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Negative to Positive Ratio from Nonrelativistic Theories of Pion Photoproduction*

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A numerical evaluation of the Low theory of charged photoproduction is given in various approximations, including also nonrelativistic recoil of the nucleon. Enhancement of only that part of the $(P\frac{3}{2},\frac{3}{2},M1)$ term which is due to interaction with the nucleon current gives reasonably good agreement with experimental data up to 440 Mev. The effect of recoil corrections is hardly discernable in the shape of the angular distribution. The differential cross section is also calculated from a modified version of the theory, called the "adjusted" theory, in which the S-wave terms are taken from perturbation theory. This version of the theory gives an equally good fit to the data. The negative to positive ratio is then calculated in the various approximations. It is found that these approximations give nearly the same ratio, although the corresponding predictions of differential cross section for the positive pion production differ greatly. This common prediction for the ratio agrees quite well with the experimental data, especially in the case of the "adjusted" theory.

I. INTRODUCTION

THE purpose of this paper is to explain the dependence on angle and energy of the negative to positive ratio in pion photoproduction. We will use the nonrelativistic theories of photoproduction which have met with considerable success for the last three years. They grew out of the realization that the formulation of a consistent and unambiguous covariant meson theory is not just around the corner, and that in the meantime much information can be obtained from theories which are willing to sacrifice some of the generality of the covariant theory. In particular, one can restrict the range of interest to low energies where nucleon pair effects can be expected to play a secondary role, and where not too many partial waves contribute in an important way. Also, one can make the theory semiempirical, containing some parameters to be determined from experiments. In return, however, one obtains a theory which is able to interpret meson phenomena up to several hundred Mev. It is the aim of this paper to show that the negative to positive ratio in photoproduction is one of those phenomena which could not be explained quantitatively by previous theories but which can be predicted essentially within experimental error by one of the forms of a nonrelativistic theory.

We shall first discuss the form of the theory in question, namely, the Low theory.[1,2] In this discussion we shall try to go beyond the fixed-source-limit approximation and add recoil corrections. We shall then apply this theory to the negative to positive ratio, and compare the predictions with experimental data. It will be evident that the agreement is quite good for most energies and angles. It will also be pointed out that recoil corrections change the predictions only very slightly, and that therefore the results do not depend essentially on our way of adding the recoil corrections. This is to be noted in view of the fact that our way of adding the recoil corrections is by no means unambiguous. It will be shown, however, that even the crudest approximation to the nonrelativistic theory gives fine agreement with the data on the ratio. The same cannot be said, however, about the angular distribution of the individual charged pion production.

II. LOW THEORY AND ITS RECOIL CORRECTIONS

All nonrelativistic theories of pion photoproduction which have been developed so far, and, in fact, all current theories in the nonrelativistic limit, give essentially the following differential cross section:

$$\begin{aligned}\frac{d\sigma}{d\Omega}=K\bigg| a\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}+b\frac{\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})\mathbf{q}\cdot\boldsymbol{\varepsilon}}{q_0k-\mathbf{q}\cdot\mathbf{k}}\\ +c[i\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})-(\boldsymbol{\sigma}\cdot\mathbf{kq}\cdot\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathbf{q}\cdot\mathbf{k})]\\ +d[2i\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})+(\boldsymbol{\sigma}\cdot\mathbf{kq}\cdot\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathbf{q}\cdot\mathbf{k})]\\ +e[\boldsymbol{\sigma}\cdot\mathbf{kq}\cdot\boldsymbol{\varepsilon}+\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathbf{q}\cdot\mathbf{k}]\bigg|^2,\quad(1)\end{aligned}$$

where K is some constant depending on energy but not angle, and $a,\cdots,e$ are energy-dependent constants which can be determined from the various theories. They are different for different charge states. The first, third, fourth, and fifth of the terms in Eq. (1) are $S\ E1$, $P\frac{1}{2}\ M1$, $P\frac{3}{2}\ M1$, and $P\frac{3}{2}\ E2$ terms, respectively. They can be written down easily by using general invariance properties. It is usually assumed that only these S- and P-wave terms will appear at low energies, and that any D-wave terms (or even higher angular momentum states) will be at most small corrections. This assumption seems to be justified for the terms arising from the interaction of the photon with the nucleon. As for the interaction with the meson, the above assumption has not been borne out, because every theory contains a term like the second in Eq. (1). This term contains all angular momentum states with

* Work performed under the auspices of U. S. Atomic Energy Commission and the Office of Naval Research.
† Part of this work is contained in the author's Ph.D. thesis, Cornell University, February, 1956 (unpublished).
[1] F. E. Low, Phys. Rev. 97, 1392 (1954).
[2] G. F. Chew and F. E. Low, Phys. Rev. 101, 1579 (1956).

strengths which decrease only very slowly with the angular momentum. The implications of Eq. (1) on the analysis of photoproduction data has been discussed elsewhere.[3]

The goal of the various theories is then to determine $a,\cdots,e$ in Eq. (1). A purely phenomenological theory would take all of the coefficients from the analysis of experimental data.[4] More basic theories predicting these coefficients are those of Chew,[5] Ross,[6] and Low.[1,2] We shall direct our attention solely to the last of these theories.

In its most general form the Low theory gives an integral equation, an explicit solution of which has not yet been obtained. It is possible, however, to approximate this integral equation for photoproduction by using the experimental pion-nucleon scattering phase shifts. In this approximation we add to the inhomogeneous part of the integral equation an "enhanced" amount of its $(P\frac{3}{2},\frac{3}{2})$ part, and the enhancement factor multiplying this part involves δ_{33}, the $(P\frac{3}{2},\frac{3}{2})$ scattering phase shift. This approximation can be justified mathematically[2] using general properties of the T matrix in the complex plane. Its physical interpretation is in terms of final state scattering. The meson, after its creation, can undergo scattering by the nucleon which will contribute in a significant way only in the $(P\frac{3}{2},\frac{3}{2})$ state in the low-energy region.

It is customary to use the nonrelativistic theories in the fixed-source approximation in which the kinetic energy (but *not* the momentum) of the nucleon is neglected, that is, in which only the terms lowest order in μ/M in each contribution are considered. Here μ is the meson mass, and M the nucleon mass. One can attempt, however, to add the so-called recoil corrections, that is, terms of higher order in μ/M. In principle, Low's derivation[1] of his equation is suitable for calculating such corrections.[7] In fact, the corrections to the inhomogeneous terms can be calculated unambiguously. If, however, we use the enhancement approximation, the recoil corrections in the enhanced part, if any, do not seem to be determined in a unique way from our procedure. This procedure is to write down the inhomogeneous part with recoil corrections, select out of it the appropriate isotopic spin and angular momentum states, and multiply that by the same enahancement factor as used in the fixed-source limit. Apart from minor ambiguities in separating out the appropriate states, there is also some uncertainty about the way of using the enhancement factor in the non-fixed-source case. There is a way of deriving photoproduction cross sections from dispersion relations[8] which gives the recoil corrections in a unique and presumably correct fashion. It is, however, not a waste of effort to treat the recoil corrections in a simple, transparent, although perhaps unjustified manner to see at least the general effect they have on the predictions. It is in this spirit that the subsequent remarks and results about recoil corrections are to be viewed. Many of our results, however, will also hold for the fixed-source limit and will therefore have a more general quantative validity.

