Assuming spins of 3/2, 5/2, 7/2 for Zr^{91}, the expected ratios are 2.3 ± 0.2, 1.00 ± 0.07, 0.56 ± 0.04, respectively, according to the formula

$$\frac{S_1}{S_2}=\frac{[mfI(I+1)\gamma H_1]_1[H_m]_2}{[mfI(I+1)\gamma H_1]_2[H_m]_1},$$

where H_1 is the amplitude of the rf field, m the concentration of the nuclei, and γ the gyromagnetic ratio. The experimentally determined ratio of the signals is therefore compatible with a spin 5/2 for Zr^{91} as measured by Arroe and Mack.

Using the ratio of the magnetic moments of H^1 and D^2 as determined by Smaller,[4]

$$\mu(H^1)/\mu(D^2)=3.2571999\pm0.0000012,$$

and the value of the proton magnetic moment,[5]

$$\mu(H^1)=2.79275\pm0.00003 \text{ nm},$$

the diamagnetically uncorrected value of the magnetic moment of Zr^{91} can be calculated as

$$\mu(Zr^{91})=-1.29802\pm0.00002 \text{ nm},$$

or, diamagnetically corrected, as

$$\mu(Zr^{91})=-1.30284\pm0.00002 \text{ nm}.$$

When one considers the proton number $Z=40$ and the neutron number $N=51$ (magic plus one), the spin and sign of the magnetic moment are in agreement with the predictions of the simple single-particle model which places the neutron in a $d_{5/2}$ state. The deviation of the magnetic moment of 32% from the Schmidt value $\mu(\text{Sch})=-1.913$ nm, however, is rather large. For this reason one may consider configurations which also include protons, particularly the last two protons outside the closed shell $Z=28$. In terms of the independent-particle model, the two protons would couple their individual spins j_p to a total proton spin $J_p=0$, 2, or 4, whereas J_p would couple with the odd-neutron spin $j_n=5/2$, to the total angular momentum $I(Zr^{91})=5/2$. Possible configurations consistent with the exclusion principle are

$$[\pi(p_{1/2})^2{}_{J_p=0};\,\nu d_{5/2}]_{5/2},\quad [\pi(g_{9/2})^2{}_{J_p=0};\,\nu d_{5/2}]_{5/2},$$
$$[\pi(g_{9/2})^2{}_{J_p=2};\,\nu d_{5/2}]_{5/2},\quad \text{and}\quad [\pi(g_{9/2})^2{}_{J_p=4};\,\nu d_{5/2}]_{5/2},$$

which yield the magnetic moments in nuclear magnetons of -1.913, -1.913, $+0.037$, and $+4.04$, respectively. This shows clearly that the ground state of Zr^{91} cannot be described by one single configuration of the type mentioned.

The observed relaxation time T_1 of Zr^{91} in aqueous solution of $(ZrF_6)^{--}$ is of the order of 10^{-3} sec and indicates an appreciable electric quadrupole moment.

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Magnetic Moment of the Mu Meson

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It is the purpose of this note to consider radiative corrections to the magnetic moment of the mu meson, under the assumption that the mu meson is a Dirac particle of spin $\frac{1}{2}$, coupled to the electromagnetic field in exactly the same way as the electron is, and not directly coupled to the electron-positron field.

We write the g factor in the form

$$g=2(1+\delta_1+\delta_2)+O(\alpha^3). \tag{1}$$

Here δ_1 represents the correction to the magnetic moment obtained by ignoring the effect of the electron-positron field, and δ_2 arises from the fact that the virtual photons emitted by the mu meson can give rise to a virtual electron-positron pair. This last effect is of order α^2, and the corresponding Feynman diagram is shown in Fig. 1.

From the work of Schwinger[1] and of Karplus and Kroll[2] on the radiative corrections to the magnetic moment of the electron we get δ_1, by noting that this correction is independent of the mass of the pair field. The correction δ_2 can be obtained most directly from formula (53), p. 546 in Karplus and Kroll[2] by a minor modification. The result is

$$\delta_1\cong\frac{\alpha}{2\pi}-\left(\frac{\alpha}{\pi}\right)^2(2.973), \tag{2}$$

$$\begin{aligned}\delta_2&=\left(\frac{\alpha}{\pi}\right)^2\int_0^1 dx\int_0^1 dy\; x^2(1-x)y^2(1-\tfrac{1}{3}y^2)\\&\qquad\times\left[x^2(1-y^2)+\frac{4m_0^2}{m_\mu^2}(1-x)\right]^{-1}\\&=\left(\frac{\alpha}{\pi}\right)^2\left[\tfrac{1}{3}\ln\left(\frac{m_\mu}{m_0}\right)-\frac{25}{36}\right]+\alpha^2 O\left(\frac{m_0}{m_\mu}\right),\end{aligned} \tag{3}$$

$$\delta_2\cong\left(\frac{\alpha}{\pi}\right)^2(1.08),$$

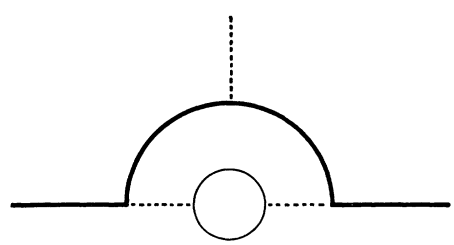

Fig. 1. Feynman diagram for the correction δ_2 to the magnetic moment of a mu meson. The heavy solid lines refer to mu mesons, the thin solid lines to electrons, and the dotted lines to photons.

