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Decay of Charged K Mesons

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The decay of the charged K meson is considered under the assumption of the existence of two parity-conjugate particles with zero spin. The effect of a weak parity-mixing interaction, which may be of environmental or intrinsic origin, is studied. Two limiting cases are examined. If the parity-mixing interaction is weaker than the usual decay interactions, then the θ and τ mesons will appear to have the same lifetime far from their point of production; however, near the point of production a difference in their lifetimes should be discernible. On the other hand, if the parity-mixing interaction is somewhat larger than the decay interactions, only a single lifetime would occur. Ordinary decay experiments will not serve to distinguish between the latter case and the existence of a single kind of K meson which decays without parity conservation.

I. INTRODUCTION

THE present experimental evidence concerning K mesons indicates the existence of two particles with zero spin, equal masses and lifetimes, but opposite parity. The equality of masses of two particles with different parity can be understood in terms of the parity conjugation symmetry suggested by Lee and Yang.[1] In general, this symmetry will not yield equal lifetimes (though this result could occur if there is an accidental numerical relationship between the coupling constants governing the decay phenomena).

Recently, Lee and Yang[2] have investigated the possibility that parity is not conserved in the weak decays. They have shown that experiments up to now are not sensitive enough to test this question.[3] If parity is not conserved in the weak interactions, then the data on the K-particle lifetimes could be understood in terms of a single K meson. However, certain strong interactions of the K meson can best be viewed in terms of a $\bar{K}K\pi$ interaction. As has been pointed out by Goldhaber[4] and Schwinger,[5] the angular distributions in the production of K^+ and K^0 and the angular distribution of scattered K^+ mesons can be understood qualitatively in terms of such an interaction. Furthermore, such a coupling would give a K-nucleon potential of sufficiently long range to account perhaps for the K hyperfragments.[6] The pseudoscalar nature of the pion field requires the existence of two K-mesons of even and odd parity (θ and τ particles) for a $\bar{K}K\pi$ structure to appear in the Lagrangian. If such an interaction does indeed exist, it is necessary to invoke the parity conjugation symmetry mentioned above in order that the θ and τ masses be equal.[7]

It is the purpose of this note to investigate what is to be expected in the decay schemes if two parity-conjugate particles exist, and if there is also a weak interaction which mixes parity. If this interaction is weaker than the usual decay interactions, then the θ and τ could appear to have the same lifetime far from their point of production; however, near the point of production a difference in the lifetimes should be discernible. If, on the other hand, the parity-mixing interaction is larger than the decay interactions (but still quite small), only a single lifetime would appear. In the latter case it would be impossible from the study of decays only to determine whether there is one meson or two.

II. ANALYSIS

We begin by considering the results concerning decay phenomena that can be obtained from a Wigner-Weisskopf line breadth analysis of the time-dependent Schrödinger equation. We assume that the Hamiltonian can be divided into two parts $H=H_s+H_w$, where H_s represents the strong interactions, and H_w produces the decays involving transitions between the eigenstates of H_s. Owing to parity conjugation, H_s has two degenerate eigenstates of opposite parity representing the θ and τ particles. Because of this symmetry any arbitrary linear combination of θ and τ forms an adequate description of the system. The weak interactions, however, disturb this symmetry and single out the θ and τ states. These states may then be expected to have slightly different energies. We take this effect of H_w into account phenomenologically by writing m_θ and m_τ for the θ and τ masses in H_s. The θ and τ will then decay each with a simple exponential law whose time constants are not expected to be equal. This is true even if the mass degeneracy is not lifted since conservation of parity does not permit matrix elements linking θ and τ.

We consider then the nature of the decay when the

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956).
[2] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[3] Much more sensitive experiments have been suggested by Lee and Yang (see footnote 2) and are now in progress at various laboratories.
[4] M. Goldhaber, Phys. Rev. **101**, 433 (1953).
[5] J. Schwinger, Phys. Rev. **104**, 1164 (1956).
[6] Fry, Schneps, and Swami, Phys. Rev. **99**, 1561 (1955).
[7] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 822 (1956), have suggested some experiments which could possibly give a direct verification of the existence of parity doublets.