The matrix element for charged photoproduction in the form we will use it arises from an equation developed from Eq. (2.2) of reference 1 in a manner analogous to the derivation of Eq. (3.9) from (1.11) of the same reference. The result is

$$
\begin{aligned}
C\mathfrak{M}^{\pm}=&\left\{\begin{matrix}+\\-\end{matrix}\right\}C\Bigg\{e\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}+e\frac{\boldsymbol{\varepsilon}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})}{q_0k-\mathbf{q}\cdot\mathbf{k}}\\
&+ik^{-1}\left(1+\frac{k}{2M}\right)^{-1}\boldsymbol{\sigma}\cdot\mathbf{q}\left\{\begin{matrix}\boldsymbol{\mu}_p\\-\boldsymbol{\mu}_n\end{matrix}\right\}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\\
&-ik^{-1}\left(1+\frac{k}{2M}+\frac{q\cos\theta}{M}\right)^{-1}\left\{\begin{matrix}\boldsymbol{\mu}_n\\-\boldsymbol{\mu}_p\end{matrix}\right\}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\boldsymbol{\sigma}\cdot\mathbf{q}\\
&-\frac{e}{M}\left\{\begin{matrix}0\\1\end{matrix}\right\}k^{-1}\left(1+\frac{k}{2M}+\frac{q\cos\theta}{M}\right)^{-1}\boldsymbol{\varepsilon}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{q}\\
&+(3q_0)^{-1}\left(k\frac{e^{i\delta_{33}}\sin\delta_{33}}{(4/3)f^2q^2}-1\right)\Bigg[\beta(2k)^{-1}\\
&\times(\boldsymbol{\varepsilon}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k}+\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}\mathbf{k}\cdot\mathbf{q})+\frac{i}{3}\left(\frac{G^{-}}{M}\gamma\left(1-\frac{q^2}{2Mq_0}\right)+\frac{\alpha}{2k}\right)\\
&\times(2\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})-i(\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{q}\cdot\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathbf{q}\cdot\mathbf{k}))\Bigg]\Bigg\},\qquad(2)
\end{aligned}
$$

where the brackets refer to $\left\{\begin{matrix}\text{positive}\\\text{negative}\end{matrix}\right\}$ meson production, $\boldsymbol{\mu}=g(e/4M)\boldsymbol{\sigma}$ with $g_p=5.59$ and $g_n=-3.83$, $G^{-}-=\frac{1}{2}(g_p-g_n)$, and α, β, and γ are constants to be explained below. $C=2\pi i\sqrt{2}f/\mu(q_0k)^{\frac{1}{2}}$, $e^2=1/137$, $f^2=0.072$. The rest of the notation is as follows: $\mathbf{k}$ is the photon momentum, $\mathbf{q}$ the meson momentum, $\boldsymbol{\sigma}$ the nucleon spin, ϵ the photon polarization, θ the meson emission angle, and q_0 the meson energy.

In the above expression the first term is a pure S wave term. It arises from the requirement of gauge invariance, or, talking in terms of graphs, from Marshak's "catastrophic interaction."[9] The second term in Eq. (2) arises from the interaction of the electromagnetic field with the meson current and

[3] M. J. Moravcsik, Phys. Rev. **104**, 1451 (1956).
[4] Watson, Keck, Tollestrup, and Walker, Phys. Rev. **101**, 1159 (1956).
[5] G. F. Chew, Phys. Rev. **89**, 591 (1953); **94**, 1749 (1954); **95**, 285, 1669 (1954).
[6] M. Ross, Phys. Rev. **104**, 1736 (1956).
[7] I am indebted to Professor Chew for several illuminating comments on the difficulties in adding recoil corrections.
[8] G. F. Chew *et al.* (to be published).
[9] R. Marshak, *Meson Physics* (McGraw-Hill Book Company, Inc., New York, 1952), p. 6.

contains S terms and higher angular momentum states. The next two terms arise from the interaction of the photon with the nucleon magnetic moments. The first of these is a pure $(P\frac{1}{2},M1)$ term corresponding to a graph where there is only a nucleon in the intermediate state and hence the total angular momentum must be $\frac{1}{2}$. The second magnetic moment term is a mixture of the $(P\frac{1}{2},M1)$ and $(P\frac{3}{2},M1)$ states and corresponds to the "cross-over" graph. The fifth term, which in the center of mass system exists only for negative pions, is a $(D\frac{3}{2},E1)$ term describing the interaction of the electromagnetic field with the nuclear charge. The third, fourth, and fifth terms also contain higher angular momentum states.

The last two of the seven terms in Eq. (2) are identical for positive and negative mesons, and represent the enhancement of the $(P\frac{3}{2},\frac{3}{2})$ state. The first of these is a $(P\frac{3}{2},\frac{3}{2},E2)$ term resulting from the meson current interaction, while the second is a $(P\frac{3}{2},\frac{3}{2},M1)$ term originating partly from the nucleon magnetic moment interaction, and partly from the meson current interaction. Low's argument justifying the "enhancement approximation" applies strictly only to the $(P\frac{3}{2},\frac{3}{2},M1)$ term of the nucleon current, since the argument is based on drawing an analogy to the scattering equation where a graph corresponding to the meson interaction graph in photoproduction does not exist, and hence $(P\frac{3}{2},\frac{3}{2},E2)$-type terms do not occur. However, there is no good *a priori* reason to believe that the $(P\frac{3}{2},\frac{3}{2},E2)$ part does not get enhanced. Actually, as we will see, the comparison with experiments[10,11] shows that the enhancement factor of the electric quadrupole term, if nonzero at all, must be much smaller than that of the magnetic dipole terms. The argument referred to above is also noncommittal about the $(P\frac{3}{2},\frac{3}{2},M1)$ part resulting from the meson current. Thus a strict Low enhancement approximation corresponds to $\gamma=1$, $\alpha=\beta=0$. We will also see, however, that an appreciable enhancement of the $(P\frac{3}{2},\frac{3}{2},M1)$ part resulting from the meson current cannot be excluded on the basis of the experimental data.

The inclusion into Eq. (2) of the nucleon recoil is effected by the terms

$$\left(1+\frac{k}{2M}\right)^{-1},\quad \left(1+\frac{k}{2M}+\frac{q\cos\theta}{M}\right)^{-1},$$
and
$$\left(1-\frac{q^2}{2Mq_0}\right). \tag{3}$$

The first two of these terms are unambiguous, while the last one arises in an ambiguous way.

