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Effect of Variable Ionic Mobility on Ambipolar Diffusion

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Recent measurements of the mobility of atomic rare gas ions in their parent gases have shown that the mobility falls with increasing field strength, becoming inversely proportional to the square root of the field at high fields. This paper presents the solution to the ambipolar diffusion problem for cylindrical geometry and with the observed field dependence of ionic mobility. Results for He^+ ions in He indicate that ambipolar diffusion in a low-pressure positive column may be reduced by a factor of two or more from that predicted by using constant mobility. The theory is also applicable to Ne^+ ions in Ne and to A^+ ions in A.

ONE of the basic assumptions made in developing the theory of ambipolar diffusion is that the ionic mobility is constant. Recent measurements of the mobilities of atomic rare gas ions in their parent gases[1,2] have shown that the mobility falls with increasing electric field strength, becoming inversely proportional to the square root of the field at high fields. This paper will present the solution to the ambipolar diffusion problem in the case of field-dependent ionic mobility. Cylindrical geometry has been used, but the method of solution is adaptable to other geometries. The theory has been worked out numerically for an analytical approximation to the field variation of the mobility which is applicable in the case of atomic rare gas ions in their parent gas, e.g., He^+ in He. The theory is then extended to the calculation of the diffusion loss from the positive column of a discharge for which the axial field is appreciable.

THEORY

The particle current density $\mathbf{\Gamma}_-$ for the electrons moving in a gas under the action of a concentration gradient and a space charge field $\mathbf{E}$ is

$$\mathbf{\Gamma}_- = -D_-\nabla n_- - \mu_-\mathbf{E}n_-; \qquad (1)$$

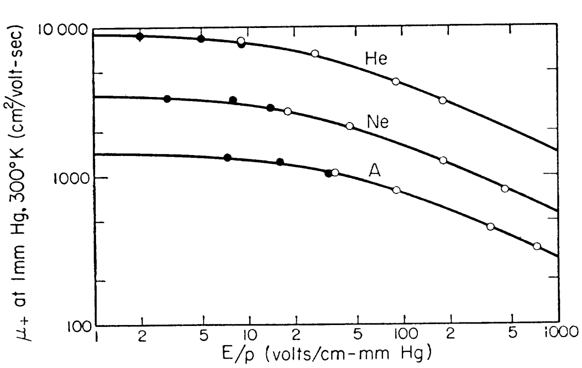

FIG. 1. Ion mobility *vs* E/p. Solid circles from reference 2; open circles from reference 1; curves represent Eq. (14) with μ_0 and a as given in Table I.

[1] J. A. Hornbeck, Phys. Rev. **84**, 615 (1951).
[2] M. A. Biondi and L. M. Chanin, Phys. Rev. **94**, 910 (1954).

the particle current density $\mathbf{\Gamma}_+$ for the ions is

$$\mathbf{\Gamma}_+ = -D_+\nabla n_+ + \mu_+\mathbf{E}n_+. \qquad (2)$$

Here n_-, n_+ are the electron and ion densities; D_-, D_+ are the diffusion coefficients, and μ_-, μ_+ are the mobilities. For sufficiently large electron densities, the difference density required to maintain the space charge field is small enough so that $n_+ - n_- \ll n$, or $n_+ \approx n_- = n$. Since no net current can continuously flow to the insulating tube walls, $\mathbf{\Gamma}_- = \mathbf{\Gamma}_+ = \mathbf{\Gamma}$. Eliminating $\mathbf{\Gamma}$ from Eqs. (1) and (2) gives the electric field as

$$\mathbf{E} = -\left(\frac{D_- - D_+}{\mu_- + \mu_+}\right)\frac{\nabla n}{n}. \qquad (3)$$

In most, if not all, gases $\mu_- \gg \mu_+$ and $D_- \gg D_+$. Thus

TABLE I. Values of the parameters μ_0 and a of Eq. (14).

Ion and gas		μ_0 (cm²/volt sec)	a (mm-cm/volt)
He^+	He	9200	0.040
Ne^+	Ne	3500	0.040
A^+	A	1460	0.0264

Eq. (3) becomes

$$\mathbf{E} = -(D_-/\mu_-)\nabla n/n = -V_e\nabla n/n, \qquad (4)$$

where V_e is defined as the characteristic energy of the electrons, D_-/μ_-. Integrating this equation, with the boundary conditions $\mathbf{E}=0$, $V=0$ and $n=n_0$ at $r=0$, we find that the electrons have a Boltzmann distribution in space with characteristic energy V_e;

$$n = n_0 e^{-V/V_e}. \qquad (5)$$

Here $-V$ is the electrostatic potential, and $\mathbf{E} = \nabla V$.

