a factor of two. For energies above 700 ev the agreement is excellent. For example, at 2000 ev the agreement is within 0.2% and gets better with increasing energy.

Comparison to the ordinary Born approximation has not been made, but it may be simply obtained from Fig. 4 and Eqs. (2) and (2a).

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Absorption of Sound in Liquid Helium II†

W. M. WHITNEY
Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts
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The coefficient of absorption of sound in liquid helium has been measured at a frequency of 11.8 Mc/sec in the temperature range from 0.2°K to 1°K, at vapor pressure, and at 8.0, 13, and 18.3 atmospheres. Careful measurements above 0.9°K at vapor pressure reveal that the two sharp maxima observed by Chase and Herlin at the top of their absorption curve were an instrumental effect. Preliminary results at the higher pressures indicate that, if allowance is made for the pressure variation of the density and the velocity of sound, the attenuation decreases exponentially with pressure at temperatures just below 1°K.

I. INTRODUCTION

CHASE and Herlin[1] have reported a measurement of the attenuation of sound in liquid helium below 1°K under the saturated vapor pressure, at a frequency of 12.1 Mc/sec. According to their results, the absorption coefficient, practically zero below 0.1°K, increases with temperature approximately as $T^{2.8}$ between 0.3°K and 0.8°K, and eventually goes through a broad peak in the vicinity of 0.9°K.

At the summit of their curve, Chase and Herlin found two secondary peaks, apparently well resolved, separated by a temperature interval of 0.1°K. This feature of their results was thought to be in qualitative agreement with a theory proposed in 1950 by Khalatnikov,[2] which attempted to account for Pellam and Squire's observation[3] that, below 2°K, the absorption increases anomalously as the temperature decreases. From a consideration of elementary collision processes involving phonons and rotons, Khalatnikov concluded that only two processes, inelastic phonon-phonon and phonon-roton collisions, had relaxation times sufficiently long that they could lead to absorption at the frequency used by Pellam and Squire (15 Mc/sec). Khalatnikov calculated the temperature dependence of the attenuation coefficient, and predicted that it would go through a maximum near 1°K. Chase[4] confirmed this prediction experimentally in 1953, and showed that Khalatnikov's equations could be adjusted to fit his data reasonably well down to a temperature of 0.95°K. The later discovery that there were two subsidiary peaks at the top of the principal absorption peak pointed to the existence of two separate relaxation mechanisms, and thereby provided further support for Khalatnikov's theory. Chase's earlier attenuation measurements had shown only a single broad maximum, but, since the temperature intervals separating his points below 1°K were quite large, it was thought the fine structure might simply have been missed.

II. ATTENUATION AT VAPOR PRESSURE ABOVE 0.9°K

Soon after the twin peaks were found, the author undertook an investigation of the effect of pressure on their relative positions and magnitudes. Early in the course of this experiment, an attempt was made to reproduce Chase and Herlin's results at vapor pressure. A cryostat was constructed in which temperatures down to 0.9°K could be reached by pumping. Temperatures in this region were measured with a mutual inductance thermometer, sensitive enough to detect changes of less than 0.001°K at 1°K.

Initially, the ultrasonic techniques and apparatus used by the author were identical with those described by Chase and Herlin. A single X-cut quartz crystal, with a resonant frequency of 11.8 Mc/sec,[5] was used

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[1] C. E. Chase and M. A. Herlin, Phys. Rev. **97**, 1447 (1955).

[2] I. M. Khalatnikov, J. Exptl. Theoret. Phys. (U.S.S.R.) **20**, 243 (1950).

[3] J. R. Pellam and C. F. Squire, Phys. Rev. **72**, 1245 (1947).

[4] C. E. Chase, Proc. Roy. Soc. (London) **A220**, 116 (1953).