The differential cross section for charged meson production is

$$d\sigma/d\Omega=2\pi\rho_F|C\mathfrak{M}^{\pm}|^2, \tag{4}$$

where $\hbar=c=1$, and ρ_F is the density of final states per unit energy interval, given by

$$\rho_F=(2\pi)^{-3}q^2\left(1+\frac{k}{M}\right)^{-1}\frac{dq}{dE_F}d\Omega$$
$$=(2\pi)^{-3}qq_0\left(1+\frac{k}{M}\right)^{-1}\left(1-\frac{q_0}{M+q_0+q^2/2M}\right)d\Omega. \tag{5}$$

In the fixed-source limit, the terms in the two brackets are to be changed to unity. For the nonfixed source case, therefore, we get

$$\frac{d\sigma}{d\Omega}=\frac{2f^2e^2}{\mu^2}\frac{q}{k}\left(1+\frac{k}{M}\right)^{-1}\left(1-\frac{q_0}{M+q_0+q^2/2M}\right)|\mathfrak{M}|^2. \tag{6}$$

The difference in the phase space factor between the fixed-source limit and the recoil case is independent of angle and therefore it does not affect the shape of the angular distribution. Since, however, we will also try to make an absolute comparison between theory and experiments, we will have to consider this difference, which in fact can be a considerable factor at the upper end of our energy range. We shall adopt the somewhat inconsistent convention of using the correct, nonfixed-source phase space factor in all our comparisons. This is reasonable if we consider that the agreement in absolute values is less likely in any case because of the experimental difficulties of calibration. Thus the differences between the fixed source limit and the full recoil case which we shall discuss will refer only to the difference in the shape of the angular distribution, i.e., the differences which arise from changes in the matrix element itself.

The experimental data which we use for comparison are from Tollestrup *et al.*,[10] Walker *et al.*,[11] and Beneventano *et al.*[12] The data in references 10 and 11 are to be multiplied by 1.07 corresponding to an alleged error in absolute calibration.[12] At 260 Mev there are also some data by Osborne.[13]

Figure 1 shows these data at 260-Mev laboratory photon energy, compared with the differential cross section as predicted by (a) only the unenhanced part of Eq. (7) ($\alpha=\beta=\gamma=0$), (b) the unenhanced part plus the enhancement of the $(P\frac{3}{2},\frac{3}{2},M1)$ part resulting from the nucleon current ($\gamma=1,\alpha=\beta=0$), (c) the unenhanced part plus the enahancement of all of the $(P\frac{3}{2},\frac{3}{2},M1)$ part ($\gamma=\alpha=1,\beta=0$), and (d) the unenhanced part plus the enhancement of all of the $(P\frac{3}{2},\frac{3}{2})$ part ($\alpha=\beta=\gamma=1$). The comparison shows that (a) and (d) are clearly incompatible with the experiments, while (b) and (c) both give a more than qualitative fit. A similar statement can be made for all energies up to

[10] Tollestrup, Keck, and Worlock, Phys. Rev. **99**, 220 (1955).

[11] Walker, Teasdale, Paterson, and Vette, Phys. Rev. **99**, 210 (1955).

[12] Beneventano, Bernardini, Carlson-Lee, Stoppini, and Tau, Nuovo cimento 4, 323 (1956).

[13] L. S. Osborne, *Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956).

440 Mev, which is the upper limit of our numerical calculations.

In making a comparison between experimental data and theory, the quantitative agreement or discrepancy must be taken with a grain of salt. Firstly, the value of the meson-nucleon coupling constant (taken to be 13 in this paper[12]) is known at best to 10%. Secondly, the absolute calibration of the measurements is in doubt also to about 10%. Finally, the relative errors of the various experimental points is also of the same order of magnitude. Thus, when making a comparison it is wiser to concentrate on the agreement of the shape of the angular distribution rather than on the agreement in absolute value. A discrepancy by an angle- and energy-independent factor should not be given too much weight. In fact, the factor does not even have to be precisely energy-independent, since the intercalibration between laboratories working in different energy ranges is also subject to errors.

The effect of including the nucleon current contribution, with and without recoil, is shown in Fig. 2. All three curves there contain the $(P\frac{3}{2},\frac{3}{2},M1)$ nucleon current part enhanced, but curve (1) does not contain any other part of the unenhanced nucleon current contribution, but only the meson current, while curve (2) contains the unenhanced nucleon current in the fixed source limit in addition to the meson current, and finally curve (3) contains the unenhanced nucleon current with recoil plus the meson current. As seen, the absence of the unenhanced nucleon current contribution results in a definite disagreement with experiments, while both versions containing the unenhanced nucleon current contribution fit the data well.

It seems from these comparisons that the experimental data are not accurate enough to make a decisive statement about the fine points of the theory, such as whether (b) or (c) is correct, or whether the recoil corrections are meaningful. Nevertheless one can make a more rigorous comparison of these theories and the data by using a recently developed method of analysis.[3] The results are given in Table I. We see that cases 1 and 4 can be eliminated already on account of the disagreement in the first two coefficients. Cases 2 and 6 agree in A, and 3 and 5 are also close. For B there is again an agreement within experimental error for cases 2 and 6, and also for 3. For C none of the previously successful approximations agree within experimental error, but cases 2, 3, and 6 agree within two experimental standard errors. Case 5 now disagrees strongly. For D the discrepancy is even more; cases 2, 3, and 6 agree only within 4 standard errors. The situation is somewhat better for E where cases 2, 3, and 6 agree within 2 standard errors. It is difficult to tell whether any of cases 2, 3, and 6 is superior to the others. However, it is not at all clear from the analysis if the experiments or the theory are responsible for the discrepancy which, we should emphasize, is relatively insignificant. It might also be worth mentioning that D and E are proportionately more significant near 0° and 180° where reliable experiments are still lacking.

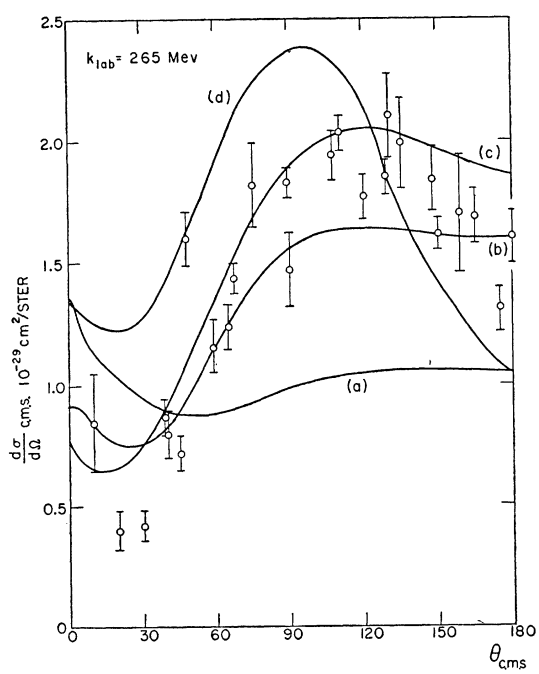

FIG. 1. Differential cross section for positive-pion production from hydrogen, at a laboratory photon energy of 265 Mev. For an explanation of the curves, see the text.