When one substitutes Eq. (4) into Eq. (2), the particle current density is given by

$$\mathbf{\Gamma} = -(\mu_+ V_e + D_+)\nabla n. \qquad (6)$$

In an active discharge $V_e \gg D_+/\mu_+$, so that we may neglect the ion diffusion current in comparison with the ion mobility current, i.e.,

$$\mathbf{\Gamma} \approx -\mu_+ V_e \nabla n = \mu_+ \mathbf{E} n. \qquad (7)$$

The remaining boundary conditions are that $n=0$ at the wall, $r=R$, and by Eq. (7), that the drift velocity, $\mu_+\mathbf{E}$, be infinite at $r=R$. This singularity results from the assumption of equal ion and electron densities. In the actual case this assumption is not valid near the wall and a space charge sheath forms.[3] Our assumptions are equivalent to the requirement that the densities of electron and ions be high enough so that the thickness of the sheath is negligible.

If the rate of ionization is directly proportional to the electron density, and if diffusion is the only loss mechanism for the electrons and ions, then the continuity equation may be written as

$$\nu n-\partial n/\partial t=\nabla\cdot\mathbf{\Gamma}, \tag{8}$$

where ν is the ionization rate per electron. Defining a diffusion loss rate per electron, β, by the equation

$$\beta n=\nabla\cdot\mathbf{\Gamma}, \tag{9}$$

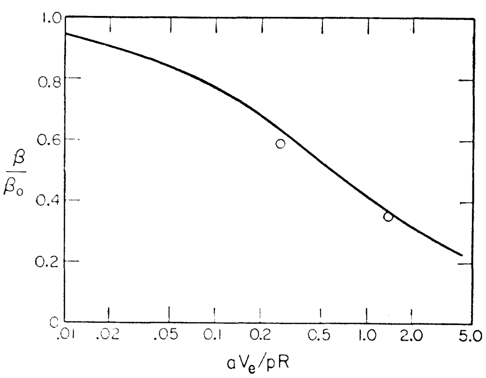

FIG. 2. Relative diffusion rate *vs* the discharge parameter aV_e/pR. The curve is for zero axial field; the open circles are points calculated including an axial field as specified in the text.

we have in a steady discharge $\beta=\nu$, and in an afterglow decaying exponentially with a time constant τ, $\beta=1/\tau$. The use of the diffusion loss rate per electron, β, makes the present theory and results applicable to both cases. Previous work[4] has shown that in the case of mobility, constant at its zero-field value, $\mu_+=\mu_0$, the solution of Eq. (9) gives

$$\beta_0=\mu_0V_e(2.405/R)^2, \tag{10}$$

for a long cylinder of radius R.

Combining Eqs. (5), (7), and (9) yields, for cylindrical geometry,

$$\beta e^{-V/V_e}=-\frac{1}{r}\frac{d}{dr}(r\mu_+Ee^{-V/V_e}). \tag{11}$$

[3] W. P. Allis and D. J. Rose, Phys. Rev. **93**, 84 (1954).

[4] A. von Engel and M. Steenbeck, *Elektrische Gasentladungen* (Verlag Julius Springer, Berlin, 1934), Vol. 2, p. 82 ff; L. B. Loeb, *Basic Processes of Gaseous Electronics* (University of California, Berkeley, 1955), p. 507.

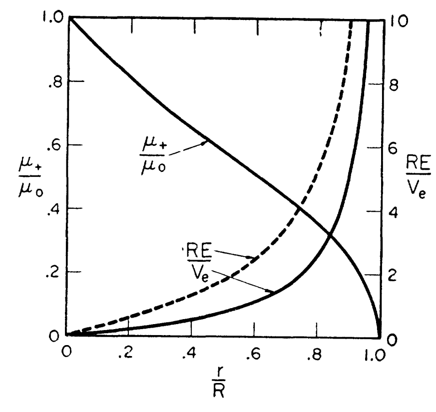

FIG. 3. Radial field and mobility *vs* radial distance. The dashed curve is for the classical case of constant ion mobility; the solid curves are for field-dependent ion mobility with $aV_e/pR=2.63$.

Introducing the dimensionless variables,

$$\rho\equiv(\beta/\beta_0)^{\frac{1}{2}}2.405r/R\equiv\rho_wr/R,\quad \eta\equiv V/V_e,$$
$$\epsilon\equiv d\eta/d\rho=RE/\rho_wV_e, \tag{12}$$

where ρ_w is the value of ρ at the wall, and performing the indicated differentiation, Eq. (11) becomes

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\epsilon\frac{\mu_+}{\mu_0}\right)=1+\epsilon^2\frac{\mu_+}{\mu_0}. \tag{13}$$

Since the ion mobility at any point in the tube is assumed to be a function only of the field at that point, we can express μ_+/μ_0 as a function of E or of the dimensionless field variable, ϵ. Once this has been done, Eq. (13) can be integrated to find the value of ρ_w, at which $\mu_+\epsilon$ becomes infinite as required by the boundary conditions at $r=R$.

