Microwave Determination of the Probability of Collision of Electrons in Neon†

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The theory for computing the microwave conductivity of a plasma discussed in the previous paper can be used for determining the collision probability for momentum transfer of slow electrons when the conductivity ratio of a plasma is measured as a function of the applied field. The conditions required for having a pressure independent conductivity ratio and steady state electron energy for a given field are investigated. The convenience of using two independent fields for measuring the conductivity and changing the electron energy is shown. Conductivity ratios measured in the afterglow of a pulsed discharge in a microwave resonant cavity are given for neon and neon contaminated with argon. The collision probability for momentum transfer in neon computed from these data as a function of the electron velocity joins Ramsauer's and Kollath's measurements for higher electron velocities.

INTRODUCTION

In a recent paper Gould and Brown[1] used a microwave method for determining the probability of collision for momentum transfer of slow electrons in helium by measuring the microwave conductivity of a plasma as a function of the electron energy. The same method has been used for neon and the results will be discussed in this paper.

We shall use the theory for the microwave conductivity of a plasma we discussed in the previous paper[2] which from now will be indicated as (I); the same symbols are used. In (I) we found that for a plasma in a quartz bottle centered in a microwave cavity the average conductivity ratio $\langle\rho\rangle$ in steady-state is given by (in this paper w means w_1):

$$\langle\rho\rangle=\omega_0\Delta(1/Q_L)/2p_0\Delta\omega_0$$
$$=\sum_{l=0}^{\infty}[\Gamma(5/2+l)/\Gamma(5/2)](1+R_l)b_l(2\bar{w}/3)^l, \quad (1)$$

where $\bar{w}$ and R_l are, respectively, a space average electron energy and a correction factor for the nonuniform field; they are given in (I) as Eqs. (28) and (44). The quantities b_l are the coefficients of an expansion of the electron collision frequency ν_m as powers of the electron energy [see Eq. (11) in (I)]. When we change the amplitude of the applied field, we change the average electron energy $\bar{w}$; if we represent the measured $\langle\rho\rangle$ *versus* $\bar{w}$ curve as an expansion in powers of $\bar{w}$ we can expect to find the coefficients b_l from (1), comparing the two expansions and from them to compute $\nu_m(v)$ or the collision probability $P_m(v)=p_0\nu_m v$.

This can be done only if R_l is independent of $\bar{w}$, as for the uniform field case. Nevertheless, the fact that R_l depends on $\bar{w}$ is not a very serious difficulty, because R_l is only a correction factor and a rapidly converging trial and error method can be used. An analogous difficulty arises because $\bar{w}$ and R_l depend on the unknown collision frequency through the parameters h and β. According to Figs. 2 and 3 of (I) we could practically ignore these dependences if $d/L<0.2$, but in this range the effect of the plasma would be (in our experimental conditions) too small for a good measurement of $\Delta(1/Q_L)$. Therefore, we used a bigger quartz bottle and found another way to limit the difficulties. The dependence on h does not present a serious obstacle if we use a trial and error method; the dependence on β is of a more difficult nature. The parameter β is proportional to the square of the mean free path, which is inversely proportional to the pressure; this means that $\langle\rho\rangle$ is also pressure dependent. This fact complicates the discussion of the experimental data, but we shall show that we can avoid this difficulty if we work in a convenient pressure range.

In the zero-pressure-limit formula (1) gives a certain expression for the conductivity ratio; the zero pressure limit corresponds to $\beta\rightarrow\infty$, $\Delta_y w$, $\Delta_z w$, c_y, $c_z\rightarrow 0$ or, in other words, a uniform electron energy is found throughout the bottle. We can determine a range of pressures from zero to a maximum value p_m in which $\langle\rho\rangle$ does not differ from the zero-pressure value of more than the experimental error we expect in the measurements. In this range $\langle\rho\rangle$ is practically pressure-independent. We have to investigate how the condition $p_0<p_m$ fits with the other limits for pressure. The range in which the theory given in (I) is correct is limited at high pressure by the condition $\nu_m^2\ll\omega^2$ and at low pressure by the conditions of no appreciable diffusion cooling and mean free path less than any dimension of the container. If the experimental conditions are such that p_m falls in this range, we have a region in which we can use the zero-pressure-limit formula and $\langle\rho\rangle$ is pressure-independent. Our conductivity measurements in neon have been performed in this region and from now on we will limit the discussion to this case.