[5] This was the same crystal used by Chase and Herlin. They measured the frequency of the pulses exciting the crystal by beating them against a continuous signal from the signal generator. It was later found, however, that the calibration of the signal-

in conjunction with a polished brass reflector to transmit and receive the ultrasonic pulses. All signals that appeared across the crystal, including an additional pulse from a signal generator, were displayed on the sweep of a Du Mont type 256D oscilloscope, after suitable amplification and detection. Echo heights were measured by adjusting the calibrated output attenuator of the signal generator until the amplitudes of the comparison pulse and the echo were equal. Since the sound pulses were transmitted over a fixed distance, only the temperature variation of the attenuation was measured directly. Absolute values of the absorption coefficient were calculated by matching the results with Chase's earlier measurements at 12.1 Mc/sec,[4] made with a variable path length. Above 1.15°K, the attenuation is proportional to the square of the frequency,[4] so it was a simple matter to correct for the slight frequency difference.

From the very outset, the author experienced difficulty in making measurements in the region of maximum absorption below 1°K. As the attenuation increased, the echoes grew smaller and became severely distorted. Often, as the temperature was varied, portions of the same pulse were seen to change both in size and shape, at quite different rates. The data strongly reflected the criterion chosen for estimating the height of the echo, and it is therefore not surprising that results obtained one day could never be duplicated the next. Minor peaks similar to those found by Chase and Herlin were often recorded, but they differed widely in temperature separation and position, as well as in magnitude.

These troubles vanished when separate crystals were used as transmitter and receiver, and so did the twin peaks. Results from one run are plotted in Fig. 1, together with smoothed values of Chase and Herlin's data (shown as a dotted line). There is little doubt that the two sets of measurements are significantly different. The variation in their curve is well outside the scatter of the data presented here, and the temperature intervals between our experimental points are small enough to preclude the possibility that the extra peak might have been missed.

The results of several other runs are in agreement with the data shown in Fig. 1, and it is now clear that the fine structure observed by Chase and Herlin was an instrumental effect. Their measurements were carried out over a range of temperature in which the absorption coefficient undergoes a considerable change in value. At low temperatures, where the attenuation was small, it was necessary for them to use ultrasonic pulses sufficiently weak that there would be no spurious absorption from finite amplitude effects. During a demagnetization, however, the pulse amplitude could not

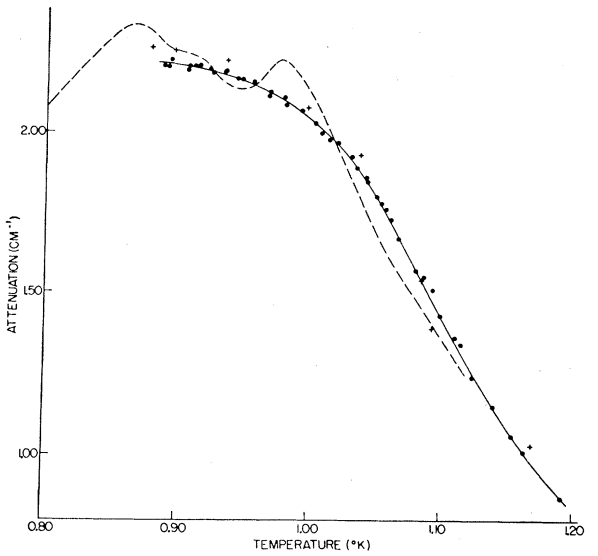

FIG. 1. The attenuation of sound in liquid helium above 0.9°K, at vapor pressure. + Chase,[4] 12.1 Mc/sec; – – – Chase and Herlin,[1] 12.1 Mc/sec (see reference 5); ·–·–· present results, 11.8 Mc/sec.

be varied; consequently, if measurements were to be made over the entire temperature range, the path length had to be sufficiently short that the received pulse would not be obscured by noise near 0.9°K, where the attenuation was large.

