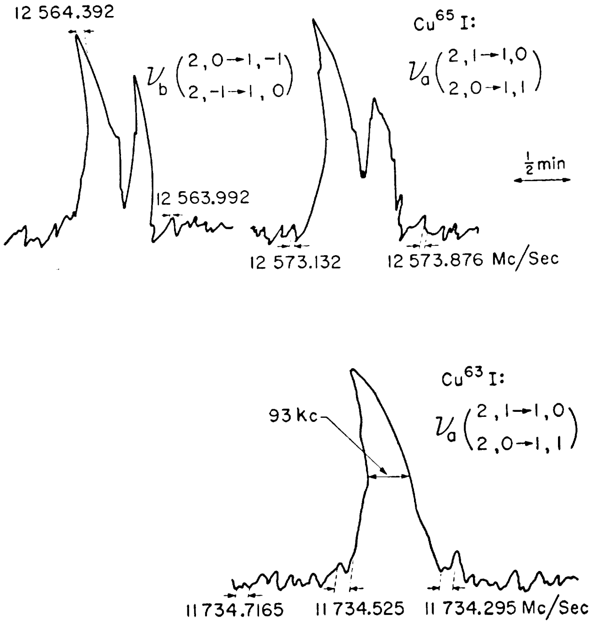

FIG. 5. Typical lines observed in the second series of measurements.

using battery operation of its heater rather than ac operation. The purity was such that the line width was of the order of 100 kc/sec, which is in agreement with previous experiences with the same apparatus and which presumably is due to inhomogeneities in the *C* field. Some typical traces are shown in Fig. 5. The lower trace shows a "normal" line of Cu^{63} of 95 kc half-width. The upper traces, however, show two lines of Cu^{65} which are split. The total half-widths of such lines were about 250 kc/sec. Since the conditions under which the two types of lines were observed were identical except for the frequency, the conclusion is suggested that the splitting was probably due to a peculiar configuration of the rf field in the wave guide at certain frequencies. In determining the centers of the split lines, the total half-widths were used. As before, the lines denoted by ν_a and ν_b in Fig. 3 were the ones used to determine $\Delta\nu$. The results are

$$\Delta\nu(\mathrm{Cu}^{63})=11\,733.820\pm0.007\ \mathrm{Mc/sec},$$
$$\Delta\nu(\mathrm{Cu}^{65})=12\,568.795\pm0.012\ \mathrm{Mc/sec},$$

where the errors are again from internal consistency considerations alone. It will be noted that these values are about 20 kc/sec less than the preceding. This is somewhat greater than the sum of the probable errors computed from the data alone. Hence there must have been some systematic error in one or both sets of measurements. This could have been in the location of the centers of the lines or in the determination of the frequency of an impure signal. In trying to assign relative weights to the two sets of measurements, the writers find that the narrower lines of the second set of measurements are more or less balanced by the large number of observations in the first set. Hence the best values are probably the simple averages of the two. We therefore quote as our final values

$$\Delta\nu(\mathrm{Cu}^{63})=11\,733.83\pm0.01\ \mathrm{Mc/sec},$$
$$\Delta\nu(\mathrm{Cu}^{65})=12\,568.81\pm0.01\ \mathrm{Mc/sec}.$$

The low-frequency line $(2,-1)\to(2,-2)$ was measured at moderate field strengths in order to determine the Landé g factor g_J of the ground state. The low-frequency line $(4,-3)\to(4,-4)$ of Cs^{133} was used to measure the field. Since the g_J of Cs appears in relations used to determine the field, the quantity measured is really the ratio $g_J(\mathrm{Cu})/g_J(\mathrm{Cs})$. In Table III are listed the data and the calculated g_J's. We find

$$g_J(\mathrm{Cu}^{63})=g_J(\mathrm{Cu}^{65})=(1.0000\pm0.0005)g_J(\mathrm{Cs}^{133}).$$