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[1] L. Gould and S. C. Brown, Phys. Rev. 95, 897 (1954).
[2] A. L Gilardini and S. C. Brown, preceding paper [Phys. Rev. 105, 25 (1956)].

We can conclude that by choosing a convenient low-pressure range and by using a trial and error method to account for the dependence of R_l on $\bar{w}$, and of R_l and $\bar{w}$ on h, we can determine the coefficients b_l from the measured conductivity ratio as a function of the applied field. Equation (11) of (I) gives the collision frequency for momentum transfer. Before presenting the use of the method for the specific case of neon we want to discuss how to satisfy experimentally the steady-state energy assumption on which the theory given in (I) is based.

STEADY STATE ENERGY CONDITIONS

Most of the assumptions made in (I) are satisfied in the afterglow of a pulsed neon discharge when measurements are performed at microwave frequencies. However, one of them needs more careful investigation. We have assumed that the energy moments of the distribution function F_0^0 are independent of time, which is equivalent to saying that the electron average energy $\bar{w}$ has reached the steady state value corresponding to the applied field at that moment. We shall derive the conditions the experiment must satisfy to make this assumption acceptable. Some rough approximations are made to simplify the problem.

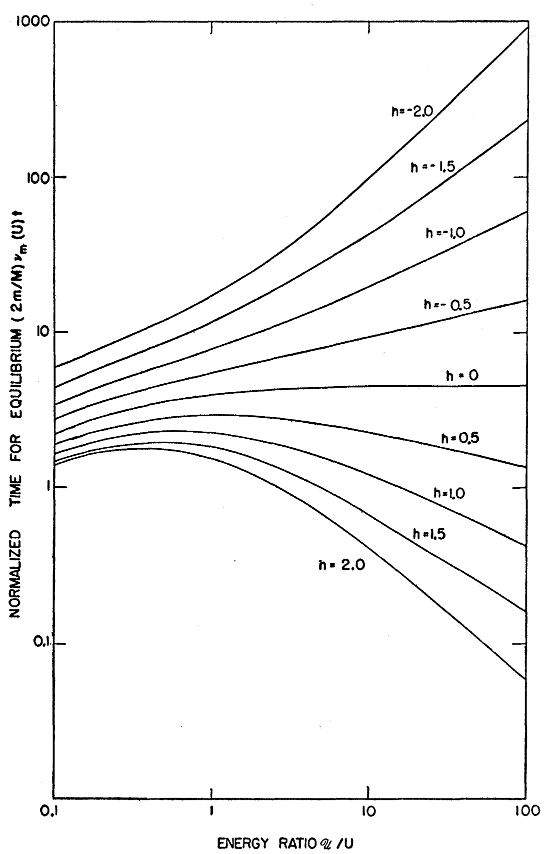

FIG. 1. Time necessary for the electrons to reach the energy 0.99 $(U+\mathfrak{U})$ starting from the gas energy U, plotted *versus* the ratio of energies $\mathfrak{U}/U$ and for various values of h. The time is measured in units of $M/2m\nu_m(U)$.

We assume a decaying plasma infinitely extended and in a uniform time varying microwave field, with no recombination and for which all the assumptions made in (I) are correct; the energy moments of the distribution function now depends on time and not on space. From Eqs. (21) and (24) of (I), for $l=1$, we obtain:

$$dw(t)/dt=[\Gamma(5/2)/\Gamma(5/2-h/2)] \times(2m/M)\nu_m(w)[U+\mathfrak{U}(t)-w(t)], \quad (2)$$

where $\nu_m(w)$ is the collision frequency at the velocity $[4w(t)/3m]^{\frac{1}{2}}$ and $\mathfrak{U}(t)=M[eE(t)/2m\omega]^2$.