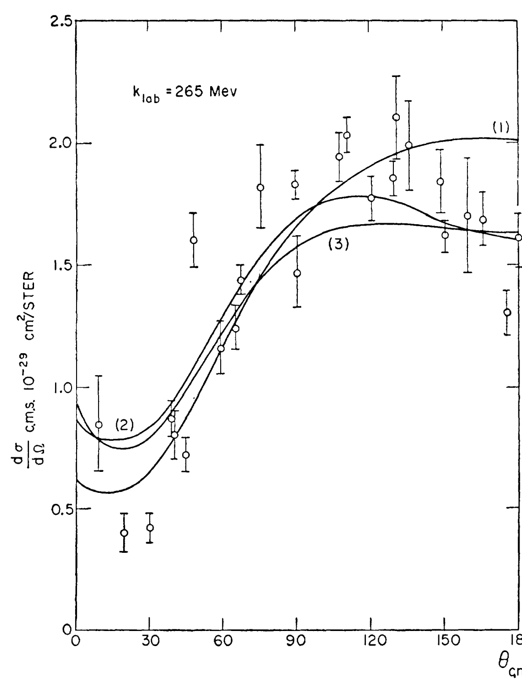

FIG. 2. The same data as in Fig. 1 compared with other curves whose explanation is given in the text.

TABLE I. Coefficients of the angular distribution analysis for positive-pion photoproduction at 260-Mev photon laboratory energy.

	A	B	C	D	E
Experiments	0.56±0.01	−1.04±0.03	0.30±0.04	0.32±0.04	−0.13±0.04
1. Fig. 1, curve (a)	0.51	−0.84	0.31	0.085	−0.024
2. Fig. 1, curve (b)	0.56	−1.02	0.38	0.17	−0.079
3. Fig. 1, curve (c)	0.58	−1.07	0.38	0.23	−0.11
4. Fig. 1, curve (d)	0.61	−0.98	0.057	0.54	−0.20
5. Fig. 1, curve (b) but no unenhanced nucleon terms	0.54	−1.08	0.59	−0.008	−0.024
6. Fig. 1, curve (b) with unenhanced nucleon term, no recoil	0.56	−1.02	0.36	0.20	−0.086

A comparison of the experimental data with the approximation involving the whole unenhanced $(P\frac{3}{2},\frac{3}{2},M1)$ nucleon current part is given at various energies by the solid curves in Fig. 3. The agreement is quite good, even at as high energies as 440 Mev. The shapes of the distributions give a very good fit, and even the agreement in absolute values is fair.

We can make the following conclusions from the above comparison:

(1) Low's "enhancement approximation," when it includes the unenhanced nucleon current terms gives a good agreement with data at least up to 440 Mev.

(2) The experimental data do not exclude the possibility of an almost full enhancement of also that part of the $(P\frac{3}{2},\frac{3}{2},M1)$ term which arises from the meson current, although the agreement of the theory including this additional enhancement appears to be slightly less satisfactory than without it.

(3) The experimental data exclude anything but a very weak enhancement of the $(P\frac{3}{2},\frac{3}{2},E2)$ part.

(4) The relative errors in the experiments have to be deduced to 5% or less before the effect of recoil can be detected on data below about 300 Mev. Above that energy the predicted difference between the fixed source limit and the recoil theory is of the order of 10%. Thus further experimental data, especially near 180°, in the higher energy range would be of considerable help in learning about the effects of recoil corrections.

It should be emphasized that although the theory discussed in this paper contains experimental data in terms of the scattering phase shifts, for any given approximation there are *no free parameters* to adjust to fit the photoproduction experiments. Thus even the moderately good agreement with the theory is gratifying.

There is at least one serious objection to the form of the theory as used above, and this objection concerns the S-wave part. The S-wave term in Eq. (7) arises from the meson current term. Therefore it has the same form whether full recoil is taken into account or not, because the meson current term does not change when we include full recoil. That this S-wave term cannot be correct is quite clear if we consider, for instance, the negative to positive pion production near threshold (see below). Equation (7) gives unity for this ratio, which disagrees both with the experiments and with all other theories. The reason for this failure might well be in the nucleon pair term effects which our theory neglects. The missing factor, as judged by experiments and by other theories, is a coefficient of the form $[1+0(\mu/M)]$, that is, a factor which, although not given by the fixed source limit calculations, can be of the same order as some of the lowest order nucleon current terms which the fixed source limit does give.

The way to obtain this missing factor is by no means clear. The Kroll-Ruderman theorem does not help, because this requires agreement with the perturbation result only to terms of order unity, and makes no prediction about the terms of order μ/M, which in fact is the order where the missing factor deviates from our result.

The procedure we followed in our calculations was to use the coefficient of the $\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}$ term as given by the perturbation theory, that is, $(1+k/2M)(1+k/M)^{-1}$ for positive mesons and $(1+k/2M)$ for negative mesons. The justification for this is twofold. Phenomenologically speaking, as we will see in Sec. III, it tends to improve the prediction of the ratio at least at low energies. More basically, there are some investigations in the development[8] which tend to indicate that at low energies the perturbation prediction for the ratio is correct even to order μ/M. The model we use, therefore, is a combination of a perturbation S wave and higher angular momentum contributions of the Low type.

On the whole, this form of the theory produces little change in the shape of the angular distribution as compared with the previous, "unadjusted" form of the theory as given by Eq. (7). The comparison is shown in Fig. 3 where the dotted curves are those given by the "adjusted" theory. As expected, the "adjusted" theory gives a smaller absolute value for positive pion production, thus producing at least at some energies a somewhat better fit with experiments. As remarked before, however, the agreement in absolute values is not a very conclusive proof of correctness because of the uncertainty in the monitoring of the beam and in the knowledge of the coupling constant.

In conclusion we might emphasize again that the various forms of the theory we have discussed are only an approximate solution of the integral equation in Low's theory, and therefore our conclusions can be used only in a very qualitative sense to argue for the validity of the rigorous Low theory.

