Isotopic Spin and Antinucleon-Nucleon Scattering*

B. J. MALENKA† AND H. PRIMAKOFF
Washington University, St. Louis, Missouri
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The total isotopic spin operator and its square are determined for the quantized nucleon-antinucleon field. In the customary configuration space representation, it is found that in the formula for the total isotopic spin of a system of nucleons and antinucleons, the antinucleons contribute terms with the transposed negatives of the $\boldsymbol{\tau}$ operators replacing the corresponding $\boldsymbol{\tau}$'s. The form that has been usually anticipated is found only after a suitable canonical transformation. The results are specifically applied to a two-particle system consisting of a nucleon and an antinucleon.

Expressions are also obtained for the isotopic spin dependence of the ratio of elastic charge exchange to elastic noncharge exchange scattering of an antiproton by a proton, and, of the ratio of elastic noncharge exchange scattering of an antiproton by a neutron and by a proton. These are discussed and evaluated under certain simplifying assumptions.

I. INTRODUCTION

IN a recent article,[1] Lepore has remarked that if the eigenstates in a certain representation of the third component of the isotopic spin operator, τ_3, for the proton and neutron are associated with eigenvalues $+1$ and -1, respectively,[2] invariance under electric charge conjugation requires that the antiproton and antineutron eigenstates in this same representation also be assigned the eigenvalues $+1$ and -1.[3,4] According to Lepore [see reference 1, Eqs. (13) and (14)], this result implies that "the rule for composition of isotopic spins is $T_3=T_3$(particles)$-T_3$(antiparticles)," rather than the intuitively expected "$T_3=T_3$(particles)$+T_3$(antiparticles)." Unfortunately, this last statement turns out to be incorrect. As shown below, the appropriate "rule for composition" of isotopic spins is indeed: $T_3=T_3$(particles)$+T_3$(antiparticles). The difficulty arises from Lepore's failure to establish first the relation between the physically meaningful total isotopic spin operator $\mathbf{T}=(T_1,T_2,T_3)$ and the no more than mathematically convenient "isotopic spin" operators $\boldsymbol{\tau}$. To do this requires a direct examination of a system of many nucleons and antinucleons.

In the present note, we will determine the isotopic spin characteristics of a system of nucleons and antinucleons on the basis of a treatment of the associated quantized nucleon-antinucleon field from the point of view of hole theory. Here, the total isotopic spin of the nucleon-antinucleon system is interpreted in terms of the eigenvalues, or equivalently, the expectation values over the eigenstates, of a suitably chosen quantized field operator. From the expressions obtained, it is then possible to find the appropriate form of the total isotopic spin operators in any particular configuration space representation.

As an application, our results are used in a simple phenomenological investigation of the isotopic spin properties of the scattering amplitude for elastic antiproton-nucleon scattering.

II. ISOTOPIC SPIN OPERATORS FOR THE QUANTIZED FIELD[5]

For the quantized nucleon-antinucleon field, it will be recalled that the operator for the total charge (in units of the proton charge) is given by

$$Q_{\mathrm{op}}=\tfrac{1}{2}\int[\bar{\psi}(\mathbf{r})\gamma_0,\,\tfrac{1}{2}(1+\tau_3)\psi(\mathbf{r})](d\mathbf{r}),\qquad(1)$$

where $\psi(\mathbf{r})$ is the Dirac nucleon quantized field amplitude operator and $\bar{\psi}(\mathbf{r})$ its Pauli adjoint. The commutator has been introduced to symmetrize the total charge operator so that its vacuum expectation value vanishes.

Only charged nucleons can contribute to Q_{op} so that the operator $\frac{1}{2}(1+\tau_3)$, with eigenvalues 1 and 0 for proton and neutron single-particle states, respectively, acts as a projection operator selecting out the proton states. Since $\psi(\mathbf{r})$ and $\bar{\psi}(\mathbf{r})$ are expanded in terms of the complete single-particle energy spectrum, these eigenvalues of $\frac{1}{2}(1+\tau_3)$ refer to protons and neutrons in negative- as well as positive-energy states. Thus, the $\frac{1}{2}(1+\tau_3)$ and so the τ_3 eigenvalues associated with a particular negative-energy state, say, a negative-energy proton state, a hole in which is interpreted as the

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† Permanent address: Department of Physics, Tufts University, Medford, Massachusetts. This author also gratefully acknowledges the hospitality of the Harvard Cyclotron Laboratory during the completion of part of this work.

[1] J. V. Lepore, Phys. Rev. **101**, 1206 (1956).

[2] The configuration space isobaric spin representation used is that of W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York, 1946), Chap. 1.

[3] Lepore, using a single-particle treatment for the nucleon, antinucleon, bases this assignment upon the demonstration that the usual Dirac C, and not $C\tau_1$, is the required charge conjugation operator for the nucleon to antinucleon transformation. In this connection, it should be recalled that in the earlier work of Pais and Jost (reference 4), it has been shown that $C\tau_1$, (their CT) is to be interpreted as the product of the operators of the charge conjugation and the charge symmetry transformations; thus $C\tau_1$, cannot be expected to describe the effects of electric charge conjugation alone.

[4] A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952).

[5] This problem has also been considered by M. Suguwara, Bull. Am. Phys. Soc. Ser. II, **1**, 304 (1956), and by J. Hamada and M. Suguwara, Progr. Theoret. Phys. Japan **8**, 256 (1952).

corresponding antiproton, are the same as the corresponding eigenvalues for a positive-energy proton state.

The form of the total charge operator suggests the appropriate (and in fact, the only consistent) definition for the total isotopic spin operator of the quantized nucleon-antinucleon field:

$$\mathbf{T}_{\text{op}}=\tfrac{1}{2}\int[\bar{\psi}(\mathbf{r})\gamma_0,\tfrac{1}{2}\boldsymbol{\tau}\psi(\mathbf{r})](d\mathbf{r}). \tag{2}$$

This definition forms the basis of the subsequent discussion.