Recent data for helium, neon, and argon[1,2] show that within the accuracy indicated by Fig. 1,

$$\mu_+=\mu_0[1+a(E/p)]^{-\frac{1}{2}}, \tag{14}$$

where a and μ_0 are constants given in Table I. In terms of our dimensionless field variable,

$$\mu_+/\mu_0=(1+A\epsilon)^{-\frac{1}{2}}, \tag{15}$$

where $A=aV_e\rho_w/pR$.

The results of the integration of Eq. (13) using Eq. (15) are shown in Fig. 2. Here $\beta/\beta_0=(\rho_w/2.405)^2$ is plotted as a function of $A/\rho_w=aV_e/pR$, so as to make the results applicable to any gas for which Eq. (14) is a satisfactory approximation. As aV_e/pR increases, the reduction in ion mobility with field is increasingly effective, resulting in a decrease in the relative rate of ion loss, β/β_0. It is found that at the largest value of aV_e/pR considered, the diffusion loss is reduced by a factor of four from the value for constant mobility.

Figures 3 and 4 show the effect of a variable ion mobility on the electric field and on the spatial variation of the ion and electron densities. The solid curves are

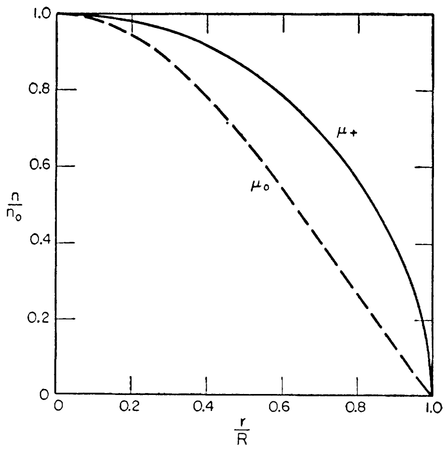

FIG. 4. Radial density distribution curve. The dashed and solid curves correspond to those in Fig. 3.

drawn for the case of $aV_e/pR=2.63$, while the dotted curves show the usual solutions for a constant mobility, i.e., $a=0$. The average electron energy, V_e, the tube radius, R, and the zero field mobility, μ_0, are the same in both cases. Figure 3 shows that the radial field is reduced in the case of variable ion mobility. The variation of ion mobility with radius is also shown. The electron density, shown in Fig. 4, is everywhere greater in the case of variable ion mobility. Calculations show that the product $n\mu_+$ is identical in the two cases to within 26%, so that by Eq. (7), the particle currents, and hence the diffusion rates, are approximately in the ratio of the radial electric fields.

SOLUTION FOR POSITIVE COLUMN

In a positive column there is an applied axial field as well as the radial space charge field. In a uniform positive column the axial field, E_a, is independent of radius and its effect on the radial diffusion can be taken into account by substituting the resultant field, $(E^2+E_a^2)^{1/2}$, into Eq. (14). Equation (15) now becomes

$$\mu_+/\mu_0=[1+A(\epsilon^2+\epsilon_a^2)^{1/2}]^{-1/2}, \tag{16}$$

where $\epsilon_a=RE/\rho_w V_e$.

Numerical integration of Eq. (13) using Eq. (16) for μ_+/μ_0, was performed for two cases: $aV_e/pR=0.270$, $E_a/p=1/2a$; and $aV_e/pR=1.40$, $E_a/p=1/a$. The results are shown in Fig. 2 as open circles. For the former case, it is seen that inclusion of the axial field reduces β/β_0 about 7% below the value calculated without axial field. We estimate that for axial fields satisfying the empirical inequality, $E_a/p\leqslant a^{-1}(aV_e/pR)^{1/2}$, the reduction in the diffusion loss rate due to the axial field is limited to about 10%, i.e., $\beta(E_a)/\beta_0\geqslant 0.90\beta(0)/\beta_0$.

DISCUSSION

The calculation of the ambipolar diffusion loss of electrons and positive ions for the case in which the ion mobility decreases with increasing electric field strength shows that the loss rate per electron may well be a factor of three lower than the rate calculated assuming the ion mobility to be constant at the zero field value. Furthermore, the calculations show that for moderate axial fields the low diffusion rate is essentially unchanged from the value obtained neglecting the axial field.

As an example of the application of this theory to a practical case, we shall consider a helium discharge operating at 0.5 mm and 20 ma in a tube of 1-cm diameter. For these conditions our experiments show that $V_e=6$ volts, so that $aV_e/pR=0.86$; from Fig. 2, we see that the diffusion loss rate is reduced to $0.44\beta_0$ by the action of the radial field on the ion mobility. The measurements give $E_a/p=17$ volts/cm-mm Hg, while the limiting value of the preceding section is 23; we therefore conclude that the existing diffusion loss rate in this case is between $0.40\beta_0$ and $0.44\beta_0$.

In this calculation we have completely neglected the space charge sheaths studied by Allis and Rose.[3] Approximate calculations show that the effects of these sheaths are important for the lower currents used in low pressure helium discharges, i.e., less than 5 ma in the case cited above. Accordingly, the results obtained in this paper should be used with caution until the complete theory has been worked out.

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