III. NEGATIVE TO POSITIVE RATIO

There are several reasons, both experimental and theoretical, why it is profitable to study the ratio of

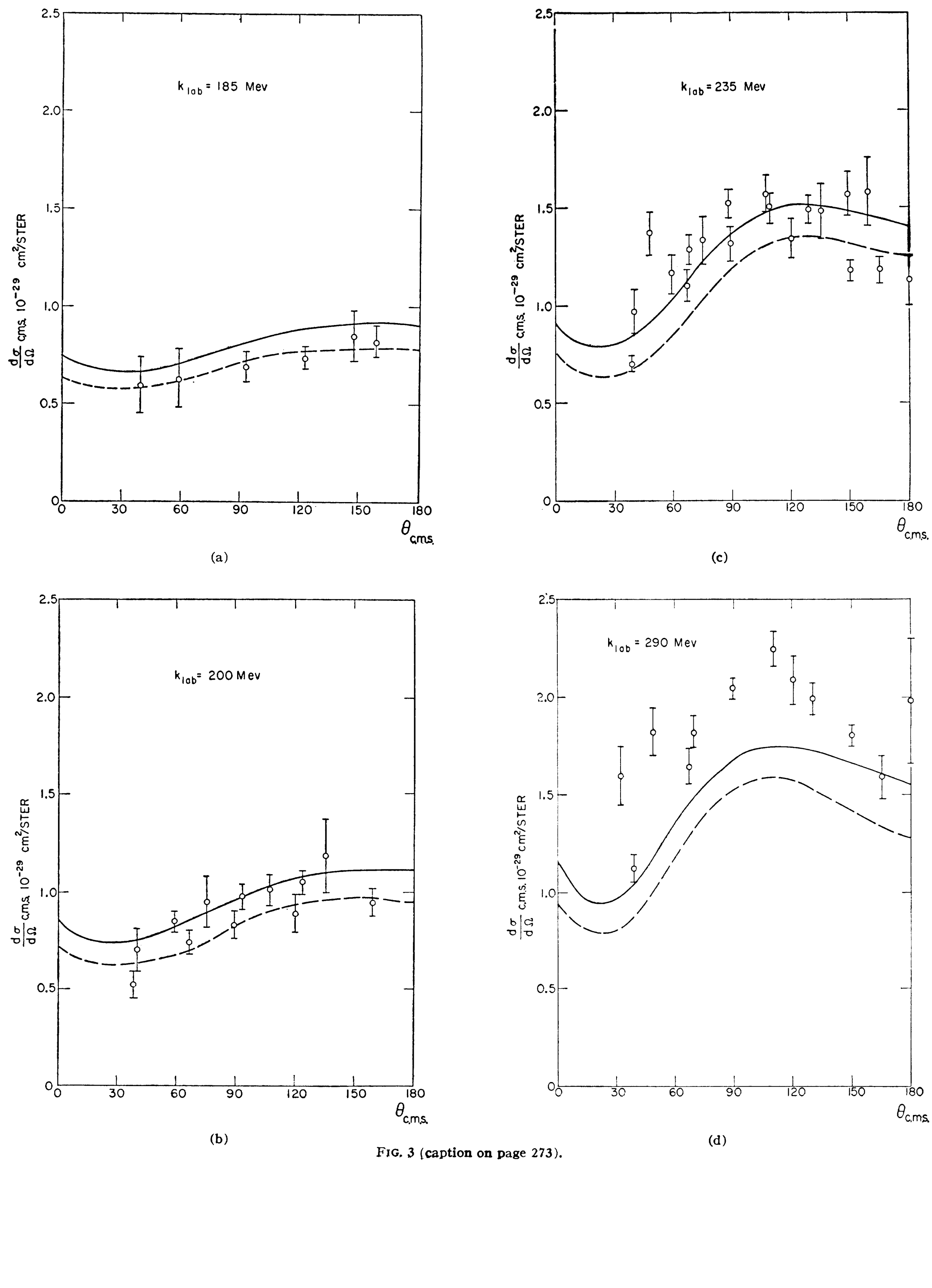

FIG. 3 (caption on page 273).

the cross sections of the two basic photoproduction reactions:

$$\gamma+p\rightarrow n+\pi^{+}, \tag{7}$$

$$\gamma+n\rightarrow p+\pi^{-}, \tag{8}$$

in addition to the individual reactions themselves. On the experimental side, it is often easier to carry out relative measurements, such as the measurement of the ratio, where corrections for the efficiency of the meas-

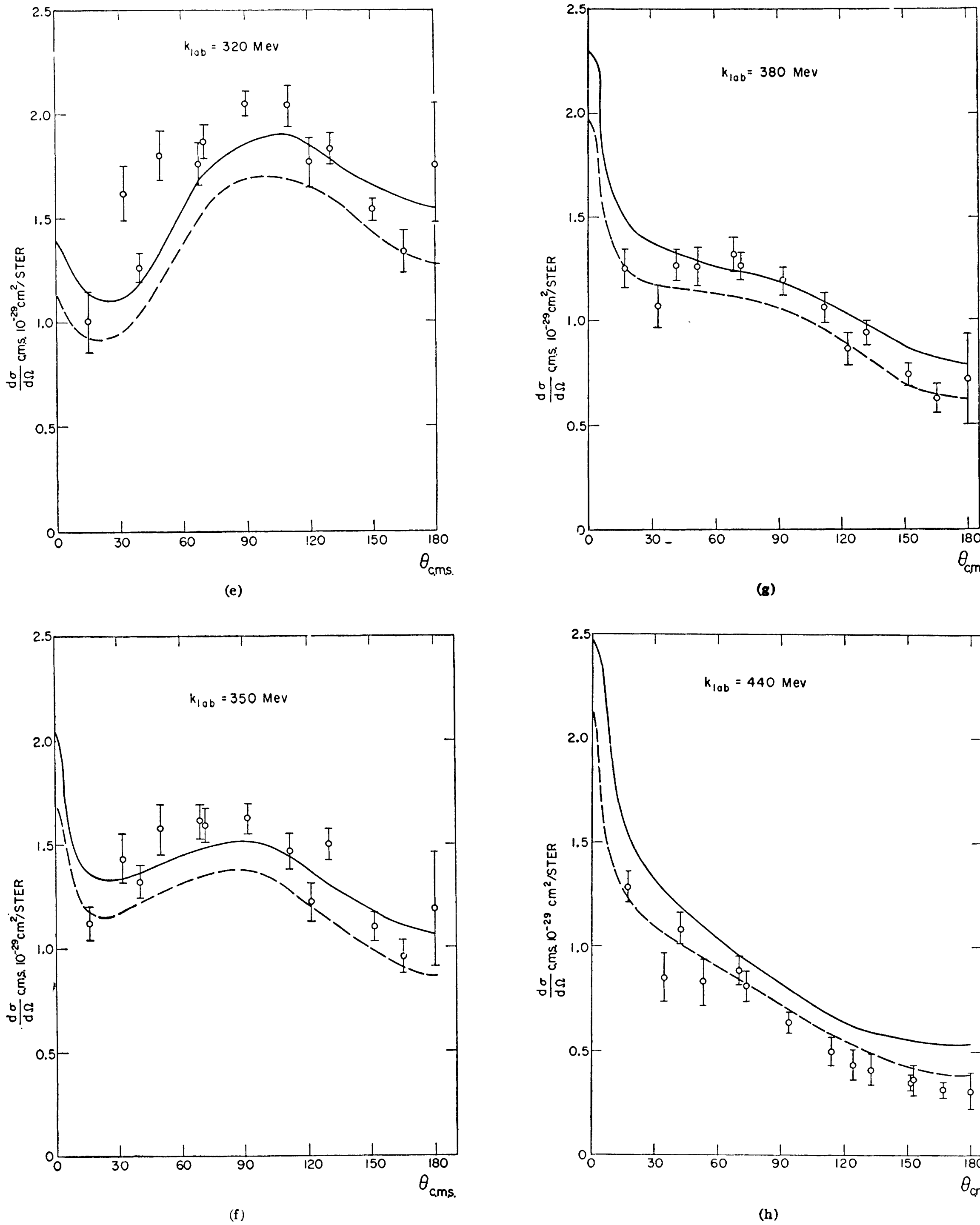

FIG. 3. Differential cross section for positive-pion production from hydrogen at various energies. The solid curves are those given by the "unadjusted" theory, the dotted ones those given by the "adjusted" theory. For explanation see the text.