For the 1.96-cm path length used by Chase and Herlin, the amplitude of the first echo near 0.9°K was still several times the noise level of the amplifier. There was additional noise visible on the oscilloscope trace, however, which arose from "ringing" of the crystal and from electronic difficulties associated with saturation of the amplifier by the large input pulse. These disturbances constituted a coherent background on which the pulses representing the ultrasonic echoes were superimposed. If the velocity of sound should vary with temperature, the phase between an echo and its background would also vary. The temperature dependence of the velocity would not have to be large to give rise to significant alterations in the shape and amplitude of the pulses; a variation of only 0.05% over a temperature interval of 0.1°K, corresponding to a phase shift of one-half wavelength, would be sufficient to account for the spurious peaks. When separate crystals are used, the background clutter is absent from the oscilloscope trace (provided that the receiver line is well shielded from the transmitter), and there is virtually no systematic distortion of the smallest signals which can be distinguished above noise.

III. ATTENUATION AS A FUNCTION OF PRESSURE ABOVE 0.2°K

Attenuation measurements have also been carried out in the temperature region 0.2°K to 1°K, at vapor

generator frequency dial was inaccurate. It is believed, therefore, that their measurements were actually made at a frequency of 11.8 Mc/sec and not 12.1 Mc/sec.

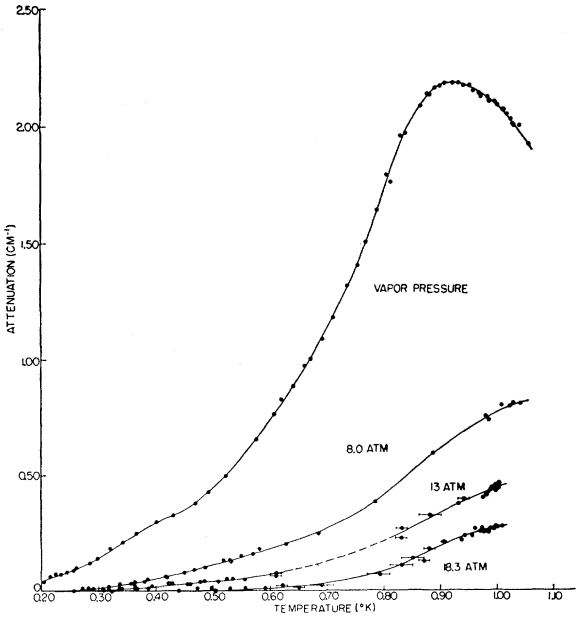

FIG. 2. The attenuation of sound in liquid helium at 11.8 Mc/sec. Temperature uncertainties greater than 0.005°K are indicated by horizontal bars.

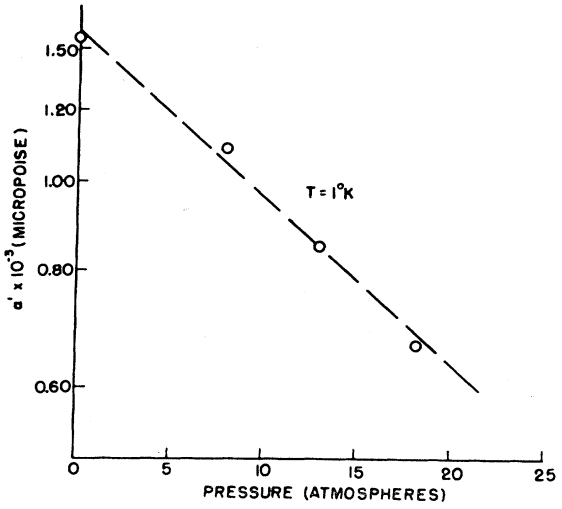

FIG. 3. The reduced attenuation of sound ($\alpha'=2\rho c^3\alpha/\omega^2$) in liquid helium at 1°K, as a function of pressure.

pressure and at 8.0, 13, and 18.3 atmospheres.[6] This work is still in progress, and only preliminary data have been obtained. Since certain features of the results are of some interest, however, they will be presented here.