If $g_J(\mathrm{Cs}^{133})$ is taken to be[9] $2(1.00125\pm0.00003)$, then we have

$$g_J(\mathrm{Cu}^{63})=g_J(\mathrm{Cu}^{65})=2(1.00125\pm0.00053).$$

V. HYPERFINE STRUCTURE ANOMALY

The hyperfine structure anomaly Δ of two isotopes is defined by

$$\frac{\Delta\nu_1}{\Delta\nu_2}=\frac{(2I_1+1)g_1}{(2I_2+1)g_2}(1+\Delta),$$

where the symbols have their usual meanings. For copper the ratio of the $\Delta\nu$'s found in the present experiment is

$$\Delta\nu_{63}/\Delta\nu_{65}=0.933567\pm0.000002.$$

The best available value for the ratio of the g's or the ratio of the magnetic moments seems to be that of Walchli.[10] He gives for the ratio μ_{63}/μ_{65} a value 0.933424 ±0.000019 as determined from the salts $\mathrm{CuCl_2}$ and $\mathrm{Cu_2Cl_2}$ and a value 0.933465 ± 0.000018 as determined from the metal powder. These two values disagree by more than their combined probable errors. Since theory indicates that magnetic shielding corrections for nuclear induction measurements are smaller in the case of salts than in the case of metals, we shall take the value of μ_{63}/μ_{65} as determined from the salts. The experimental value of the hyperfine structure anomaly then becomes

$$\Delta=(\Delta\nu_{63}\cdot\mu_{65})/(\Delta\nu_{65}\cdot\mu_{63})-1=+(15\pm2)\times10^{-5}.$$

According to the theories of Rosenthal and Breit[1,2] and of Bohr and Weisskopf,[3,4] the hfs anomaly may be attributed to deviations of the nucleus from a point charge and a point magnetic dipole. With a distributed nuclear charge, Rosenthal and Breit say that the Coulomb potential in which the electrons move cuts off at the nuclear boundary. This gives rise to a change in the electronic wave function from that which would

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[9] Brix, Eisinger, Lew, and Wessel, Phys. Rev. **92**, 647 (1953).

[10] H. E. Walchli, Oak Ridge National Laboratory Report ORNL-1469, 1955 (unpublished), Suppl. II.

exist if the nucleus were a point charge. The magnetic interaction energy between the nucleus and the electrons is consequently changed by a factor $(1+\Delta_{BR})$, where Δ_{BR} is given by

$$\Delta_{BR}=-\frac{2\rho(j-\rho)(2\rho+1)}{(2j-1)[\Gamma(2\rho+1)]^2}(py_0)^{2\rho-1}.$$

Here $j=-1,\ +1,\ -2,\ +2\cdots$ for $s_{\frac{1}{2}}$, $p_{\frac{1}{2}}$, $p_{\frac{3}{2}}$, $d_{\frac{3}{2}}\cdots$ electrons; $\rho=(1-Z^2\alpha^2)^{\frac{1}{2}}$ for s electrons; $y_0=2Zr_0/a_H$; r_0 = radius of the nucleus; and a_H = radius of first Bohr orbit for hydrogen. The parameter p is, according to Rosenthal and Breit, approximately 2. However, Crawford and Schawlow[3] find that the value $p\approx 1$ gives a better fit between theory and experiment. We shall take $p=1$. With different isotopes of a given element a differential correction occurs since the nuclear radius and hence the cutoff point of the Coulomb potential is different. On the average, the radius of a nucleus is given by $r_0=1.2\times 10^{-13}\,A^{\frac{1}{3}}$ cm. The analysis of isotope shifts, however, indicates that the increase in nuclear radius with the addition of two neutrons is substantially less than that predicted by the $A^{\frac{1}{3}}$ law. On the basis of a compressible model of the nucleus, Wilets, Hill, and Ford[11] give for $Z=29$

$$\delta r_0/r_0=0.8\delta N/3A,$$

where δr_0 is the increment in the nuclear radius corresponding to an increment δN in the neutron number. Using this and substituting in the appropriate values for the quantities in the expression for Δ_{BR}, we find for the net Breit-Rosenthal correction for the copper isotopes:

$$\Delta_{BR}{}^{(63)}-\Delta_{BR}{}^{(65)}=6\times 10^{-5}.$$

This is about a third of the observed anomaly.