uring equipment or the uncertainty in some parameters might be eliminated, or at least reduced. On the theoretical side, one could imagine that in taking the ratio some features of the theory might cancel out, thus simplifying the theoretical treatment. (To take a trivial example, the coupling constant in the lowest order perturbation theory drops out in the ratio.) On the other hand, precisely because of the above possibility of cancellation, one would expect that certain other features of the theory may become prominent, in which case the study of the ratio might reveal information about them.

There is another important theoretical argument favoring the study of the ratio instead of the individual reactions. Process (8) cannot be studied directly since we have no free neutron target available. Thus we may either use experiments from various complex nuclei and develop a theory[14] of the modification of (7) and (8) by the nuclear structure, which effect can then be eliminated from the ratio. Alternatively, we may use the data from the reactions

$$\gamma+d-2n+\pi^{+}, \tag{9}$$

$$\gamma+d-2p+\pi^{-}, \tag{10}$$

and apply a correction factor specifically calculated for deuterium. We shall choose the second alternative to study (7) and (8) because it is the more precise one at the present time.

In calculating the correction factor for deuterium, our work is greatly simplified by the fact that we consider the ratio and not the individual cross sections. In fact, if we assume charge symmetry it is evident that the purely nuclear effects in the final state will cancel out and that we shall have to be concerned only with the Coulomb effects. Although the complete correction including nuclear and Coulomb effects, to be applied to deuterium reactions has been attempted,[15] it is not too accurate and thus the above-mentioned simplification is quite welcome.

Various approximate estimates for the Coulomb correction have been given in the past.[16] These estimates will use the argument involving the Coulomb penetration factor to calculate the correction; then, admitting that this method tends to give too large a correction, this calculated factor is reduced by an arbitrary factor. These estimates give

$$\left(\frac{\sigma(n\rightarrow\pi^{-})}{\sigma(d\rightarrow\pi^{-})}\right)_{\text{Coul}}=1+\epsilon,\quad \epsilon=0.1-0.3, \tag{11}$$

at moderate energies. Clearly, such a factor must be applied only to the negative-pion production since in the case of the positive-pion production there is only one charged particle in the final state, and hence, there is no Coulomb interaction.

A somewhat more elaborate treatment of ϵ has been given by Chew and Low[17] who calculated all three interactions which exist in the final state between the two protons and the negative pion. They used the Coulomb penetration factor for the interaction of the meson with the nucleon from which it was created, the Born approximation for the interaction for the meson with the other nucleon, and the sum rule method for the interaction of the two protons. Using an $r^{-1}\exp(-\alpha r)$ internal deuteron wave function, they obtain quite a small ϵ. Moreover, except very near threshold, it is opposite in sign to the previous estimates. It decreases in absolute magnitude as the energy increases, and at 180-Mev γ-ray energy in the laboratory system, it is already decreased to only 5.5%. Since most of the experimental data have errors larger than 6%, we shall simply neglect this correction factor altogether and shall consider an agreement within 10% a satisfactory one. There is one more effect which will make the ratios obtained from deuterium slightly different from those one would expect from the individual nucleons. Owing to the internal motion of the nucleons in the deuterium the kinematic relationship between $\mathbf{k}$, q, and θ will not be a unique one, but for instance a range of k values can contribute to the production of pions with momentum q and emission angle θ. This range of k will be the same for reactions (9) and (10), but since the ratio is a function of angle and energy, the deuterium data of a certain angle and energy will in fact correspond to a superposition of energies and angles for reactions (7) and (8). We shall assume that this rather complicated effect is small and shall neglect it. Since the angle and energy dependence of the ratio is not very strong, and since the ratio is near unity anyway, such an assumption is quite justified.

The experimental data for the π^{-}/π^{+} ratio are by now abundant.[18–29] In general, the results at lower energies tend to have larger estimated errors than those at higher energies, which are the more recent measurements. The resolution in angle and energy is often poor, and sometimes the errors are not given. The data are

[14] K. W. McVoy, Cornell thesis, June, 1956 (unpublished).
[15] For a summary and references, see H. A. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson, and Company, Evanston, 1955), Vol. II, p. 165 ff.
[16] For a summary and references, see reference 15, pp. 164, 278.
[17] G. F. Chew (private communication).
[18] R. M. Littauer and D. Walker, Phys. Rev. **82**, 746 (1951) and **83**, 206 (1951).
[19] Lebow, Feld, Frisch, and Osborne, Phys. Rev. **85**, 681 (1952).
[20] White, Jakobson, and Schultz, Phys. Rev. **85**, 770 (1952).
[21] R. M. Littauer and D. Walker, Phys. Rev. **86**, 838 (1952).
[22] White, Jacobson, and Schultz, Phys. Rev. **88**, 836 (1952).
[23] Palfrey, Luckey, and Wilson, Phys. Rev. **91**, 468 (1953).
[24] Jenkins, Luckey, Palfrey, and Wilson, Phys. Rev. **95**, 179 (1954).
[25] Sands, Teasdale, and Walker, Phys. Rev. **95**, 592 (1954).
[26] Beneventano, Carlson-Lee, Stoppini, Bernardini, and Goldwasser, Nuovo cimento **12**, 156 (1954).
[27] Sands, Teasdale, and Walker, Phys. Rev. **96**, 850 (1954).
[28] Beneventano, Bernardini, Carlson-Lee, Stoppini, and Tau, "Proceedings of the International Conference on Elementary Particles, Pisa, 1955," Nuovo cimento (to be published).
[29] Watson, Keck, Tollestrup, and Walker, Phys. Rev. **101**, 1159 (1956).

usually tabulated in terms of meson energy and angle, and the conversion of these data to a function of photon energy and angle is done assuming free nucleon kinematics. These data are shown in Fig. 4.

