Statistical Fluctuations in Energy Losses of 37-Mev Protons*

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The frequency distributions of energy losses of 37-Mev protons has been measured for a wide range of thin absorbers. The purpose of this experiment was to compare the measured distributions with the theory of Landau and Symon. The measurements required the application of both scintillation and proportional counter techniques. A simple method of unfolding the inherent resolution of the scintillation counter was developed. The observed distributions were found to be in good agreement with the Landau-Symon theory.

INTRODUCTION

THE problem of fluctuations in energy losses of heavy charged particles traversing thin absorbers was first considered by Bohr[1] in 1915. He showed that for α particles incident on moderately thin absorbers the distribution function would be approximately Gaussian.

In 1944, Landau[2] provided a rigorous mathematical treatment of this problem. Symon[3] has extended the range of these calculations to include mesons from 1 Mev to 1 Bev and protons from 10 Mev to 10 Bev for absorber thicknesses up to 85% of the range of the particle. Symon also calculated the mean and most probable energy losses in this range.

The theory of Landau and Symon predicts that for heavy particles traversing thin absorbers a skew bell-shaped curve with a pronounced high-energy tail will be obtained for the distribution of energy losses. The width of the distribution will be considerably broader than a Poisson distribution appropriate to the statistical fluctuations in the number of ion pairs formed in the absorber (at 25 ev per ion pair). Physically the broad distribution with the high-energy tail is due to the non-negligible probability of collisions in which the charged particle imparts appreciable kinetic energy to the electrons of the stopping material. The skew-shaped distribution is observed when the maximum kinetic energy that can be transferred to an electron in a single collision is comparable to, and particularly when greater than, the mean energy lost by the particle in the finite thickness of the absorber. For nonrelativistic particles this maximum kinetic energy is just

$$T_{\max}=T_0(4m/M),$$

where T_0 = kinetic energy of incident particle, M = mass of incident particle, and m = electron mass.

For thicker absorbers, where the mean energy loss is greater than $T_{\max}$, the skew-shaped distribution has the form of a broad symmetrical distribution.

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[1] N. Bohr, Phil. Mag. **30**, 581 (1915).
[2] L. Landau, J. Phys. (U.S.S.R.) **8**, 201 (1944).
[3] K. R. Symon, thesis, Harvard University, 1948 (unpublished). Parts of this work appear in B. Rossi, *High Energy Particles* (Prentice-Hall Inc., New York, 1952).

Previous experimental work using 31-Mev protons,[4] 15-Mev electrons,[5] and μ mesons[6] has shown fairly good agreement with the theory but these measurements have in general been restricted to one absorber thickness at a particular energy.

In this experiment energy loss distributions are measured over a wide range of absorber thicknesses from the Landau region to a region where one would expect a symmetrical distribution.

EXPERIMENTAL PROCEDURE

The 40-Mev beam from the Minnesota proton linear accelerator is magnetically deflected through a system of diaphragms and a thin aluminum window into a scattering chamber. To reduce the beam intensity the protons are elastically scattered from helium into the counters, which are fixed at 30° to the incident beam. Consequently, the energy of the protons at the absorber face in which the energy losses are measured is 37 Mev. The experimental arrangement for the scintillation counter experiment is shown in Fig. 1.

The output of the thick NaI counter (E) which measures the total energy of the protons is used to trigger the coincidence gate of a 10-channel pulse-height analyzer. The coincidence threshold is set to accept only elastically scattered protons which have traversed the dE/dx crystal and dissipated their remaining energy in the (E) counter. Pulse-height distributions from the dE/dx counter are then observed in coincidence with these protons.

The range of absorber thicknesses used in this experiment required the use of both scintillation and proportional counter techniques for the dE/dx detector. The scintillation counter was used for the thicker absorbers and a proportional counter for the thin absorbers, with a small overlap to insure continuity of the measurements.[7]

[4] Igo, Clark, and Eisberg, Phys. Rev. **89**, 879 (1953).
[5] Hanson, Goldwasser, and Mills, Phys. Rev. **86**, 617 (1952).
[6] A. Hudson and R. Hofstadter, Phys. Rev. **88**, 589 (1952).
[7] It should be noted that the experimentally measured quantity will be equal to the theoretically predicted energy loss only if there is no net transfer of energy, by energetic delta rays, into or out of the absorber. In the case of the proportional counter, the sensitive volume was 3 cm from the entrance and exit windows, and in general out of range of the maximum energy delta rays from the entrance window. Consequently, approximately equal numbers enter and leave the sensitive volume. In the case of the