Hamiltonian contains a small added contribution H_m which mixes θ and τ. The appropriate eigenstates are then (see Appendix):

$$\psi_a=c_1\theta+c_2\tau,\quad \psi_b=c_1^*\tau-c_2^*\theta,\quad |c_1|^2+|c_2|^2=1. \quad (1)$$

These are the states that decay with a simple exponential law provided the line breadths are smaller than the total energy separation of states a and b. The particles produced by the Bevatron can be described by linear combinations of ψ_a and ψ_b with arbitrary phases and amplitudes. Averaging over these parameters yields a result which is equivalent to the production of equal amounts of the "particles" a and b which subsequently decay in a statistically independent fashion. Thus, if the above criteria for simple exponential time dependence is valid, the usual radioactive parent-daughter decay equations hold. The states a and b decay according to

$$\dot{N}_a=-\lambda_aN_a,\quad \dot{N}_b=-\lambda_bN_b, \quad (2)$$

where N_a, N_b are the number of a, b particles and λ_a, λ_b are their respective lifetimes:

$$\begin{aligned}\lambda_a&=|c_1|^2(\lambda_\theta+\mu_\theta+\nu_\theta)+|c_2|^2(\lambda_\tau+\mu_\tau+\nu_\tau),\\ \lambda_b&=|c_1|^2(\lambda_\tau+\mu_\tau+\nu_\tau)+|c_2|^2(\lambda_\theta+\mu_\theta+\nu_\theta).\end{aligned} \quad (3)$$

Here $\lambda_\theta, \lambda_\tau$ are the transition probabilities for $\theta\rightarrow2\pi$, $\theta\rightarrow3\pi$, respectively; $\mu_{\theta,\tau}$ for θ, $\tau\rightarrow K_{\mu2}$; and $\nu_{\theta,\tau}$ for $\tau,\tau\rightarrow K_{\mu3}$, K_{e3}. The rate of appearance of $K_{\pi2}$ is given by

$$\begin{aligned}\dot{N}_{2\pi}&=\lambda_\theta(|c_1|^2N_a+|c_2|^2N_b)\\ &=\lambda_\theta(|c_1|^2e^{-\lambda_at}+|c_2|^2e^{-\lambda_bt}). \quad (4)\end{aligned}$$

Similarly,

$$\begin{aligned}\dot{N}_{3\pi}&=\lambda_\tau(|c_2|^2e^{-\lambda_at}+|c_1|^2e^{-\lambda_bt}),\\ \dot{N}_{\mu2}&=(|c_1|^2\mu_\theta+|c_2|^2\mu_\tau)e^{-\lambda_at}\\ &\quad+(|c_2|^2\mu_\theta+|c_1|^2\mu_\tau)e^{-\lambda_bt}.\end{aligned} \quad (5)$$

III. COMPARISON WITH EXPERIMENT

For simplicity we consider two limiting cases for comparison with the available data.

Case A: $|c_2|^2/|c_1|^2\ll1$

In this case the parity-mixing interaction is small compared to the decay interactions. We assume here that $1/\lambda_\theta$ is of the order of the θ^0 lifetime as might be expected from phase space considerations. Hence $\lambda_\theta\gg\lambda_\tau$ though it is possible that $|c_2|^2\lambda_\theta\sim|c_1|^2\lambda_\tau$. Under these circumstances $\lambda_b\gg\lambda_a$, and only the b particle will occur in the K-meson beam, accounting for the observed single lifetime $1/\lambda_b$. The a particle will decay near the point of production and predominantly into the $K_{\pi2}$ mode because $|c_1|^2/|c_2|^2$ is large. In this case the $K_{\pi2}$ decay curves would show a compound structure with a break occurring at a time $t\sim[\ln(|c_1|^2/|c_2|^2)]/\lambda_a$ after production. (This mixed exponential behavior in the other decay modes would not be readily observable.) The validity of the lifetime analysis depends upon the smallness of $\lambda/\delta E$; here δE is the energy shift between the a and b levels which in this case comes predominantly from the mass shift caused by the decay processes. From the values of λ obtained below, a mass shift of $\sim10^{-4}$ ev would suffice to satisfy this condition. Such an order of magnitude does not seem unreasonable though, of course, practically nothing is understood or known about this quantity.