Bohr and Weisskopf have calculated the effect of a finite distribution of nuclear magnetism on the hyperfine structure. By considering the nuclear moment as consisting of two intrinsically different parts, a spin moment and an orbital moment, they have been able to account satisfactorily for the observed anomalies in certain elements.

The theory necessarily involves an assumed model for the nucleus. We shall calculate the Bohr-Weisskopf correction ϵ for copper first with an extreme single particle model and then with Bohr's asymmetric model. The Bohr-Weisskopf correction ϵ is given by[4]

$$\epsilon=-\{(1+0.38\zeta)\alpha_s+0.62\alpha_L\}b\left[\left(\frac{R}{R_0}\right)^2\right]_{\mathrm{Av}},$$

TABLE III. Summary of data for the determination of g_J. All frequencies are in Mc/sec.

Nominal C field	50 gauss	62 gauss
Cs^{133} frequency	17.755 ± 0.005	22.180 ± 0.010
Cu^{63} frequency	35.370 ± 0.005	44.102 ± 0.010
Cu^{65} frequency	35.334 ± 0.005	44.053 ± 0.020
$g_J(Cu^{63})/g_J(Cs)$	1.0003 ± 0.0005	0.9994 ± 0.0007
$g_J(Cu^{65})/g_J(Cs)$	0.9999 ± 0.0005	0.9991 ± 0.001

where ζ is an asymmetry factor defined by Bohr,[4] α_s, α_L denote the fractions of the nuclear moment of the spin type and orbital type, respectively, b is a function of Z and R_0 tabulated in reference 3, and $(R/R_0)^2$ is esentially the density distribution of the odd particle to which the spin and magnetic moment of the nucleus is ascribed. Formulas for α_s, α_L, and ζ are given by Bohr for different refinements of the shell model. For $[(R/R_0)^2]_{\mathrm{Av}}$, a reasonable value is, according to Bohr and Weisskopf,[3] 0.8.

According to the shell model, the odd proton in Cu^{63} and Cu^{65} is in the $p_{\frac{3}{2}}$ state. Let us first consider the case where this proton is assumed to possess the entire angular momentum of the nucleus (extreme single-particle model) and where we take $g_L=1$ and g_s adjusted to yield the empirical value of g_I. From

$$g_I=\frac{\langle\mathbf{S}\cdot\mathbf{I}\rangle}{I(I+1)}g_s+\frac{\langle\mathbf{L}\cdot\mathbf{I}\rangle}{I(I+1)}g_L$$

we find $g_s=0.22664$ for Cu^{63} and 0.38473 for Cu^{65}. Hence for Cu^{63}, $\alpha_s=0.551$, $\alpha_L=0.4491$ and for Cu^{65}, $\alpha_s=0.5807$, $\alpha_L=0.4193$. For a $p_{\frac{3}{2}}$ proton, $\zeta=0.2$. With $b=0.36\%$ from Bohr and Weisskopf, we find

$$\Delta_{BW}=\epsilon_{63}-\epsilon_{65}=+4\times 10^{-5}.$$

This correction when added to the Breit-Rosenthal correction yields a total anomaly of

$$\Delta=\Delta_{BR}+\Delta_{BW}=+10\times 10^{-5},$$

which compares favorably with the observed 15×10^{-5}. It may be pointed out that if we had taken for the ratio μ_{63}/μ_{65} the value as found from the metal powder, the observed anomaly would have been 10×10^{-5}.

If in the extreme single particle model we take g_s to be the same as that for a free proton and adjust g_L to give the empirical value of g_I, we find,

$$\Delta_{BW}=\epsilon_{63}-\epsilon_{65}=-11\times 10^{-5}.$$

This value is opposite in sign to the observed and suggests that $g_s=g$ (of a free proton) is a poor assumption.