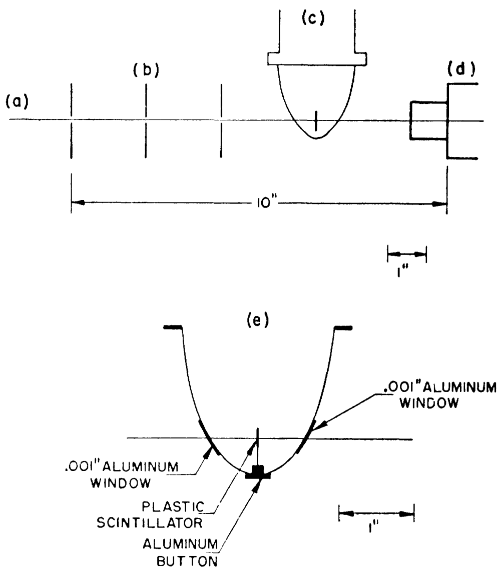

FIG. 1. Schematic diagram of counter arrangement (a) 37-Mev proton beam elastically scattered from He^4, (b) collimators, (c) dE/dx scintillation counter, (d) NaI total energy counter—"E," (e) details of parabolic dE/dx mounting. Both counters have DuMont 6292 photomultipliers.

SCINTILLATION COUNTER EXPERIMENT

In this experiment plastic scintillators[8] were used in thicknesses of from 0.005 in. to 0.050 in. in the dE/dx counter, corresponding to an energy loss from 200 kev to 2 Mev, for 37-Mev protons. To maximize the light collection efficiency the scintillators were mounted at the focus of a small aluminum paraboloid as shown in Fig. 1.

The distribution in magnitude of the pulses from the dE/dx counter is due to the Landau-Symon fluctuations coupled with the inherent photoelectron statistics of the counter, due to the finite number of photoelectrons emitted from the photocathode.

To unfold the Landau-Symon fluctuations from the inherent counter statistics, the following simple technique was developed.

The face of the photomultiplier was covered with a mask, made by punching small holes in a piece of black paper. The transmission of the mask was measured using a small Po α source sandwiched between two thin pieces of plastic scintillator mounted in the paraboloid, and by then observing relative pulse heights for different masks. The dE/dx spectrum was observed later as a function of the transmission of the mask.

In the energy loss region covered by the measurements with the plastic scintillators, the Landau-Symon effect is approximately Gaussian. Since the photoelectron statistics are also approximately Gaussian the over-all distribution should be Gaussian in shape with a full width at half-maximum[9] given by

$$F^2=L^2+R^2, \tag{1}$$

where

F = total % full width,

L = % full width due to Landau-Symon effect,

R = % full width due to photoelectron statistics.

To a good approximation R is given by an equation $R=\text{constant}/n^{\frac{1}{2}}$, where n = number of photoelectrons from photocathode. Since n is proportional to the transmission T of the mask, we have

$$F^2=L^2+(\text{constant}/T). \tag{2}$$

The results of this simple analysis are shown in Fig. 2 for two scintillators 0.025-in. and 0.050-in. thicknesses.

From Eq. (2) we have that intercept at $1/T=0$ gives the Landau-Symon width. The slopes of the lines are proportional to the photoelectron statistics. The slopes of the lines are in the ratio of 2 to 1 which is in agree-

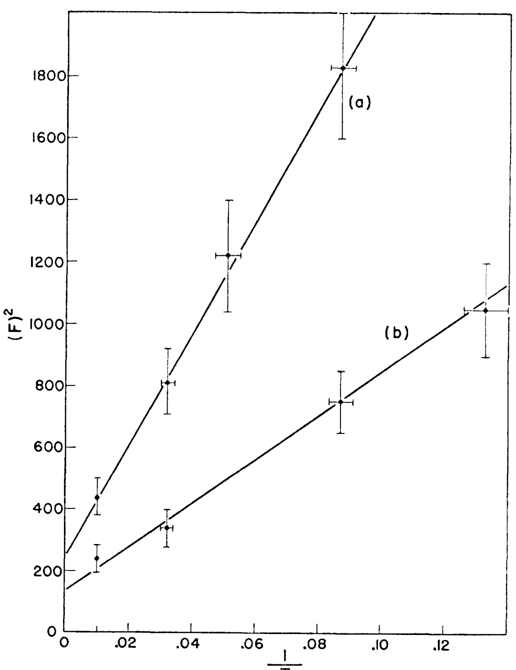

FIG. 2. (Total % full width)² vs reciprocal of % mask transmission for scintillation counter experiment. (a) Results for scintillator thickness 0.025 in., (b) results for scintillator thickness 0.050 in. The intercept at $1/T=0$ gives Landau-Symon width; the slope of the line is proportional to the inherent photoelectron statistics of the counter.

scintillators very few delta rays enter the front face, but it is possible that a considerable number leave the rear face. However, the scintillators are thick in comparison to the maximum energy delta rays so that the effect will be small. Further, the effect will only distort the tail of the distribution and not affect the full width, since the full width is primarily due to the numerous low-energy delta rays.