A simple case of interest would be when the field is applied at $t=0$ and is constant thereafter; at $t=0$ we assume $w=U$. Solving Eq. (2) for this case we have:

$$(2m/M)\nu_m(U)t=[\Gamma(5/2-h/2)/\Gamma(5/2)] \times\int_1^{w/U}\zeta^{-h/2}(1+\mathfrak{U}/U-\zeta)^{-1}d\zeta, \quad (3)$$

which gives the time t in which the electron energy reaches the value w. In Fig. 1 the quantity $(2m/M)\nu_m(U)t$ for $w=0.99(U+\mathfrak{U})$ has been plotted *versus* $\mathfrak{U}/U$ for different values of the exponent h. From these curves we know how much time must elapse after the field has been applied before the electron energy has reached the steady state value within one percent. Actually, in this specific case the Maxwellian hypothesis used in (I) to compute Eq. (24) is not very satisfactory, and we cannot expect the curves to give more than an approximate value for the time.

Unfortunately our experimental conditions are much more involved since the power absorbed by the cavity changes with time. The field frequency is constant, but the cavity resonant frequency changes due to the change in the electron density during the decay. We measure the conductivity ratio at the moment $t=0$ when these two frequencies coincide. The electron energy at that moment $w(0)$ has to be very near to $U+\mathfrak{U}(0)$, in order to insure the validity of the assumed conditions. To find when this is true we must solve Eq. (2) with the proper law for $\mathfrak{U}(t)$. Assuming the unloaded Q a constant in time, which represents a good approximation to the actual experimental case, we can write the equation for $\mathfrak{U}(t)$ as the equation for the power in a detuned cavity:

$$\mathfrak{U}(t)=\mathfrak{U}(0)/\{1+[2Q\delta\omega_0(t)/\omega]^2\}. \quad (4)$$

Here $\delta\omega_0(t)$ is the detuning, or the difference between the field frequency and the resonant frequency. When the density decay is controlled by ambipolar diffusion, and we neglect higher order diffusion modes, we can approximately express the detuning as:

$$\delta\omega_0(t)=\omega-\omega_0(t)=\Delta\omega_0(0)\left\{1-\exp\left[-\int_0^t\gamma(t)dt\right]\right\}, \quad (5)$$

where $\Delta\omega_0(0)$ is the resonant frequency shift at $t=0$ from the condition without plasma and γ is the density decay constant in a plasma with the electron energy w. For γ we use Eq. (34) of (I), which at the zero-pressure limit becomes:

$$\gamma(t)=(\pi/d)^2(2\mu_+/e)[U+w(t)]. \quad (6)$$

When Eqs. (4), (5), and (6) are substituted into (2), this becomes an integro-differential equation for $w(t)$, which can be written using dimensionless variables in the following form:

$$d\zeta/d\xi+[\Gamma(5/2)/\Gamma(5/2-h/2)]\zeta^{h/2}\left\{\zeta-1-A\left[1+B\left(\exp\left[-C\int_0^\xi(\zeta+1)d\xi\right]-1\right)^2\right]^{-1}\right\}=0, \quad (7)$$

where the dimensionless variables are:

$$\begin{gathered}\xi=(2m/M)\nu_m(U)t, \quad \zeta=w/U,\\ A=\mathfrak{U}(0)/U, \quad B=[2Q\Delta\omega_0(0)/\omega]^2, \quad (8)\\ C=(M/2m)(\sqrt{3}\pi/p_0d)^2(D_+p_0)/[\nu_m(U)/p_0].\end{gathered}$$

We assume also the boundary condition $\zeta=1$ when $t\rightarrow-\infty$, which means energy equilibrium between electrons and gas in the absence of the applied field. We can now restate our problem as follows. Given the range of values of A and C over which measurements have to be made, find the maximum value of B for which $\zeta(0)$ differs from $(1+A)$ of a given fractional amount (we shall assume one percent). This can be solved numerically and in the experiment we shall choose Q and $\Delta\omega_0$ small enough to make B less than this maximum value. For neon one finds that this can be accomplished only by choosing a rather low value for Q, when actually we would like to have a high Q cavity for a good measurement of ρ. This difficulty can be solved[1] by using two independent fields corresponding to two different modes of the cavity: one with a low Q for the purpose of controlling the electron energy (heating mode), the other with a high Q, but much less power than the first, for measuring the conductivity ratio (measuring mode).