It is advantageous to obtain an alternate expression for $\mathbf{T}_{\text{op}}$ by expanding the quantized field amplitude operators $\psi(\mathbf{r})$ and $\bar{\psi}(\mathbf{r})$ in terms of a complete set of orthonormal single particle eigenstates $\psi_{\kappa\tau}(\mathbf{r})$, $\bar{\psi}_{k\tau}(\mathbf{r})$ where

$$\begin{aligned}\psi(\mathbf{r})&=\psi^{(+)}(\mathbf{r})+\psi^{(-)}(\mathbf{r})\\&=\textstyle\sum_{\kappa\tau}a_{k\tau}\psi_{k\tau}(\mathbf{r})+\sum'_{\kappa\tau}b_{k\tau}{}^{\dagger}\psi_{k\tau}(\mathbf{r}),\\ \bar{\psi}(\mathbf{r})&=\bar{\psi}^{(-)}(r)+\bar{\psi}^{(+)}(r)\\&=\textstyle\sum_{\kappa\tau}a_{\kappa\tau}{}^{\dagger}\bar{\psi}_{k\tau}(\mathbf{r})+\sum'_{k\tau}b_{k\tau}\bar{\psi}_{k\tau}(\mathbf{r}),\end{aligned} \tag{3}$$

and $\sum_{k\tau}$ and $\sum'_{k\tau}$ refer to the respective sums over the positive- and negative-energy single-particle nucleon states $k\tau$. The subscript k denotes all quantum numbers other than isotopic spin while the notation $\tau=p,n$ corresponding to $\tau_3=1,\,-1$ for a nucleon state and $\tau=\bar{p},\bar{n}$ corresponding to $\tau_3=1,\,-1$ for an antinucleon state is also employed. As usual, the $a_{k\tau}{}^{\dagger}$, $a_{k\tau}$ and $b_{k\tau}{}^{\dagger}$, $b_{k\tau}$ are interpreted as the creation, annihilation operators in the indicated states for a nucleon and an antinucleon respectively. Substituting the expansions of Eq. (3) into Eqs. (2) and (1), we have

$$\begin{aligned}\mathbf{T}_{\text{op}}&=\textstyle\sum_{k\tau\tau'}a_{k\tau}{}^{\dagger}a_{k\tau'}(\tau|\tfrac{1}{2}\boldsymbol{\tau}|\tau')_k\\&\qquad-\textstyle\sum'_{k\tau\tau'}b_{k\tau}{}^{\dagger}b_{k\tau'}(\tau'|\tfrac{1}{2}\boldsymbol{\tau}|\tau)_k\\&=\textstyle\sum_{k\tau\tau'}a_{k\tau}{}^{\dagger}a_{k\tau'}(\tau|\tfrac{1}{2}\boldsymbol{\tau}|\tau')_k\\&\qquad-\textstyle\sum'_{k\tau\tau'}b_{k\tau}{}^{\dagger}b_{k\tau'}(\tau|\tfrac{1}{2}\boldsymbol{\tau}^T|\tau')_k,\end{aligned} \tag{4}$$

$$\begin{aligned}Q_{\text{op}}&=[\text{same as in Eq. (4) but with }\tfrac{1}{2}\boldsymbol{\tau},\tfrac{1}{2}\boldsymbol{\tau}^T\text{ replaced by}\\&\qquad\tfrac{1}{2}(1+\boldsymbol{\tau}_3),\tfrac{1}{2}(1+\boldsymbol{\tau}_3{}^T)=\tfrac{1}{2}(1+\boldsymbol{\tau}_3)].\\&=-\tfrac{1}{2}\textstyle\sum_{k\tau}a_{k\tau}{}^{\dagger}a_{k\tau}-\tfrac{1}{2}\sum'_{k\tau}b_{k\tau}{}^{\dagger}b_{k\tau}+(T_{\text{op}})_3,\end{aligned} \tag{5}$$

where the matrix elements involve just the isotopic spin parts of $\psi_{k\tau}(\mathbf{r})$, $\psi_{k\tau'}(\mathbf{r})$ and their Pauli adjoints, and where $\boldsymbol{\tau}^T$ is the transpose of $\boldsymbol{\tau}$.[6]

[6] Equation (3) can also be written in terms of positive-energy antinucleon single-particle states $\psi_{k\tau}{}^{\text{anti}}$ related to the negative-energy nucleon single-particle states $\psi_{k\tau}$ by $\psi_{k\tau}{}^{\text{anti}}=C\psi_{k\tau}$. With use of the $\psi_{k\tau}{}^{\text{anti}}$, $\mathbf{T}_{\text{op}}$ takes on the same form as in Eq. (4), but with $(\tau|\tfrac{1}{2}\boldsymbol{\tau}^T|\tau')_k$ replaced by $(\tau|C\tfrac{1}{2}\boldsymbol{\tau}^TC^{-1}|\tau')_k$. However, according to Pais and Jost (reference 4), invariance under charge conjugation of the interaction Lagrangian of the nucleon-antinucleon and (π) meson fields demands that C be independent of the $\boldsymbol{\tau}$ so that $C\boldsymbol{\tau}^TC^{-1}=\boldsymbol{\tau}^T$ immediately above. Thus, Eq. (4) is again obtained.