In Bohr's asymmetric model,[4] the nuclear core as well as the odd nucleon contributes to the magnetic moment. The fractions α_s and α_L then depend on the coupling between the core and the odd nucleon. If we follow the procedure described in Sec. V of Bohr[4] for a certain coupling case intermediate between his B_1 and B_2, we find, assuming $g_s=5.5854$, $g_L=1$, $g_R=Z/A=0.45$, $[(R/R_0)^2]_{\mathrm{Av}}=0.8$

$$\Delta_{BW}=\epsilon_{63}-\epsilon_{65}=+30\times 10^{-5}.$$

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[11] Wilets, Hill, and Ford, Phys. Rev. **91**, 1488 (1953), Table I(a).

This is twice as large as the observed anomaly even without including the Breit-Rosenthal correction. It appears that best agreement between theory and experiment is obtained when the total angular momentum of the nucleus is considered to be possessed by the odd proton and when g_L is set equal to unity, with g_s adjusted to give the observed g_I.

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X-Ray Spectra of Polonium Atomic Number 84

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Two samples, each containing an estimated 10 curies (2 mg) of polonium-210, were purified by vacuum distillation and evaporated onto oblique sections of x-ray targets made of copper. Four lines of the *K* series and ten lines of the *L* series of the x-ray spectrum of polonium were recorded with a one-meter transmission crystal spectrograph and a 25-centimeter Bragg spectrograph, respectively. Measured wavelengths agree approximately with values predicted by extrapolation of Moseley's law and with those reported by Hulubei. Decay of polonium-210 (half-life = 138 days) and growth of lead-206 (stable) were observed over a period of 180 days.

INTRODUCTION

UNTIL recently, no record of the *K* spectrum of polonium and only incomplete information of the *L* spectrum[1] could be found in the literature.[2] This lack of information was largely caused by the scarcity of the natural element with its short half-life of 138.26 days.[3]

SAMPLE PREPARATION

The polonium to be used for these experiments was electroplated from a hydrochloric acid solution onto platinum gauzes.[4] Each gauze carried approximately 10 curies of material. The high vapor pressure of polonium[5] permitted the application of a fractional volatilization technique[6] to purify the polonium from impurities deposited during the electroplating process and from any lead that had accumulated as a result of radioactive decay.

Platinum gauzes carrying a total of approximately 30 curies of polonium were inserted as far as possible into an all-quartz purification tube, and the tube was attached to a vacuum system. After the tube was evacuated to 5×10^{-6} millimeter of mercury, the region surrounding the gauzes was heated strongly with a hand gas-oxygen torch. As this heating proceeded, the pressure rose, and the polonium distilled from the gauzes and condensed in the form of a mirror on the cooler portions of the tube. The flame was removed, and pumping was continued until the pressure had again reached 5×10^{-6} millimeter of mercury. The tube was then sealed from the vacuum system.

The tube containing the polonium and the x-ray target was placed in a vacuum chamber where the sample was heated and the polonium was deposited on the water-cooled target. It was estimated that between 10 and 15 curies were deposited.

K SPECTRUM

The x-ray tube, high-voltage power supply, and transmission type spectrograph have been described in a previous report.[7] Several modifications were made to extend the wavelength coverage of the spectrograph. Before the exposures were started, the loaded x-ray tube was maintained at a maximum pressure of 2×10^{-5} millimeter of mercury. Shortly after the high voltage was turned on, the pressure decreased to 5×10^{-6}

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[1] Hulubei, Cauchois, and Cotelle, Compt. rend. **207**, 1204 (1938).

[2] F. Sachs, Report Y-B4-49, Carbide and Carbon Chemicals Company, Oak Ridge, 1951 (unpublished).

[3] W. H. Beamer and W. E. Easton, J. Chem. Phys. **17**, 1298 (1949).

[4] C. R. Maxwell, J. Chem. Phys. **17**, 1288 (1949).

[5] L. S. Brooks, Mound Laboratory Report MLM-189, September 13, 1948 (unpublished).

[6] I. Curie and F. Joliot, J. Chem. Phys. **28**, 202 (1931).

[7] Burkhart, Peed, and Spitzer, Phys. Rev. **75**, 86 (1949); Peed, Spitzer, and Burkhart, Phys. Rev. **76**, 143 (1949).