Rigorously the discussion just given is not sufficient to insure that the energy steady state is reached in our experiment. An additional condition has to be satisfied, namely that the time constant for energy redistribution due to density and energy gradients is small compared to the time during which the heating field is effective. The time constant for the energy redistribution τ_w can be computed from Eq. (24) of (I) and the result is, in the zero pressure limit:

$$\tau_w=\frac{\Gamma(5/2-h/2)}{5(3-h)\Gamma(5/2-h)}\left(\frac{3d}{2\pi}\right)^2\frac{m\nu_m}{\bar{w}}. \quad (9)$$

This last condition is always well satisfied in our experiment, the time constant being of the order of a microsecond or less.

CONDUCTIVITY RATIO WHEN TWO FIELDS ARE PRESENT

When the power in the measuring mode is much less than the power in the heating mode all the formulas of (I) from (26) to (38) are correct, provided $\mathfrak{U}_c$ is referred to the heating mode. When we average over the cavity, $\langle\rho\rangle$ is still given from (1), but R_l is no longer Eq. (44) of (I).

Here we give the formulas for the average energy $\bar{w}$ and the correction factor R_l in the zero pressure limit, and when the heating mode has the electric field parallel to the x axis and the measuring mode has its electric field parallel to the z axis.

$$\begin{aligned}\bar{w}=U+\{&\Phi(d/L_y)\Phi(d/L_z)+(1-h/5)(3-h)^{-1}\\ &\times[\Psi(d/L_y)\Phi(d/L_z)+\Phi(d/L_y)\Psi(d/L_z)]\}\mathfrak{U}_c, \quad (10)\end{aligned}$$

$$\begin{aligned}\bar{w}R_l=&[\Gamma(5/2)/5(3-h)\Gamma(5/2-h)]\\ &\times\{[(5-h)\delta_l-\eta_l/2-(\eta_l/2)\Theta_1(d/L_y)/\Theta_0(d/L_y)]\\ &\times\Psi(d/L_y)\Phi(d/L_z)+[(5-h)\delta_l-\eta_l/3]\\ &\times\Phi(d/L_y)\Psi(d/L_z)\}\mathfrak{U}_c. \quad (11)\end{aligned}$$

EXPERIMENTAL RESULTS FOR NEON

The method discussed has been used in the determination of the collision probability in neon. The experimental procedure for this kind of measurements has been described in detail in the paper by Gould and Brown[1]; no significant modifications have been introduced. The geometrical dimensions of the cavity and of the bottle are $L_x=6.90$ cm, $L_y=7.51$ cm, $L_z=6.28$ cm, $d=2.82$ cm; the resonant frequency for the heating mode is 3000 Mc and the resonant frequency for the measuring mode is 2840 Mc. The electron densities in the plasma were of the order of 10^8 to 10^9 cm^{-3}.

According to the analysis presented in the introduction, the pressure must be bigger than 1 mm Hg to prevent diffusion cooling[3] and less than the pressure corresponding to the most stringent of the two conditions $\nu_m^2\ll\omega^2$ and $\beta>3$. For a given $\langle\rho\rangle$ and a given $\Delta\omega_0$, the change $\Delta(1/Q_L)$ is proportional to the pressure [formula (1)]; this means that it is better to work at the highest pressures in the allowed range. The measurements have been made at pressures from 13 to 5 mm Hg, the lowest for the highest electron energies. In this pressure range, except at high electron energies, the decay in pure neon is controlled by recombination; we found in (I) that it is difficult to determine the electron density distribution for this case, and we prefer to work when diffusion is controlling the decay. Dissociative recombination is prevented in neon by adding a small amount of argon[4] (from 0.01 to 0.1 percent); this

[3] M. A. Biondi, Phys. Rev. 93, 1136 (1954).
[4] M. A. Biondi, Phys. Rev. 83, 1078 (1951).