The operator corresponding to the square of the total isotopic spin can next be defined as

$$\mathbf{T}_{\text{op}}{}^2=\mathbf{T}_{\text{op}}\cdot\mathbf{T}_{\text{op}}, \tag{6}$$

and can be explicitly expressed in terms of creation and annihilation operators by Eq. (4). It is readily shown by direct calculation that $\mathbf{T}_{\text{op}}{}^2$ commutes with $(T_{\text{op}})_3$ and that both $\mathbf{T}_{\text{op}}{}^2$ and $(T_{\text{op}})_3$ commute with the Hamiltonian operator

$$H_{\text{op}}=\textstyle\sum_{k\tau}E_{k\tau}a_{k\tau}{}^{\dagger}a_{k\tau}+\sum'_{k\tau}|E_{k\tau}|b_{k\tau}{}^{\dagger}b_{k\tau}, \tag{7}$$

where $E_{k\tau}$ is the energy eigenvalue associated with $\psi_{k\tau}(\mathbf{r})$.

As a first and very simple application of Eqs. (4)–(6), consider a one-particle system consisting of a proton, or a neutron, or an antiproton, or an antineutron described by the eigenstates $\Psi_p=a_{k,p}{}^{\dagger}\Psi_0$, $\Psi_n=a_{k,n}{}^{\dagger}\Psi_0$, $\Psi_{\bar{p}}=b_{k,\bar{p}}{}^{\dagger}\Psi_0$, $\Psi_{\bar{n}}=b_{k,\bar{n}}{}^{\dagger}\Psi_0$, where Ψ_0 is the vacuum eigenstate. Equation (4) then gives

$$(T_{\text{op}})_3\begin{bmatrix}\Psi_p\\\Psi_n\\\Psi_{\bar{p}}\\\Psi_{\bar{n}}\end{bmatrix}=\begin{bmatrix}+\frac{1}{2}\Psi_p\\-\frac{1}{2}\Psi_n\\-\frac{1}{2}\Psi_{\bar{p}}\\+\frac{1}{2}\Psi_{\bar{n}}\end{bmatrix}, \tag{8}$$

while the eigenvalue of $\mathbf{T}_{\text{op}}{}^2$ for all four cases is $\frac{3}{4}$. It is to be noted that our $(T_{\text{op}})_3$ eigenvalues for the antiproton and the antineutron eigenstates, $-1/2$ and $+1/2$, coincide with the third component of isotopic spin "values" which are assigned to the antiproton and the antineutron in discussions of elementary particle transformations and interactions.

Consider now the above results as applied to the two-particle system. The eigenstate of the system when both particles are nucleons is given by

$$\Psi_{N,N'}=a_{k\tau}{}^{\dagger}a_{k'\tau'}{}^{\dagger}\Psi_0;$$

while in the case that one particle is a nucleon and the other an antinucleon, the eigenstate is given by $\Psi_{N,\bar{N}}=a_{k\tau}{}^{\dagger}b_{k'\tau'}{}^{\dagger}\Psi_0$. The eigenvalues of $\mathbf{T}_{\text{op}}{}^2$ and $(T_{\text{op}})_3$ or, for convenience in calculation, the equivalent expectation values over the associated eigenstates, are determined in the subspace of the $\Psi_{N,N'}$, $\Psi_{N,\bar{N}}$, by use of Eqs. (4), (6):

Eigenstates of $\mathbf{T}_{\text{op}}{}^2$, $(T_{\text{op}})_3$	$\langle\mathbf{T}_{\text{op}}{}^2\rangle$	$\langle(T_{\text{op}})_3\rangle$	
$\Psi_{N,N';1,1}=\Psi_{p,p'}$:	2	1	(9)
$\Psi_{N,N';1,0}=2^{-\frac{1}{2}}(\Psi_{p,n'}+\Psi_{n,p'})$:	2	0	
$\Psi_{N,N';1,-1}=\Psi_{n,n'}$:	2	−1	
$\Psi_{N,N';0,0}=2^{-\frac{1}{2}}(\Psi_{p,n'}-\Psi_{n,p'})$:	0	0	
$\Psi_{N,\bar{N};1,1}=\Psi_{p,\bar{n}}$:	2	1	(10)
$\Psi_{N,\bar{N};1,0}=2^{-\frac{1}{2}}(\Psi_{p,\bar{p}}-\Psi_{n,\bar{n}})$:	2	0	
$\Psi_{N,\bar{N};1,-1}=\Psi_{n,\bar{p}}$:	2	−1	
$\Psi_{N,\bar{N};0,0}=2^{-\frac{1}{2}}(\Psi_{p,\bar{p}}+\Psi_{n,\bar{n}})$:	0	0	

As anticipated, we observe, both in the N,N' and $N,\bar{N}$ cases, that $\langle \mathbf{T}_{op}^2 \rangle = \{\mathbf{T}_{op}^2\}_{eigenvalue} = T(T+1)$ with $T=1$, 0, corresponding to the isotopic triplet and singlet states of a two-particle system. However, we also observe by comparing Eqs. (9) and (10) that the simple symmetry relations whereby the triplet state is symmetric and the singlet is antisymmetric under isotopic spin exchange, no longer hold in the usual sense for the $N,\bar{N}$ case. It is of some interest therefore to consider how the above results appear in the physically more vivid configuration-space description, and how they are related to our more intuitive ideas about isotopic spin.

III. ISOTOPIC SPIN OPERATORS IN CONFIGURATION SPACE

From Eq. (4), it is readily verified that the appropriate form for the total isotopic spin operator in configuration space for m nucleons and μ antinucleons is given by

$$\mathbf{T} = \tfrac{1}{2}\Big[\sum_{k=1}^{m} \boldsymbol{\tau}^k - \sum_{\kappa=1}^{\mu} \boldsymbol{\tau}^{T\kappa}\Big], \tag{11}$$

so that

$$\mathbf{T}^2 = \tfrac{1}{4}\Big[3m+3\mu+2\sum_{k>l}^{m} \boldsymbol{\tau}^k\cdot\boldsymbol{\tau}^l + 2\sum_{\kappa>\lambda}^{\mu} \boldsymbol{\tau}^{T\kappa}\cdot\boldsymbol{\tau}^{T\lambda} - 2\sum_{k}^{m}\sum_{\kappa}^{\mu} \boldsymbol{\tau}^k\cdot\boldsymbol{\tau}^{T\kappa}\Big], \tag{12}$$