From the experimental lifetime $1/\lambda_b=1.2\times10^{-8}$ sec and the abundance ratios,[8] information on the values of λ_θ, λ_τ, etc., may be obtained:

$$\lambda_\tau=0.83\times10^7\ \text{sec}^{-1},\qquad (|c_2|^2/|c_1|^2)\lambda_\theta=1.74\times10^7\ \text{sec}^{-1},$$
$$\mu_\tau=5.4\times10^7\ \text{sec}^{-1},\qquad \nu_\tau=0.33\times10^7\ \text{sec}^{-1}.$$

Here we have assumed that $\mu_\theta\sim\mu_\tau$ and $\nu_\theta\sim\nu_\tau$ and have used the fact that $|c_1|^2\simeq1$ for this case (see Appendix). Taking $\lambda_\theta\sim10^{10}$ sec^{-1} (as suggested by the θ^0 lifetime) yields $|c_2|^2/|c_1|^2\sim2\times10^{-3}$; thus the smallness of this parameter is verified. With these numbers one obtains $\lambda_a\cong\lambda_\theta\sim10^{10}$ sec^{-1}.[9]

The value obtained for μ_τ is in agreement with that to be expected from the universal axial vector coupling for $\mu+\nu$ decays suggested by Bludman and Ruderman.[10] From the results obtained in the Appendix for the special form of H_m chosen there, the coupling constant g_m appearing in H_m is given by

$$g_m=\frac{4\delta m}{m}\left|\frac{c_2}{c_1}\right|\sim\frac{1}{30}g_F^2, \quad (6)$$

where we have used $\delta m\sim10^{-4}$ ev, and $g_F^2\sim10^{-12}$ is the usual Fermi weak decay constant. The axial vector coupling constant is also smaller than g_F. It is conceivable that these two effects have a common origin.

The assumption of parity conjugation invariance implies the existence of two Λ^0 particles of opposite parity, Λ_1 and Λ_2. From phase space considerations, one would expect Λ_1 and Λ_2 to have comparable partial lifetimes and hence Λ^0 should show a compound decay curve. Presumably there should soon exist enough experimental data to clarify this point. It should be noted, however, that if the Λ_1, say, decays through a scalar coupling while the Λ_2 decays through a γ_5 coupling, then the partial lifetime of the Λ_2 will be ~200 times smaller than that of the Λ_1. Under these circumstances, the situation will be similar to that of the θ and τ decays, though here it is the short lifetime that is being observed experimentally.

[8] Smith, Heckman, and Barkas, University of California Radiation Laboratory Report UCRL-3289 (unpublished).

[9] Some indication exists for the short K^+ lifetime in cloud-chamber work: Arnold, Balam, and Reynolds, Phys. Rev. **100**, 295 (1955); see also G. H. Trilling and R. B. Leighton, Phys. Rev. **100**, 1468 (1955).

[10] S. A. Bludman and M. A. Ruderman, Phys. Rev. **101**, 910 (1956).

Case B: $|c_2|^2/|c_1|^2 \cong 1$

This case requires a parity-mixing interaction somewhat bigger than the weak interactions. In particular, as shown in the Appendix, $|c_2|^2 \cong |c_1|^2$ holds provided that $\frac{1}{4}g_m\delta m/m \gg 1$. Under these circumstances the criteria for the validity of the lifetime analysis, $\lambda/\delta E \sim \lambda/g_m m \ll 1$, is satisfied. The states ψ_a and ψ_b whose decays obey simple exponential laws are now given by

$$\psi_a = (\theta+\tau)/\sqrt{2}, \quad \psi_b = (\theta-\tau)/\sqrt{2}. \tag{7}$$

It follows from Eq. (3) that $\lambda_a = \lambda_b = \frac{1}{2}(\lambda_\theta+\mu_\theta+\nu_\theta+\lambda_\tau+\mu_\tau+\nu_\tau)$ so that only one lifetime exists. From the experimentally observed abundance ratios one finds in this case that $\lambda_\tau = 1.7\times10^7$ sec^{-1}, $\lambda_\theta = 3.5\times10^7$ sec^{-1}, $\mu_\tau+\mu_\theta = 10.8\times10^7$ sec^{-1}, and $\nu_\tau+\nu_\theta = 0.7\times10^7$ sec^{-1}.

The value of λ_θ obtained here is considerably smaller than that expected from phase space considerations. Such a result would be compatible with the selection rule $\Delta T = \pm\frac{1}{2}$ discussed by Wentzel.[11] If $\mu_\tau = \mu_\theta$, this rate is again in agreement with that obtained from the universal axial vector (and equivalent vector) interactions of Bludman and Ruderman.[10]

It should be noted that in this case decay experiments on a beam that is not parity-polarized will yield the same results as would be obtained if there existed a single meson which decays without parity conservation.