Previous attempts to explain the observed π^-/π^+ ratio have been made by Brueckner and Goldberger,[30] Brueckner,[31] Kaplon,[32] and Watson.[29] The theory of Brueckner and Goldberger is entirely classical. Brueckner, in reference 31, elaborates on the classical argument, also gives general quantum-mechanical argument, and finally calculates the ratio by the first-order covariant perturbation theory, using pseudoscalar coupling. Kaplon also uses the perturbation method with pseudoscalar and pseudovector couplings, but includes also the anomalous magnetic moments of the nucleons. Finally, Watson's theory is purely phenomenological, containing numerous parameters which are adjusted to fit experiments.

We shall not concern ourselves with the last theory. The results of the other theories can be summarized as follows.

Classical arguments using current interactions, or the perturbation theory without anomalous magnetic moments, all give the same result for the ratio, namely,

$$\frac{\sigma^-(\theta)}{\sigma^+(\theta)}=\left[1-\frac{q_0}{q}\left(1-\frac{q}{q_0}\cos\theta\right)\right]^{-2}. \tag{12}$$

Equation (12) predicts much larger ratios than observed. For instance, at 180° and 240-Mev photon energy in the laboratory system, the prediction is 2.2, while the experiments give 1.3. Essentially the same result is obtained from Kaplon's calculations with pseudoscalar coupling.

General quantum-mechanical arguments which take into account interaction through magnetic moments alone, give a much smaller ratio. In fact, now the prediction is much too small, yielding 1.06 in the above quoted example. A similar reduction (at least at threshold) is obtained from Kaplon's calculations using pseudovector coupling.[33] None of the above theories gives, therefore, a good fit to the experimental data.

Now let us turn to the nonrelativistic theory we discussed in Sec. II. Let us first consider the "unadjusted" form of the theory. Calculating the ratios in the various approximations, we obtain the noteworthy result that all approximations give very nearly the same ratio, however different predictions they may give for the positive-pion differential cross sections. The variations among the ratios given by the various approximations increase with increasing angle and energy. The only exceptions to this agreement are those approximations in which the nucleon interaction through the magnetic moments is completely neglected, and where the ratio is nearly unity at all energies and angles. Even without any enhancement one gets the same ratio, although in that case the prediction for the positive-pion differential cross section strongly disagrees with the experimental data on hydrogen. We shall return to the discussion of this fact later.

The comparison of this common prediction for the ratio with experimental data yields in general a good agreement even at 340-Mev laboratory photon energy. The predictions of the "unadjusted" theory are shown in Fig. 4 by the solid lines. The three lines correspond to the upper and lower limits of the predictions given by the various approximations, and to the prediction of that approximation which is illustrated in Fig. 3. There are only two areas of disagreement. One is at low energies and is quite definite. There, our predictions fall clearly below the experimental points for all angles. As we have seen the deuteron correction is not large enough to account for this discrepancy which amounts to as much as 30% at the lowest energies. However, the explanation of this disagreement is not difficult. At low energies, the S-wave contribution becomes important, and our "unadjusted" theory is not to be trusted in its S-wave part. We will see how the adjusted theory remedies this defect.

The other discrepancy is not as definite as the one at low energies. There is some indication, however, that at all energies our predictions fall below the experimental points at the smallest angles ($\theta<40°$). In that region experiments are scarce, so that this disagreement is not well established and thus we shall not attempt to explain it.

It should be mentioned that the behavior of the negative to positive ratio at very low energies (within 25 Mev from threshold) is a complicated problem which is at the present time in a state of flux. In that region the corrections due to the interaction in the final state of the deuterium become important. In addition to this, the experimental evidence is also hazy, and different extrapolations give different values for the ratio at threshold. The Panofsky ratio can also be used to determine the value of the ratio at threshold, but the value of the Panofsky ratio has also been changed by new experiments. We shall not discuss this complex problem in detail, and in fact shall refer to it only indirectly through the assumption which underlies the "adjusted" version of our theory. One simple observation, however, might turn out to be relevant in this respect. This is, that the measured values of the ratio at 90° c.m. below 200 Mev deviate considerably from any smooth curve one can draw through the experimental ratios at other angles and at the same energy. In reference 12 these 90° data play an important role in the extrapolation to determine the ratio very near threshold.

[30] K. A. Brueckner and M. L. Goldberger, Phys. Rev. **76**, 1725 (1949).

[31] K. A. Brueckner, Phys. Rev. **79**, 641 (1950).

[32] M. F. Kaplon, Phys. Rev. **83**, 712 (1951).

[33] Kaplon's statement that the inclusion of the anomalous magnetic moments does not change the ratio seems to hold only for the pseudoscalar coupling. That the two couplings give different results is not surprising since the often quoted equivalence theorem does not hold for photoproduction.