where we have used the Greek letters to distinguish the antinucleons. The principal characteristic of these relations is that the isotopic spin operator $\boldsymbol{\tau}$ is replaced by $-\boldsymbol{\tau}^T$ in the usual addition formula for $\mathbf{T}$ when referring to an antinucleon. If we take the third component of $\mathbf{T}$ in Eq. (11), we immediately obtain the expression

$$T_3 = \tfrac{1}{2}\sum_{k=1}^{m} \tau_3^k - \tfrac{1}{2}\sum_{\kappa=1}^{\mu} \tau_3^\kappa, \tag{13}$$

since $\tau_3^T = \tau_3$. In the present configuration space representation, Eq. (13) unambiguously expresses the relation between the T_3 operator and the τ_3 operators. We see that for a system consisting of only m nucleons, $T_3(\text{particles}) = \frac{1}{2}\sum_{k=1}^{m} \tau_3^k$ whereas for a system consisting of only μ antinucleons, $T_3(\text{antiparticles}) = -\frac{1}{2}\sum_{\kappa=1}^{\mu} \tau_3^\kappa$, so that corresponding to Eq. (13), Lepore's "rule of composition" mentioned above should be corrected to read $T_3 = T_3(\text{particles}) + T_3(\text{antiparticles})$ as intuitively expected.

The isotopic spin properties in configuration space of the one- and two-particle nucleon systems are well known and will not be discussed. However, for the one-particle antinucleon and the two-particle nucleon-antinucleon systems, the above equations reduce to

$$T_3 = -\tfrac{1}{2}\tau_3^T = -\tfrac{1}{2}\tau_3, \tag{14a}$$

$$\mathbf{T}^2 = \tfrac{3}{4}, \tag{14b}$$

and

$$T_3 = \tfrac{1}{2}[\tau_3^a - \tau_3^{T\alpha}] = \tfrac{1}{2}[\tau_3^a - \tau_3^\alpha], \tag{15a}$$

$$\mathbf{T}^2 = \tfrac{1}{2}[3 - \boldsymbol{\tau}^a\cdot\boldsymbol{\tau}^{T\alpha}] = \tfrac{1}{2}[3 - \tau_3^a\tau_3^\alpha - 2\tau_+^a\tau_+^\alpha - 2\tau_-^a\tau_-^\alpha], \tag{15b}$$

respectively, where τ_+ and τ_- are the isotopic spin raising and lowering operators. The a and α superscripts refer to the nucleon and the antinucleon respectively. If we denote the τ_3 eigenstates[2] associated with the individual proton, antiproton by $p(a)$, $\bar{p}(\alpha)$ and with the individual neutron, antineutron by $n(a)$, $\bar{n}(\alpha)$,

$$\left[\text{where } p(a) = \begin{pmatrix}1\\0\end{pmatrix}_a, \quad \bar{p}(\alpha) = \begin{pmatrix}1\\0\end{pmatrix}_\alpha, \quad n(a) = \begin{pmatrix}0\\1\end{pmatrix}_a, \quad \bar{n}(\alpha) = \begin{pmatrix}0\\1\end{pmatrix}_\alpha\right],$$

the isotopic spin eigenvalues and eigenstates of $\mathbf{T}^2$, T_3 for the two-particle nucleon-antinucleon system are given by

Eigenstates of $\mathbf{T}^2$, T_3	$\mathbf{T}^2$	T_3	$\boldsymbol{\tau}^a\cdot\boldsymbol{\tau}^{T\alpha}$
$\chi_{1,1}(a,\alpha) = p(a)\bar{n}(\alpha)$:	2	1	-1
$\chi_{1,0}(a,\alpha) = 2^{-\frac{1}{2}}[p(a)\bar{p}(\alpha) - n(a)\bar{n}(\alpha)]$:	2	0	-1
$\chi_{1,-1}(a,\alpha) = n(a)\bar{p}(\alpha)$:	2	-1	-1
$\chi_{0,0}(a,\alpha) = 2^{-\frac{1}{2}}[p(a)\bar{p}(\alpha) + n(a)\bar{n}(\alpha)]$:	0	0	3

(16)

These eigenvalues and the form of the associated configuration space eigenstates correspond to the expectation values of the $\mathbf{T}_{op}^2$ of Eq. (6) and $(T_{op})_3$ of Eq. (4) with respect to the eigenstates of Eq. (10).

An alternative expression of the above results and one that is probably more in line with our usual intuitive concepts regarding isotopic spin can be obtained by introducing a "transformed" configuration space representation[7] via the canonical transformation

$$S = \prod_{\text{antinucleons}:\,\kappa} (i\tau_2^\kappa), \tag{17}$$

so that for nucleons

$$\boldsymbol{\tau}^k \rightarrow \{\boldsymbol{\tau}^k\} = S\boldsymbol{\tau}^k S^{-1} = \boldsymbol{\tau}^k, \tag{18}$$

and for antinucleons

$$-\boldsymbol{\tau}^{T\kappa} \rightarrow \{-\boldsymbol{\tau}^{T\kappa}\} = S(-\boldsymbol{\tau}^{T\kappa})S^{-1} = \boldsymbol{\tau}^\kappa \tag{19}$$

where the curly bracket indicates a transformed operator or eigenstate. Under this canonical transfor-

[7] T. D. Lee and C. N. Yang, Nuovo cimento 3, 749 (1956).