where m_μ is the mass of the mu meson and m_0 is the mass of the electron. Numerically the term δ_2 is seen to be of the same order of magnitude as the usual fourth-order correction.

This note was stimulated by recent advances in experimental techniques for the measurement of the magnetic moment of the mu meson. It does not seem inconceivable that it will be possible to measure both the mass and the magnetic moment of the mu meson with an accuracy sufficient to test these radiative corrections. The experimental accuracy will almost certainly be sufficient to test the correction of order α.

In this connection we wish to draw attention to a note by Berestetskii, Krokhin, and Khlebnikov[3] concerning the effect on the magnetic moment of the mu meson of a modification of quantum electrodynamics at small distances.

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We use this opportunity to express our gratitude to The Institute for Advanced Study and its Director, Dr. Robert Oppenheimer for kind hospitality shown us during our stay here.

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Magnetic Moment of the μ Meson

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IN the very recent past, the experimental g value—and thus the magnetic moment—of the μ^+ meson was still so uncertain that it did not allow one even to decide whether its spin was $\frac{1}{2}$ or $\frac{3}{2}$. Now, new and powerful methods, due to Garwin, Lederman, and Weinrich,[1] have already determined it to be $+2.00 \pm 0.1$. Moreover these authors have designed a magnetic resonance experiment to determine the magnetic moment to $\sim 0.03\%$. This is only one order of magnitude bigger than the α^2 corrections to this moment, and it seems to be worthwhile, owing to these rapid improvements of the experimental situation, to look into the predictions of quantum electrodynamics.

For the μ meson, with spin $\frac{1}{2}$, the results of Schwinger[2] and Karplus and Kroll[3] can be applied, but one has to consider, in the fourth-order corrections, one more term, the contribution of which is not negligible. It is due to the vacuum polarization effect by electrons during the virtual photon propagation. Its contribution to the magnetic moment is given, in units of $e\hbar(2Mc)^{-1}$, by the integral

$$\mu_P = \frac{\alpha^2}{\pi^2}\int_0^1 du \int_0^1 dv \frac{u^2(1-u)v^2(1-v^2/3)}{u^2(1-v^2)+\lambda(1-u)},$$

with $\lambda = 4m^2/M^2$, m and M being the electron and the μ-meson masses, respectively.

This yields

$$\mu_P = \frac{\alpha^2}{\pi^2}\left[\tfrac{1}{6}\ln(1/\lambda) + \tfrac{1}{3}\ln 2 - \frac{25}{36} + \epsilon\right],$$

the error ϵ being shown to be less than $O(\lambda^{\frac{1}{2}})$. With $M = 207.2m$, the numerical value is

$$\mu_P = (\alpha^2/\pi^2)(1.08),$$

and together with the results of the previous authors, the magnetic moment of the μ meson amounts to

$$\mu = \left[1 + \frac{\alpha}{2\pi} - \left(\frac{\alpha^2}{\pi^2}\right)1.89\right](e\hbar/2Mc).$$

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K^+ Production in p-p Collisions at 3.0 Bev*

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FOR some time K^+ particle beams emanating from heavy nuclei have been observed at the Cosmotron and Bevatron,[1–5] first by emulsion and then by counter techniques. The direct observation of strange particle production by π^- mesons of kinetic energy ~ 1.4 Bev incident on hydrogen has been studied by the Brookhaven hydrogen diffusion cloud chamber group[6] and by other groups,[7] and it has been found that, of the total $\pi^- + p$ inelastic cross section of ~ 25 millibarns, about 1 millibarn corresponds to strange particle production of the type

$$\pi^- + p \rightarrow \text{hyperon} + K \text{ meson}.$$

The observation[3–5] of K^+ mesons produced in heavy nuclei at various angles (60–90°) and lab momenta (300–500 Mev/c) gave relative cross sections, expressed in terms of the K^+/π^+ ratio at the target, of $\sim 1/20$ to 1/100.

Using the known order of magnitude cross sections for production of high-energy pions and the previously stated cross section of ~ 1 millibarn for the $\pi^- + p$ interaction leading to strange particle production, one