IV. DISCUSSION

The considerations of this paper are subject to further experimental tests. For example, the validity of Case A depends strongly on the existence of a short-lived component for the charged K mesons; this matter will be clarified as more events are studied, particularly by means of bubble chambers. The experiments recently suggested by Lee and Yang[7,2] may test the underlying assumptions of this investigation, namely, parity conjugation and parity mixing. These experiments involve an asymmetry in the distribution of the decay products of hyperons with respect to directions occurring in the strong interactions in which the hyperons are produced. The possibility of observing an asymmetry which shows that parity is not conserved is independent of whether or not there are parity conjugate K particles. Similarly, the asymmetry involved in the experiment which would verify the existence of parity doublets may appear whether or not parity is conserved in the weak decays.[12]

If parity mixing does exist, it may be of an environmental or intrinsic nature. At present it is somewhat difficult to see what environmental factors could cause sufficient parity mixing for particles of zero spin. On the other hand, if parity nonconservation is of an intrinsic nature, it may be connected with the weak decay interactions themselves.

[11] G. Wentzel, Phys. Rev. **101**, 1215 (1956).

[12] However, if parity is not conserved, the time dependence of the angular distribution is different from that given by Lee and Yang. In particular, the asymmetry in question may still be observed even if δm is larger than the level width.

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APPENDIX

We consider here the effects of the mixing Hamiltonian H_m on the eigenstates of H_s. H_s has two eigenstates $|\theta)$ and $|\tau)$ with energies $\omega_\theta = (p^2+m_\theta^2)^{\frac{1}{2}}$ and $\omega_\tau = (p^2+m_\tau^2)^{\frac{1}{2}}$ representing the θ and τ mesons. (We have included phenomenologically the mass shifts due to H_w in our formulas.) The states of H_s+H_m [where H_m has matrix elements $H_{\theta\tau}$ between $|\theta)$ and $|\tau)$] can be represented by

$$\psi = c_1\,|\theta)+c_2\,|\tau). \tag{A.1}$$

The Schrödinger equation, $(H_s+H_m)\psi = E\psi$, yields the equations

$$\begin{aligned} c_1(\omega_\theta-E)+c_2H_{\theta\tau}&=0,\\ c_2(\omega_\tau-E)+c_1H_{\tau\theta}&=0. \end{aligned} \tag{A.2}$$

The energy eigenvalues are then found to be

$$E_{a,b} = \tfrac{1}{2}(\omega_\theta+\omega_\tau)\pm[(\delta\omega/2)^2+|H_{\theta\tau}|^2]^{\frac{1}{2}}, \tag{A.3}$$

where $\delta\omega = \omega_\theta-\omega_\tau$ is the energy shift due to H_w, and the plus (minus) sign refers to the state $\psi_a(\psi_b)$. From Eq. (A.2), one easily sees that

$$\left(\frac{c_2}{c_1}\right)_a = -\left(\frac{c_1}{c_2}\right)_b^* = \frac{H_{\tau\theta}}{\frac{1}{2}\delta\omega+[(\delta\omega/2)^2+|H_{\theta\tau}|^2]^{\frac{1}{2}}}, \tag{A.4}$$

while the total energy shift between the two eigenstates of the system is

$$\delta E = E_a-E_b = [\delta\omega^2+4|H_{\theta\tau}|^2]^{\frac{1}{2}}. \tag{A.5}$$

For H_m weak, i.e., $H_{\theta\tau}\ll\delta\omega$, one obtains $(c_2/c_1)_a \cong H_{\tau\theta}/\delta\omega$ while for $H_{\theta\tau}\gg\delta\omega$ one gets $(c_2/c_1)_a \cong H_{\tau\theta}/|H_{\tau\theta}|$ (which is just a pure phase factor).

The parity-mixing interaction H_m can be characterized in a simple way by assuming

$$H_m = \tfrac{1}{2}m_\theta m_\tau g_m\int d^3x[\bar{\theta}(x)\tau(x)+\bar{\tau}(x)\theta(x)], \tag{A.6}$$

where $\theta(x)$ and $\tau(x)$ are the θ and τ field operators, and g_m is the coupling constant. A straightforward calculation yields

$$H_{\theta\tau} = (\theta|H_m|\tau) = \tfrac{1}{4}g_m m_\theta m_\tau(\omega_\theta\omega_\tau)^{-\frac{1}{2}}. \tag{A.7}$$

For the situation where $(c_2/c_1)_a$ is small, this gives

$$\left(\frac{c_2}{c_1}\right)_a = \frac{H_{\tau\theta}}{\delta\omega} \cong \frac{1}{4}\frac{g_m m}{\delta m}, \quad \delta m = m_\theta-m_\tau, \quad m = \tfrac{1}{2}(m_\theta+m_\tau) \tag{A.8}$$

to first order in δm.