mation, Eqs. (11) and (12) for m nucleons and μ antinucleons assume the more anticipated form:

$$\mathbf{T} \to \{\mathbf{T}\} = S\mathbf{T}S^{-1} = \tfrac{1}{2}\sum_{k=1}^{m+\mu} \boldsymbol{\tau}^k, \tag{20}$$

$$\mathbf{T}^2 \to \{\mathbf{T}^2\} = S\mathbf{T}^2S^{-1} = \tfrac{1}{4}[3(m+\mu)+2\sum_{k>l}^{m+\mu} \boldsymbol{\tau}^k\cdot\boldsymbol{\tau}^l]. \tag{21}$$

We note that the transformed version of Eq. (14a) becomes $\{T_3\} = +\frac{1}{2}\tau_3$ so that $\{T_3\}$ for a single antinucleon now bears the same relation to τ_3 as it does for a single nucleon. We further note that the eigenstates, $p(a)$, $n(a)$, are unchanged in form by the canonical transformation S; while on the other hand, under S

$$\bar{p}(\alpha) = \begin{pmatrix}1\\0\end{pmatrix}_\alpha \to \{\bar{p}(\alpha)\} = (i\tau_2)\bar{p}(\alpha) = -\begin{pmatrix}0\\1\end{pmatrix}_\alpha,$$
$$\bar{n}(\alpha) = \begin{pmatrix}0\\1\end{pmatrix}_\alpha \to \{\bar{n}(\alpha)\} = (i\tau_2)\bar{n}(\alpha) = \begin{pmatrix}1\\0\end{pmatrix}_\alpha. \tag{22}$$

In the "transformed" representation, it is also seen that the $\mathbf{T}^2$, T_3 eigenstates of Eq. (16) take on a form exhibiting isotopic spin exchange symmetry properties analogous to those of the $\mathbf{T}^2$, T_3 eigenstates of the more familiar two-nucleon system. For example,

$$\chi_{0,0}(a,\alpha) = 2^{-\frac{1}{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_a\begin{pmatrix}1\\0\end{pmatrix}_\alpha + \begin{pmatrix}0\\1\end{pmatrix}_a\begin{pmatrix}0\\1\end{pmatrix}_\alpha\right] \to$$
$$\{\chi_{0,0}(a,\alpha)\} = -2^{-\frac{1}{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_a\begin{pmatrix}0\\1\end{pmatrix}_\alpha - \begin{pmatrix}0\\1\end{pmatrix}_a\begin{pmatrix}1\\0\end{pmatrix}_\alpha\right],$$

so that $\{\chi_{0,0}(a,\alpha)\}$ now appears in a form antisymmetric under the exchange of the isotopic spin coordinates of a and α in the same manner as the isotopic singlet eigenstate of the two-nucleon system.

Because of the similarity of description in the "transformed" representation of the isotopic spin of an antinucleon and the ordinary spin of a positron, experience with electron-positron systems indicates that the "transformed" representation should be particularly convenient for nucleon-antinucleon systems in problems involving configuration space treatment of conservation laws and selection rules.[7] However, in quantized field theoretic calculations on nucleon-antinucleon systems, e.g., in those involving computation of scattering transition probabilities, there appears to be no obvious advantage in employing the "transformed" representation in any passage to a configuration space description. This point is specifically demonstrated in the example considered below.

IV. ELASTIC ANTINUCLEON-NUCLEON SCATTERING

In principle, the cross section for the elastic scattering of an antinucleon by a nucleon can be calculated from the charge independent interaction of the quantized (π) meson and nucleon-antinucleon fields:

$$g\tfrac{1}{2}[\bar{\psi},\Theta_N\boldsymbol{\tau}\psi]\cdot(\Theta_\pi\boldsymbol{\phi}), \tag{23}$$

where, e.g., $\Theta_N=\gamma_5$, $\Theta_\pi=1$, or $\Theta_N=\gamma_5\gamma_\mu$, $\Theta_\pi=\partial_\mu$. Thus, the scattering transition operator can only be composed of terms of the form

$$(\bar{\psi}^{(+)}A\psi^{(-)})(\bar{\psi}^{(-)}B\psi^{(+)}), \tag{24a}$$

$$(\bar{\psi}^{(+)}C\boldsymbol{\tau}\psi^{(-)})\cdot(\bar{\psi}^{(-)}D\boldsymbol{\tau}\psi^{(+)}), \tag{24b}$$

$$(\bar{\psi}^{(-)}A'\psi^{(-)})(\bar{\psi}^{(+)}B'\psi^{(+)}), \tag{24c}$$

$$(\bar{\psi}^{(-)}C'\boldsymbol{\tau}\psi^{(-)})\cdot(\bar{\psi}^{(+)}D'\boldsymbol{\tau}\psi^{(+)}), \tag{24d}$$

where A, B, $\cdots D'$ are functionals of the operators Θ_N, Θ_π. The matrix elements of these terms between initial and final states of the type considered in Eq. (10), then leads to an expression for the scattering amplitude,[8] $(N_f\bar{N}_f|\mathcal{T}|N_i\bar{N}_i)$. Here,

$$\mathcal{T} = \alpha_{fi}M(1)+\beta_{fi}M(\boldsymbol{\tau})+\alpha_{fi}'N(1)+\beta_{fi}'N(\boldsymbol{\tau}) \tag{25}$$

is an operator in isotopic spin configuration space with the numerical coefficients α_{fi}, β_{fi}, α_{fi}', α_{fi}' depending on the nature of the nucleon-meson coupling in Eq. (23), i.e., on g, Θ_N, Θ_π, and on the spins, energies, and momenta of the nucleon and antinucleon in the initial (i) and final (f) states. The operators $M(1)$, $M(\boldsymbol{\tau})$, $N(1)$, $N(\boldsymbol{\tau})$ are defined via the relations

$$(N_f\bar{N}_f|M(1)|N_i\bar{N}_i) = (\bar{N}_i|1|\bar{N}_f)(N_f|1|N_1) = (\bar{N}_f|1|\bar{N}_i)(N_f|1|N_i), \tag{26a}$$