The data shown in Fig. 2 were obtained with two matched X-cut quartz crystals, resonant at 11.8 Mc/sec, separated by a path length of 1.48 cm. The results at vapor pressure were adjusted to fit Chase's 1953 data.[4] Calculations of the absorption coefficient at higher pressures were based on the assumption that the attenuation falls to zero at sufficiently low temperatures. Chase and Herlin's work shows that this assumption is valid at vapor pressure, and it is reasonable to suppose it holds at other pressures as well.

The curve at vapor pressure is in agreement with the measurements published by Chase and Herlin, if allowance is made for temperature uncertainties of 5% in their experiment, and approximately 3% in the present one. Between 0.4°K and approximately 0.8°K the attenuation is proportional to $T^{2.9}$ at vapor pressure, and $T^{2.8}$ at 8.0 atmospheres. At 13 and 18.3 atmospheres, the attenuation appears to increase somewhat more rapidly than T^3; however this conclusion must be regarded as tentative. The warm-up rates from 0.6°K to 0.9°K were, unfortunately, quite rapid for the demagnetizations that were made at these pressures, and the temperature uncertainties for the points taken in this region are therefore quite large.

Above 0.9°K, measurements both of the temperature and the signal size can be considered fairly reliable, so we can estimate the pressure dependence of the attenuation. According to Khalatnikov,[7] the absorption coefficient is given for liquid helium, as for ordinary liquids, by the expression

$$\alpha=(\omega^2/2\rho c^3)[(4/3)\eta+\zeta_2],$$

where ω is the angular frequency, ρ the density, c the velocity of sound, η the coefficient of ordinary (shear) viscosity, and ζ_2 the coefficient of second viscosity, including relaxation effects. The attenuation has a substantial pressure dependence that arises solely from the factor $(1/\rho c^3)$. We therefore calculate the quantity $\alpha'=2\rho c^3\alpha/\omega^2$, using the data of Keesom and Keesom for the density,[8] and of Atkins and Stasior for the velocity of sound.[9] Values of α' are plotted in Fig. 3 for a temperature of 1°K. It can be seen that α' decreases approximately exponentially with pressure. At 0.9°K, the decline is somewhat more rapid, but it is still exponential.

One other feature of the data deserves brief comment. Chase and Herlin found a knee, or change in slope in their curve at a temperature of approximately 0.3°K, below which the attenuation fell to zero much more

[6] Measurements of the attenuation as a function of pressure above 1.2°K have been made at 14.4 Mc/sec by J. A. Newell, in collaboration with J. Wilks [J. A. Newell, *Conférence de Physique des Basses Températures* (Centre National de la Recherche Scientifique and UNESCO, Paris, 1956)]. At the end of his report, Newell mentions measurements at lower temperatures. No details are given, but it is stated that: (1) a smooth maximum was found at vapor pressure in place of Chase and Herlin's twin peaks; (2) the sum of the first and second viscosities, calculated from the attenuation, decreases with pressure; and (3) there is a "shift in the position of the maximum of up to 0.14°K."

Note added in proof.—Newell and Wilks have now published a further account of their measurements below 1°K [Phil. Mag. **1**, 588 (1956)]. They present curves of the normalized attenuation-*versus*-temperature which extend down to 0.4°K, at 8, 16, and 25 atmospheres and at vapor pressure.

[7] I. M. Khalatnikov, J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 21 (1952).

[8] W. H. Keesom, *Helium* (Elsevier Publishing Company, Inc., Amsterdam, London, New York, 1942), p. 240.

[9] K. R. Atkins and R. A. Stasior, Can. J. Phys. **31**, 1156 (1953).

rapidly than T^3. A log-log plot of the present data reveals that there is a similar knee in the curve at vapor pressure, and perhaps also at 8.0 atmospheres. The knee is located at 0.4°K, rather than at 0.3°K, however, and we found no further increase in slope at lower temperatures.