[8] Pilot scintillator B, Pilot Chemicals Inc., Wolton, Massachusetts.

[9] Defined as $\Delta\epsilon/\epsilon_p$, where $\Delta\epsilon$ = width of distribution at half-maximum, ϵ_p = most probable energy loss.

TABLE I. Comparisons of total and partial full widths for plastic scintillators of various thicknesses. The theoretical full widths are calculated from formulas of Symon (reference 3).

Plastic scintillator thickness	Mean energy loss at 37 Mev	Total % full width	% full width due to photoelectron statistics	% full width due to Landau-Symon effect Experimental	Theoretical
0.050 in.	2.0 Mev	15±1	8.5±1.5	12.5±2.5	12±1
0.025 in.	1.0 Mev	21±1.5	13.5±1.5	16 ±2.5	17±1.5
0.010 in.	400 kev	32±2.5	20 ±4.5	25 ±4	25±2
0.005 in.	200 kev	41±3	27 ±6	29.5±3	31±3

ment with this simple theory since the number of photoelectrons is also in ratio of 2 to 1 for the two scintillators.

Owing to the experimental difficulties arising from the magnitude of the pulse obtained from the 0.005-in. and 0.010-in. scintillators, the mask technique could not be employed to determine their photoelectron statistics. For these crystals the photoelectron statistics was calculated from the measurements shown in Fig. 2.

Since R, the percentage full width due to photoelectron statistics, is proportional to $1/n^{\frac{1}{2}}$ and n is proportional to t, where t= thickness of the scintillator, we have

$$R \propto 1/t^{\frac{1}{2}}. \quad (3)$$

It should be noted that since the Landau-Symon effect is approximately Gaussian in this region, the only significant parameter is the full width. Table I shows the theoretical and experimental full widths for the plastic scintillators used.

The errors quoted in Table I were determined as follows. In column three the errors were estimated graphically from the accuracy of drawing a smooth curve through the experimental points. For column four, and lines one and two of column five, the errors quoted represent the uncertainty in the slope of the straight lines which could be drawn through the experimental points shown in Fig. 2. For lines three and four of column five the errors were calculated by using error formulas appropriate to equations three and four.

PROPORTIONAL COUNTER EXPERIMENT

The experimental arrangement is the same as in Fig. 1 with the dE/dx scintillation counter replaced by a proportional counter. The construction of the proportional counter is shown in Fig. 3.

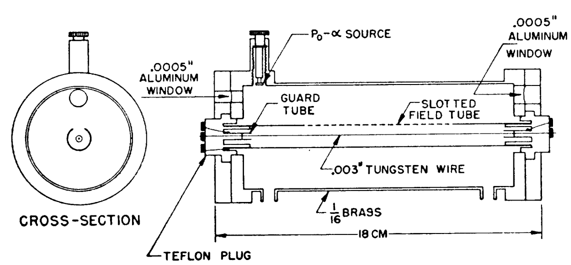

FIG. 3. Schematic cross section of the proportional counter.

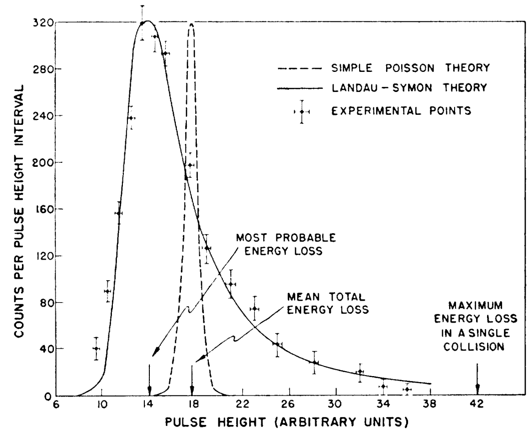

FIG. 4. Frequency distribution of energy losses of 37-Mev protons traversing a 10-cm argon proportional counter at a pressure of 0.2 atmos. The theoretical distribution is calculated from the formulas of Symon (reference 3). The dashed curve is a Poisson distribution based on the statistics of the number of ion pairs formed in track.

The protons traversed the counter parallel to the wire and 2.5 cm away. To minimize end effects in the counter, a system of guard tubes and a field tube was employed.