$$(N_f\bar{N}_f|M(\boldsymbol{\tau})|N_i\bar{N}_i) = (\bar{N}_i|\boldsymbol{\tau}|\bar{N}_f)\cdot(N_f|\boldsymbol{\tau}|N_i) = (\bar{N}_f|\boldsymbol{\tau}^T|\bar{N}_i)\cdot(N_f|\boldsymbol{\tau}|N_i), \tag{26b}$$

$$(N_f\bar{N}_f|N(1)|N_i\bar{N}_i) = (N_f|1|\bar{N}_f)(\bar{N}_i|1|N_i) = (\bar{N}_f|1|N_f)(\bar{N}_i|1|N_i), \tag{26c}$$

$$(N_f\bar{N}_f|N(\boldsymbol{\tau})|N_i\bar{N}_i) = (N_f|\boldsymbol{\tau}|\bar{N}_f)\cdot(\bar{N}_i|\boldsymbol{\tau}|N_i) = (\bar{N}_f|\boldsymbol{\tau}^T|N_f)\cdot(\bar{N}_i|\boldsymbol{\tau}|N_i). \tag{26d}$$

The matrix elements of 1, $\boldsymbol{\tau}$, and $\boldsymbol{\tau}^T$ in Eq. (26) are evaluated in Table I, and, using the eigenstates of Eq. (16), it is easily verified that in the N, $\bar{N}$ isotopic spin configuration space, these matrix elements of Eq. (26) may be reproduced by assigning operators to $M(1)\cdots$, $N(\boldsymbol{\tau})$ as follows:

$$M(1)=1, \tag{27a}$$

$$M(\boldsymbol{\tau})=3-2\mathbf{T}^2, \tag{27b}$$

$$N(1)=2-\mathbf{T}^2, \tag{27c}$$

$$N(\boldsymbol{\tau})=\mathbf{T}^2. \tag{27d}$$

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[8] Initial and final states of the type of Eq. (10), must be here interpreted as referring to the presence of two noninteracting physical or dressed particles, i.e., a physical nucleon and a physical antinucleon. Similarly, the $\bar{\psi}^{(-)}(\mathbf{r})$, $\psi^{(-)}(\mathbf{r})$, $\psi^{(+)}(\mathbf{r})$, $\bar{\psi}^{(+)}(\mathbf{r})$ in Eq. (24) are to be viewed as quantized field creation, annihilation operators for physical nucleons and antinucleons at $\mathbf{r}$. On the other hand, the interaction (23) refers to bare particles and the unrenormalized g.

TABLE I. The calculated values of the matrix elements defined in Eq. (26).

$N_i\bar{N}_i \to N_f\bar{N}_f$	$(N_f\bar{N}_f\|M(1)\|N_i\bar{N}_i)$	$(N_f\bar{N}_f\|M(\boldsymbol{\tau})\|N_i\bar{N}_i)$	$(N_f\bar{N}_f\|N(1)\|N_i\bar{N}_i)$	$(N_f\bar{N}_f\|N(\boldsymbol{\tau})\|N_i\bar{N}_i)$
$p_i\bar{n}_i \to p_f\bar{n}_f$	1	−1	0	2
$n_i\bar{p}_i \to n_f\bar{p}_f$	1	−1	0	2
$p_i\bar{p}_i \to p_f\bar{p}_f$, $n_i\bar{n}_i \to n_f\bar{n}_f$	1	1	1	1
$p_i\bar{p}_i \to n_f\bar{n}_f$, $n_i\bar{n}_i \to p_f\bar{p}_f$	0	2	1	−1
$2^{-\frac{1}{2}}(p_i\bar{p}_i-n_i\bar{n}_i) \to 2^{-\frac{1}{2}}(p_f\bar{p}_f-n_f\bar{n}_f)$	1	−1	0	2
$2^{-\frac{1}{2}}(p_i\bar{p}_i+n_i\bar{n}_i) \to 2^{-\frac{1}{2}}(p_f\bar{p}_f+n_f\bar{n}_f)$	1	3	2	0

Thus, $\mathcal{T}$ may be written in the N, $\bar{N}$ isotopic spin configuration space as

$$\mathcal{T}=(\alpha_{fi}+3\beta_{fi}+2\alpha_{fi}')-(2\beta_{fi}+\alpha_{fi}'-\beta_{fi}')\mathbf{T}^2, \quad (28)$$

or alternatively from Eq. (15)

$$\mathcal{T}=\tfrac{1}{2}(2\alpha_{fi}+\alpha_{fi}'+3\beta_{fi}')+\tfrac{1}{2}(2\beta_{fi}+\alpha_{fi}'-\beta_{fi}')\boldsymbol{\tau}^N\cdot\boldsymbol{\tau}^{TN}. \quad (29)$$

The forms of $\mathcal{T}$ in Eqs. (28) or (29) now show that the nucleon-antinucleon elastic scattering process conserves the total isotopic spin. This conservation law is, of course, a special case of the conservation of the total isotopic spin of the nucleon-antinucleon-pion system with a charge-independent interaction of the kind given in Eq. (23).[9]

In view of the current Berkeley Bevatron experiments with antiprotons, it is of interest to examine the expressions for some cross sections immediately deducible from the $\mathcal{T}$ of Eqs. (28) or (29) and the eigenstates of Eq. (16). In particular, we may obtain the scattering amplitudes corresponding to the elastic antiproton-proton scattering without and with charge exchange, and elastic antiproton-proton scattering. These are given by

$$(p_f\bar{p}_f|\mathcal{T}|p_i\bar{p}_i)=\alpha_{fi}+\beta_{fi}+\alpha_{fi}'+\beta_{fi}', \quad (30)$$

$$(n_f\bar{n}_f|\mathcal{T}|p_i\bar{p}_i)=2\beta_{fi}+\alpha_{fi}'-\beta_{fi}', \quad (31)$$

$$(n_f\bar{p}_f|\mathcal{T}|n_i\bar{p}_i)=\alpha_{fi}-\beta_{fi}+2\beta_{fi}'. \quad (32)$$