Chase and Herlin suggested that the effect might be associated with the rapid increase of the phonon mean free path as the temperature decreases. Assuming that the knee in our curve has the same origin, we might have expected to find it at a higher temperature, since we used a shorter path length (1.48 cm). However, their measurements below 0.7°K were actually made at two different path lengths—1.96 and 3.94 cm—and the increase in power law apparently occurred at 0.3°K for both sets of data. It seems unreasonable, on the basis of their interpretation, that a path length increase of 0.5 cm would depress the temperature of the knee by 0.1°K, when a further increase of 2 cm apparently produces no change at all. Measurements now being made at two other frequencies, and with different path lengths, may enable us to determine whether the effect is instrumental, or whether it actually represents a change in the attenuation at these low temperatures.

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Calculation of the Viscosity of Gaseous He^3 and He^4 at Low Temperatures*

WILLIAM E. KELLER
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico
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The viscosity coefficients for gaseous He^3 and He^4 have been calculated to the second order approximation from 0°K to 40°K according to the quantum-mechanical theory of transport phenomena. Results are presented for two intermolecular potentials, the Lennard-Jones 12-6 (LJ1) and the exp-6 (MR5) functions. Contrary to expectation, LJ1 provides a better over-all fit to experimental data. A brief survey is given of the theory, previous calculations, and existing experimental data, with the conclusion that the low-temperature transport properties of the gaseous helium isotopes are not yet described with certainty by either experiment or theory.

I. INTRODUCTION

RECENTLY we have calculated[1,2] the second virial coefficients, B, of gaseous He^3 and He^4 below 60°K using two different intermolecular potential functions: the Lennard-Jones 12-6 function,

$$\varphi(r)=4\epsilon[(\sigma/r)^{12}-(\sigma/r)^6)]$$

with constants σ and ϵ given by de Boer and Michels[3] (designated as LJ1), and an exp-6 potential,

$$\varphi(r)=\frac{\epsilon}{1-6/\alpha}\left[\left(\frac{6}{\alpha}\right)\exp\{\alpha(1-r/r_m)\}-\left(\frac{r_m}{r}\right)^6\right]$$

with constants ϵ and r_m derived from low-temperature He^4 B values[4] and α from high-temperature gaseous He^4 viscosity data[5] (designated as MR5). Comparison of measured P-V-T data[4,6] for He^3 and He^4 with the calculations indicated that MR5 gives a better over-all fit to $B(T)$ than does LJ1. With this conclusion and because one of three MR5 potential constants was determined from transport data, the expectation was that MR5 could yield better values of the low temperature gaseous transport coefficients of the helium isotopes than LJ1.

There have been several calculations of the thermal conductivity and viscosity coefficients of He^4 gas below 5°K,[7] and de Boer and Cohen[8] and Buckingham and Scriven[7] have calculated these quantities for He^3. De Boer and co-workers have based all computations on LJ1, while Buckingham and co-workers have used

* Work done under the auspices of the U. S. Atomic Energy Commission.

[1] Kilpatrick, Keller, Hammel, and Metropolis, Phys. Rev. **94**, 1103 (1954).
[2] Kilpatrick, Keller, and Hammel, Phys. Rev. **97**, 9 (1955).
[3] J. de Boer and A. Michels, Physica **5**, 945 (1938).
[4] W. E. Keller, Phys. Rev. **97**, 1 (1955).
[5] See E. A. Mason and W. E. Rice, J. Chem. Phys. **22**, (1954) for use of viscosity data in determining the repulsive steepness of the potential.
[6] W. E. Keller, Phys. Rev. **98**, 1571 (1955).
[7] Buckingham, Hamilton, and Massey, Proc. Roy. Soc. (London) **A179**, 103 (1941); J. de Boer, Physica **10**, 348 (1943); R. A. Buckingham and R. A. Scriven, Proc. Phys. Soc. (London) **B65**, 376 (1952).
[8] J. de Boer and E. G. D. Cohen, Physica **17**, 993 (1951).