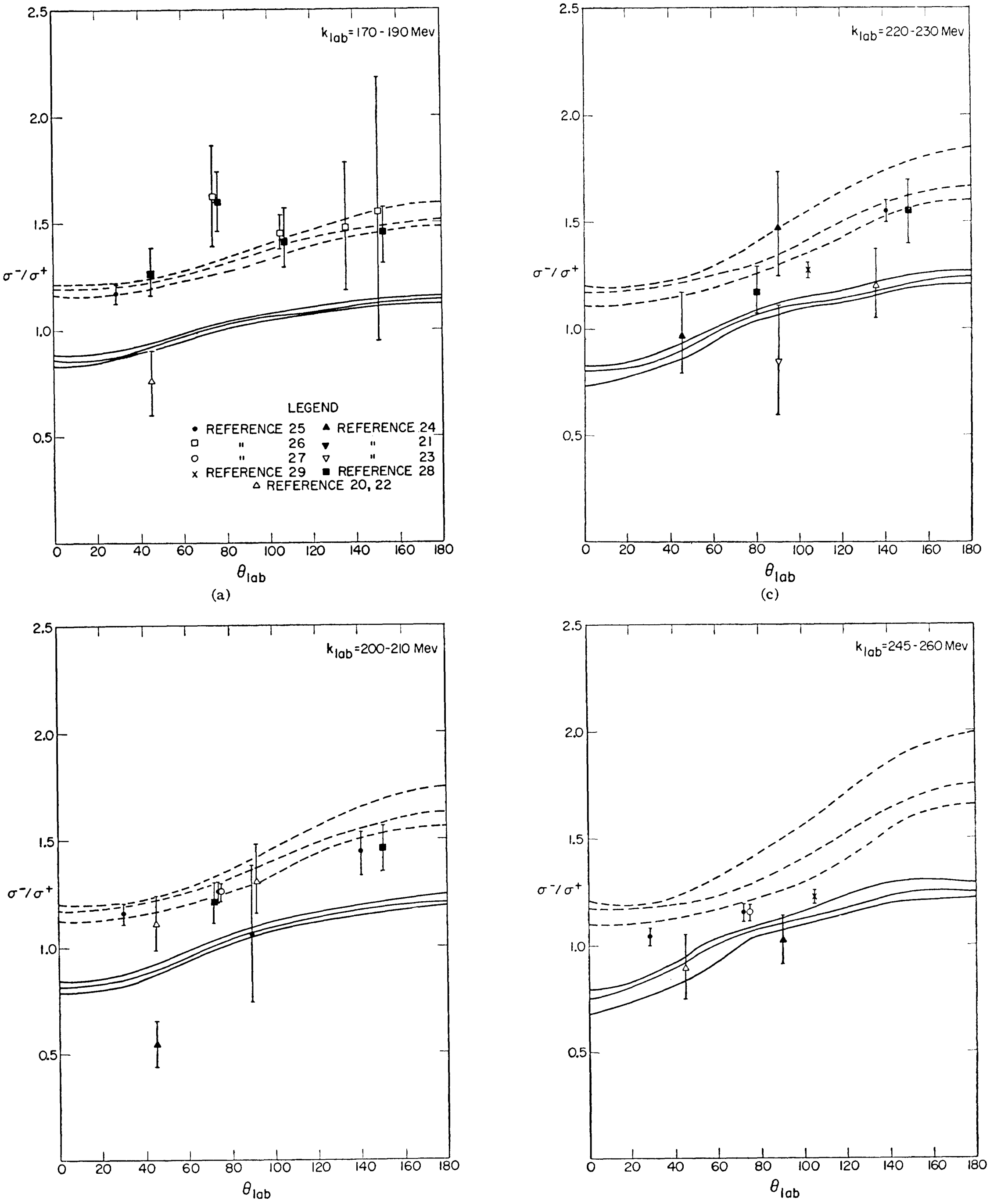

FIG. 4 (caption on page 277).

Now let us turn to the "adjusted" form of the theory. Here the S-wave part is taken from the lowest order perturbation theory, and is asymmetric for positive and negative pions so as to give a ratio of about 1.3 at threshold. With this modification the predicted ratio agrees now quite well up to about 225 Mev. Above that energy, up to 380 Mev, the predictions give somewhat larger ratios than the observed ones. There is still a surprisingly good agreement among the predictions of the various approximations, although the variation from one approximation to another is somewhat larger than it was in the "unadjusted" theory. The dependence of the predicted ratio on the individual approximation again increases with increasing angle and energy. The predictions of the "adjusted" theory are indicated in Fig. 4 by the dotted lines. The meaning of the three lines is the same as it was in the case of the "unadjusted" theory.

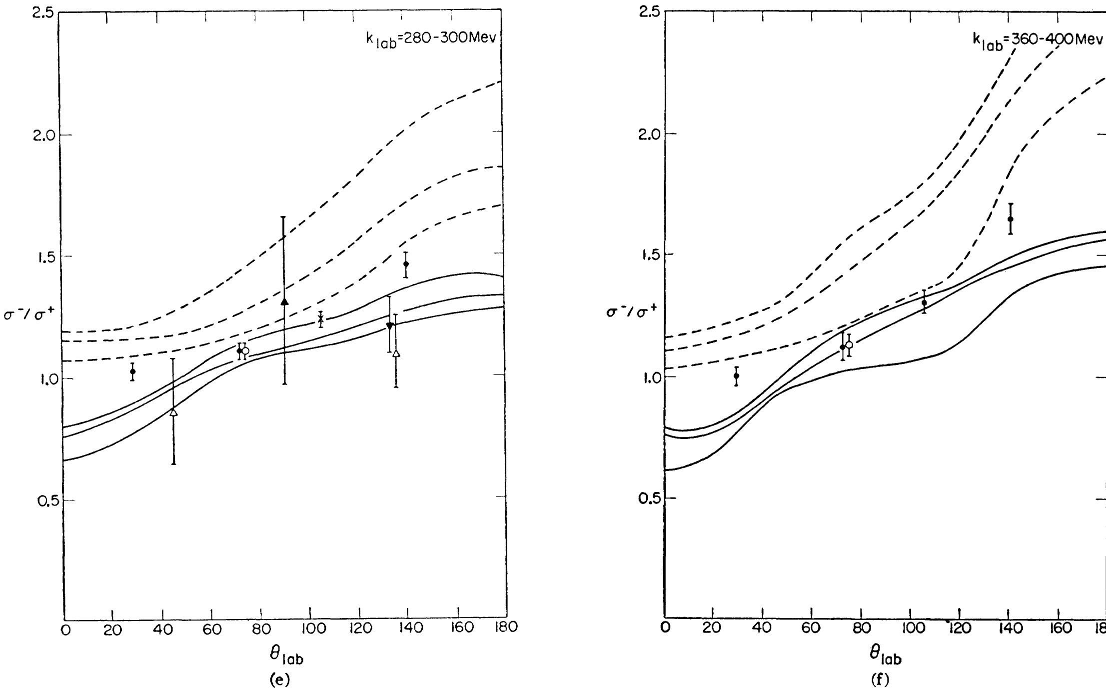

FIG. 4. The ratio of negative to positive pions in photoproduction at various energies, *vs* angle in the laboratory system. The experimental points are those obtained from deuterium. The key to the origin of the various data is indicated in (a). The solid curves are predictions of the "unadjusted" theory, while the dotted lines indicate the results of the "adjusted" theory. In both cases the upper and lower lines correspond to the upper and lower limits of the predictions coming from the various approximations, while the middle line is the result of that approximation whose predictions for the positive-pion angular distribution are plotted in Fig. 3

The agreement between experiments and theoretical predictions is better for the "adjusted" theory than for the "unadjusted" one, which in turn is much better than the agreement of any of the previous theories. Still, one could require an even better fit at higher energies than that which the "adjusted" theory gives. One procedure for accomplishing this seems to be evident from the above analysis, although actual numerical calculations have not been carried out in this direction. This procedure would take the S wave in the ratio given by perturbation theory *at threshold* and add to this the P and higher waves as given by Low's theory. The difference between this version and the "adjusted" theory would therefore be that the energy dependence of the S-wave terms, as given by the perturbation theory, would not be included. It is not difficult to see that such a theory would give about the same ratio as the "adjusted" theory at low energies and somewhat lower values at higher energies. The predictions for the angular distribution of positive pions should also be good, since it is expected to be somewhere between the values predicted by the "adjusted" and "unadjusted" theories, both of which give a reasonable agreement with experiments. The theoretical foundations of such a theory, however, would be more phenomenological than either the "adjusted" or the "unadjusted" theories.

Perhaps the most interesting question arising from the above analysis is why the various approximations, which give such different predictions for the angular distribution of positive pions, agree so well in the prediction of the ratio. A qualitative answer may be given by pointing out that the ratio is very nearly unity, and it deviates from it only in terms of order μ/M. The values of these terms are determined mainly by the magnitude of the magnetic moments of the proton and the neutron, and these moments enter all approximations in a similar manner. Thus the ratios should be very nearly the same. That this common prediction agrees so well with the experiments is therefore an argument more for the soundness of the nonrelativistic approach in general than for the validity of any of the particular forms we have used.

IV. ACKNOWLEDGMENTS

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