The above amplitudes suffice to express the following ratio of cross sections

$$R_{\bar{n}\bar{p}}=\frac{\sigma(\bar{p}p\to\bar{n}n)}{\sigma(\bar{p}p\to\bar{p}p)}=\frac{\sum_{fi}|2\beta_{fi}+\alpha_{fi}'-\beta_{fi}'|^2}{\sum_{fi}|\alpha_{fi}+\beta_{fi}+\alpha_{fi}'+\beta_{fi}'|^2}, \quad (33)$$

$$R_{n\bar{p}}=\frac{\sigma(\bar{p}n\to\bar{p}n)}{\sigma(\bar{p}p\to\bar{p}p)}=\frac{\sum_{fi}|\alpha_{fi}-\beta_{fi}+2\beta_{fi}'|^2}{\sum_{fi}|\alpha_{fi}+\beta_{fi}+\alpha_{fi}'+\beta_{fi}'|^2}, \quad (34)$$

where the $\sum_{fi}\cdots$ runs over all the experimentally indistinguishable initial and final states. In general, the magnitudes of $R_{\bar{n}\bar{p}}$ and $R_{n\bar{p}}$ will depend on the values of all of the α_{fi}, β_{fi}, α_{fi}', β_{fi}' coefficients. However, there are some features of current theories of the pion-nucleon-antinucleon interaction that may lead to simple results for $R_{\bar{n}\bar{p}}$ and $R_{n\bar{p}}$. For example, if at the scattering energies in question, a lowest order calculation is sufficient to describe the antinucleon-nucleon interaction with reasonable accuracy, then effectively the α_{fi} and α_{fi}' are zero since to order g^2 the interaction (23) can only give rise to terms with $M(\boldsymbol{\tau})$ and $N(\boldsymbol{\tau})$ in the $\mathcal{T}$ of Eq. (25). The cross-section ratios then would reduce to

$$R_{\bar{n}\bar{p}}\approx\frac{\sum_{fi}|2\beta_{fi}-\beta_{fi}'|^2}{\sum_{fi}|\beta_{fi}+\beta_{fi}'|^2}, \quad (35)$$

$$R_{n\bar{p}}\approx\frac{\sum_{fi}|-\beta_{fi}+2\beta_{fi}'|^2}{\sum_{fi}|\beta_{fi}+\beta_{fi}'|^2}. \quad (36)$$

To order g^2, the term in $\mathcal{T}$ with $M(\boldsymbol{\tau})[N(\boldsymbol{\tau})]$ and hence the $\beta_{fi}[\beta_{fi}']$ corresponds to the so-called nonannihilation [annihilation] "force" illustrated in Fig. 1(a) [1(b)]. The above ratios give estimates of their relative importance. These ratios will depend critically on the character of the nucleon-meson coupling. For instance, a pure γ_5 coupling can only connect the "large" and "small" components of the $\bar{\psi}_{k\tau}(\mathbf{r})$, $\psi_{k'\tau'}(\mathbf{r})$, so that to order g^2, $\beta_{fi}=0$ and only the annihilation "force" is present.[10] The cross-section ratios then reduce to the simple numerical values

$$R_{\bar{n}\bar{p}}\approx 1, \quad R_{n\bar{p}}\approx 4. \quad (37)$$

Of course, if the higher order terms in g^2 are important, the γ_5 theory would require that we use the more general forms of Eqs. (33) and (34).

Another possibility that would simplify the results is that the hypothesis of "(virtual) nucleon pair suppression" approximately applies to the antinucleon-nucleon interaction at the scattering energies in question so that the nonannihilation "force" is predominant. Then, the important contribution to the scattering amplitude would come from terms in Eq. (24) generated by an effective interaction $\frac{1}{2}g[\bar{\psi},\sigma_i\boldsymbol{\tau}\psi]\cdot\partial_i\boldsymbol{\phi}$ which does not connect the "large" and "small" components of the $\bar{\psi}_{\kappa\tau}(\mathbf{r})$, $\psi_{k'\tau'}(\mathbf{r})$; here one would have $\alpha_{fi}'\ll\alpha_{fi}$ and

[9] H. A. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, 1955), Vol. II, pp. 55 ff.

[10] A calculation of the cross section in the γ_5 theory for antinucleon-nucleon scattering to order g^2 in the scattering amplitude has been made by K. A. Johnson, Phys. Rev. **96**, 1659 (1954). The factor multiplying the cross section in his Eq. (17) is given as 1 for $T_3=\pm1$ and 2 for $T_3=0$. According to our Eq. (27d), it should be corrected to read 2 for $T=1$, $T_3=\pm1$, 0, and 0 for $T=0$, $T_3=0$.

$\beta_{fi}'\ll\beta_{fi}$ and the cross-section ratios become

$$R_{\bar{n}\bar{p}}\approx\sum_{fi}|2\beta_{fi}|^2/\sum_{fi}|\alpha_{fi}+\beta_{fi}|^2, \tag{38}$$

$$R_{n\bar{p}}\approx\sum_{fi}|\alpha_{fi}-\beta_{fi}|^2/\sum_{fi}|\alpha_{fi}+\beta_{fi}|^2. \tag{39}$$

In this case, if again only the lowest order term in g^2 is important so that $\alpha_{fi}\ll\beta_{fi}$, we would find that

$$R_{\bar{n}\bar{p}}\approx 4, \quad R_{n\bar{p}}\approx 1. \tag{40}$$

It should be noted that the limiting results for $R_{\bar{n}\bar{p}}$, $R_{n\bar{p}}$ in Eqs. (37), (40) are particularly simple. If future experimental determinations of these ratios should tend to either of the limiting values, we would have a possible indication of the dominance in the nucleon-antinucleon interaction at the relevant scattering energies of the lowest order annihilation or nonannihilation "forces."