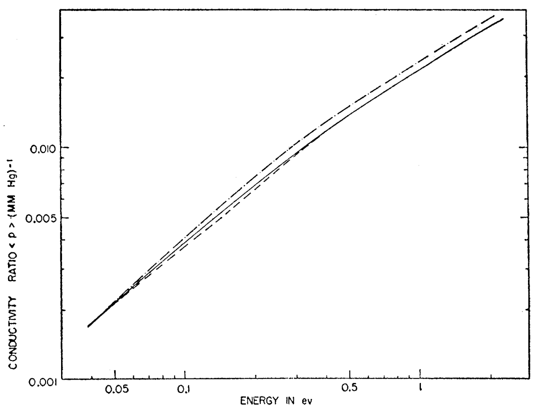

FIG. 2. Average conductivity ratio $\langle\rho\rangle$ as a function of electron energy. Solid line: $\langle\rho\rangle$ *vs* $U+\mathfrak{U}_c$, the electron energy at the center of the cavity in an equivalent uniform plasma, for neon contaminated with argon. Short-dash line: $\langle\rho\rangle$ *vs* $U+\mathfrak{U}_c$ for pure neon. Long-and-short-dash line $\langle\rho\rangle$ *vs* $\bar{w}$, an average electron energy through the cavity. The average scattering of the individual data from the curves is two percent.

amount is so small that no appreciable change in electron collision probability is expected.

Equation (7) has been discussed numerically to determine the conditions for which energy steady state has been reached within one percent at the time of measurements. Having $D_{+}p_0=150$ cm^2 sec^{-1} mm Hg, $C<0.027$ and $1.5<h<1.9$, we found that B must be less than 3 for $1<A<10$, and less than 1.5 for $10<A<50$. Consequently we chose $Q=220$ and $\Delta\omega_0<11$ Mc for the first range, $\Delta\omega_0<8$ Mc for the second one.

The neon used was a commercial "spectroscopically pure" sample obtained from Air Reduction Sales Co. When we improved its purity using the method of cataphoresis[5] we found that $\langle\rho\rangle$ was a function of the post-discharge time, as if the neon metastables excited during the discharge would remain for a long time in the afterglow and by ionization would produce electrons with higher energies than the other plasma electrons. Only without the heating field the effect could be observed.

The measured $\langle\rho\rangle$ is plotted as a function of $U+\mathfrak{U}_c$ in ev in Fig. 2, both for neon and for neon with argon. The average scattering of the individual data from the curves is two percent. The measurements have been performed at post-discharge time larger than 1 millisecond to insure energy equilibrium between electrons and gas in the absence of the applied field. The two curves coincide for high values of $U+\mathfrak{U}_c$, because there diffusion becomes the dominant loss process also in pure neon. At low energies the curve for pure neon deviates from that for neon plus argon as if the density was uniform through the cavity, in agreement with a recombination decay. By applying the method of this paper, the collision probability in neon has been computed from the measurements in neon plus argon. In Fig. 2 an intermediate step is also shown, the $\langle\rho\rangle$ *versus* $\bar{w}$ curve; this curve is stopped at W, which in our case is 2 ev.

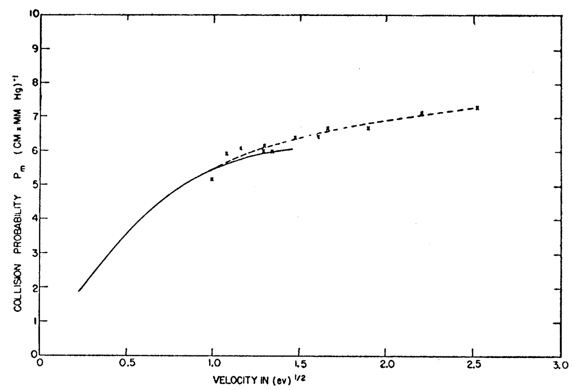

FIG. 3. Electron collision probability for momentum transfer in neon as a function of electron velocity. Solid line: curve determined by the microwave method. Crosses: measurements by Ramsauer and Kollath. Dashed line: average curve through their data.

In Fig. 3 the computed collision probability as a function of electron velocity is shown as a solid curve. The curve has been limited between the velocity for which the integrand $\nu_m v(\partial F_0{}^0/\partial v)$ in the conductivity ratio formula [see Eq. (10) in (I)] is a maximum at $\bar{w}=U$ and the velocity for which the same expression is a maximum at $\bar{w}=W$. The crosses represent the results computed by Barbiere[6] from measurements by Ramsauer and Kollath,[7] using an electron beam method; the dotted line is their average. The agreement between the results obtained with the microwave and the electron beam method can be considered satisfactory.

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[6] D. Barbiere, Phys. Rev. **84**, 653 (1951).

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