Background was reduced by using a continuous field tube, slotted as shown in Fig. 3. The field tube was slotted 3 cm from the window to minimize effects of knock-on electrons from the window. The effective length of the counter was 10 cm and was filled with 96% argon and 4% CO_2 to pressures up to 1.2 atmos. When an internal Po α source was used, the resolution

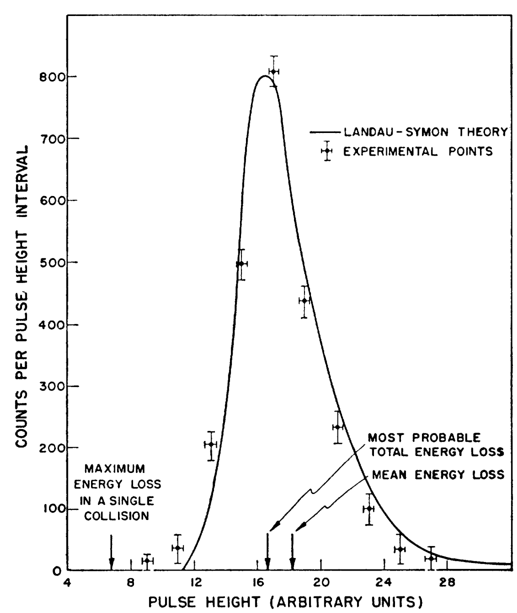

FIG. 5. Frequency distribution of energy losses of 37-Mev protons traversing a 10-cm argon proportional counter at a pressure of 1.2 atmos.

Table II. Theoretical and experimental full widths for various proportional counter pressures.

Proportional counter thickness cm argon	Mean energy loss	% full width due to Landau-Symon effect Theoretical	Experimental
12.0	220 kev	34±3	32.5±3
5.0	85 kev	40±3	45 ±3
2.0	34 kev	45±3	50 ±4

was found to be 13%. Since a part of this is due to the energy spread of the nonuniform thickness α source, the inherent resolution of the counter is better than 13%, and certainly adequate to investigate the observed distributions.

The highest pressure proportional counter measurement was equivalent in terms of absorber thickness to the 0.005-in. plastic scintillator. This affords a check on the shape of the distribution in this region. Figures 4 and 5 show the distribution of pulses produced in the proportional counter in the Landau region and symmetrical region, respectively. Also shown are the theoretical curves calculated from the formulas of Symon, and a Poisson distribution appropriate to the statistical fluctuations in the number of ion pairs formed in the absorber. The Poisson distribution is arbitrarily normalized to the same maximum value as the Landau distribution. Table II shows theoretical and experimental full widths for three proportional counter pressures. The errors quoted in this table for the measured percent full widths were estimated graphically from the accuracy of drawing a smooth curve through the experimental points.

CONCLUSIONS

The frequency distribution of energy losses of 37-Mev protons was found to be in good agreement, both in shape and full width, with the predictions of Landau and Symon, for absorber thicknesses from 34 kev to 2 Mev.

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Principle of Minimum Entropy Production*

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Prigogine has shown that in the steady state in which certain macroscopic affinities $F_1, F_2, \cdots F_k$ are fixed and other macroscopic affinities $F_{k+1}, F_{k+2}, \cdots F_r$ are unconstrained, the values assumed by the unconstrained affinities are such as to minimize the rate of production of entropy. We here show that the complete *microscopic* density matrix of the system is that which minimizes the rate of entropy production subject to the imposed constraints. All magnetic fields are assumed to be zero.

It is shown that the kinetic coefficients connecting Casimer's α-type and β-type variables always vanish. The validity of the minimum entropy production theorem in the absence of a magnetic field depends on this fact. The limitations on the validity of the minimum entropy production theorem in the presence of a magnetic field are briefly discussed.

Calculations on particular models by Klein and Meijer and by Klein corroborate the theorem here proved. An analysis of magnetic resonance by Wangsness suggests certain modifications necessary in this case of a nonzero, nonstationary, magnetic field.

INTRODUCTION

The essential foundation of the theory of irreversible thermodynamics is the Onsager reciprocity theorem.[1] This theorem establishes a symmetry in the mutual interference between two simultaneous, linear, irreversible processes. In particular, if $\{J_k\}$ is a set of "fluxes," and if $\{F_k\}$ is a set of associated affinities so defined that the rate of entropy production $\dot{S}$ is

$$\dot{S}=\sum_k F_k J_k, \tag{1}$$

and if the phenomenological equations between fluxes and affinities are linear

$$J_k=\sum_i L_{ik} F_i, \tag{2}$$

then the Onsager theorem states that

$$L_{ik}(H)=L_{ki}(-H). \tag{3}$$

Here H signifies a magnetic field.

Attempts have been made to rephrase the Onsager

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[1] For general references see S. R. deGroot, *Thermodynamics of Irreversible Processes* (Interscience Publishers Inc., New York, 1951).