Note added in proof.—Certain conclusions, similar to the foregoing, have recently been obtained by D. Amati and B. Vitale [Nuovo cimento **4**, 145 (1956)]. In order to compare our results and theirs, we note that the various nucleon-antinucleon elastic scattering amplitudes, $(N_f\bar{N}_f|\mathcal{T}|N_i\bar{N}_i)$, may be expressed in terms of the scattering amplitudes, $a^{(1)}{}_{fi}$, $a^{(0)}{}_{fi}$, associated with the nucleon-antinucleon isotopic triplet and singlet states. Thus Eqs. (16), (28), and Eqs. (30)–(32) yield (with $t=1, 0, -1$),

$$\begin{aligned}a^{(1)}{}_{fi}&=([\chi_{1,t}]_f|\mathcal{T}|[\chi_{1,t}]_i)=(\alpha_{fi}-\beta_{fi}+2\beta'_{fi})\\&=(n_f\bar{p}_f|\mathcal{T}|n_i\bar{p}_i)=(p_f\bar{p}_f|\mathcal{T}|p_i\bar{p}_i)-(n_f\bar{n}_f|\mathcal{T}|p_i\bar{p}_i)\end{aligned} \tag{41a}$$

and

$$\begin{aligned}a^{(0)}{}_{fi}&=([\chi_{0,0}]_f|\mathcal{T}|[\chi_{0,0}]_i)=(\alpha_{fi}+3\beta_{fi}+2\alpha'_{fi})\\&=(p_f\bar{p}_f|\mathcal{T}|p_i\bar{p}_i)+(n_f\bar{n}_f|\mathcal{T}|p_i\bar{p}_i)\end{aligned} \tag{41b}$$

so that the various cross sections become:

$$\begin{aligned}\sigma(\bar{p}p\to\bar{p}p)&=\tfrac{1}{4}b\sum_{fi}|a^{(0)}{}_{fi}+a^{(1)}{}_{fi}|^2\\\sigma(\bar{p}p\to\bar{n}n)&=\tfrac{1}{4}b\sum_{fi}|a^{(0)}{}_{fi}-a^{(1)}{}_{fi}|^2\\\sigma(\bar{p}n\to\bar{p}n)&=b\sum_{fi}|a^{(1)}{}_{fi}|^2\end{aligned} \tag{42}$$

where b is the appropriate common factor ($b=1$ with suitable normalization). The formulas for the cross sections in Eq. (42) are seen to be identical with those

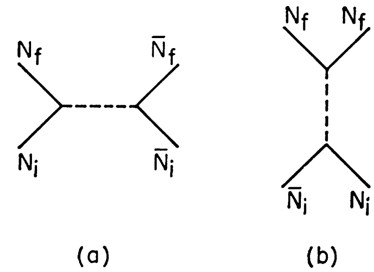

FIG. 1. Feynman diagrams for (a) nonannihilation and (b) annihilation antinucleon-nucleon scattering in lowest order.

in Eqs. (33), (34) when the $a^{(0)}{}_{fi}$, $a^{(1)}{}_{fi}$ are expressed [via Eq. (41)] in terms of the α_{fi}, β_{fi}, α'_{fi}, β'_{fi}.

Equations (35)–(37) and Eqs. (38)–(40) show that the approximation of dominance of the lowest order annihilation or nonannihilation "forces" which corresponds to $\beta'_{fi}\gg\beta_{fi}$, α_{fi}, α'_{fi}, or, to $\beta_{fi}\gg\beta'_{fi}$, α_{fi}, α'_{fi}, from Eq. (41), also corresponds to $a^{(1)}{}_{fi}\gg a^{(0)}{}_{fi}$, or, to $a^{(1)}{}_{fi}\approx-\frac{1}{3}a^{(0)}{}_{fi}$. Again, I. Pomeranchuk [Soviet Phys. JETP **3**, 306 (1956); J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 423 (1956)] has recently given an argument which indicates that at very high energies: $|a^{(1)}{}_{fi}-a^{(0)}{}_{fi}|\ll|a^{(1)}{}_{fi}|$. It is also worth mentioning that Eq. (42) predicts that the various cross sections must be restricted by:

$$\begin{aligned}\sigma(\bar{p}p\to\bar{p}p)+\sigma(\bar{p}p\to\bar{n}n)&=\tfrac{1}{2}\{\sigma(\bar{p}n\to\bar{p}n)+b\sum_{fi}|a^{(0)}{}_{fi}|^2\}\\&\geqq\tfrac{1}{2}\sigma(\bar{p}n\to\bar{p}n)\end{aligned} \tag{43}$$

where, as in the relations in Eqs. (33), (34), (42), the relation in Eq. (43) is a direct consequence of the conservation of total isotopic spin. Equation (43) thus constitutes a restriction, arising from charge independence, on corresponding experimental values of $\sigma(\bar{p}p\to\bar{p}p)$, $\sigma(\bar{p}p\to\bar{n}n)$, $\sigma(\bar{p}n\to\bar{p}n)$. This restriction is reminiscent of an analogous restriction, also arising from charge independence, which has been derived in the theory of nucleon-nucleon elastic scattering [by B. A. Jacobsohn, Phys. Rev. **89**, 881 (1953)]:

$$\begin{aligned}\sigma(np\to np)&+\sigma(np\to pn)\\&=\tfrac{1}{2}\{\sigma(pp\to pp)+b\sum_{fi}|[a^{(0)}{}_{fi}]_{np}|^2\}\\&\geqq\tfrac{1}{2}\sigma(pp\to pp).\end{aligned} \tag{44}$$