Bremsstrahlung Spectra Corrected for Multiple Scattering in the Target*

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The spectral distribution in the forward direction of bremsstrahlung from a platinum target is evaluated for some typical cases, taking into account the multiple scattering of the incident electrons in the target. It is found that the proportion of high-energy photons is larger than in Schiff's intrinsic integrated spectrum. Tables are given of the correction factors to be applied to Schiff's spectrum for some typical target thicknesses and electron energies from 10–300 Mev. For incident electrons of energies 10–70 Mev it is found that the intrinsic differential spectrum for $\vartheta=0$ gives a better approximation to the spectrum corrected for multiple scattering than the integrated spectrum does.

1. INTRODUCTION

THE x-ray spectrum produced by high-energy electrons hitting a target is of interest both theoretically and because of the extensive use of such x-rays in the investigation of photonuclear reactions.

Bethe and Heitler[1] have calculated the intrinsic differential cross section for the production of photons of energy k as a function of the angles of the photon and scattered electron. Assuming an exponential screening law, Schiff[2] has integrated this formula over all angles of the electron, and has obtained an analytical expression for the intrinsic angular distribution of photons of energy k. Integrating again over all angles of the photons, he obtains the intrinsic integrated energy spectrum. His angular distribution is not valid for large angles because of the use of the Born approximation. The error which this approximation introduces into the spectral distributions is discussed by Bethe, Maximon, and Davies[3,4] and by Olsen.[5]

For a target of finite thickness, various secondary effects have to be taken into account. The effect of energy loss and absorption in the target and the double radiation process have been computed by several authors.[6–9] These effects should, however, be negligible as long as the target thickness is less than 0.1 radiation length.[10]

Another correction comes from the multiple elastic scattering which the electrons undergo in the target before radiating. In the first place the multiple scattering will have the effect of broadening the angular distribution of the x-rays. Approximate expressions for the corrected angular distribution of the bremsstrahlung irrespective of the energy of the photons have been calculated by Schiff[11] and Lawson.[10,12,13] Sirlin[14] has computed formulas for the angular distribution of the different spectral components.

From the experimental point of view, the radiation emitted from the target in the forward direction is of special interest. The spectrum of this forward radiation corrected for multiple scattering in the target, we shall call the "corrected spectrum." In order to derive an expression for the corrected spectrum, it is usual to neglect the variation of the logarithmic term in the intrinsic bremsstrahlung formula with angle and energy.[2,10,14] It is then found that the shape of the corrected spectrum is the same as that of the intrinsic integrated spectrum. In the present article we take the variation of the logarithmic term into account, and find the corrected spectrum for some typical cases by numerical integration.

The corrected spectrum computed in this paper is valid for experiments in which the angular aperture of the beam or of the x-ray detector is small (less than or of the order of the rest energy of the electron divided by the total energy of the primary electron). Several of the published measurements have used such small apertures.[7,15–23] In the case of larger angular apertures, the spectrum will be more nearly equal to the intrinsic integrated spectrum.

2. INTRINSIC BREMSSTRAHLUNG DISTRIBUTION

We shall use the following notation: E_0= total energy of the primary electron, E= total energy of the electron after it has emitted a photon, $k=E_0-E$= energy of

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[1] H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).
[2] L. I. Schiff, Phys. Rev. **83**, 253 (1951).
[3] H. A. Bethe and L. C. Maximon, Phys. Rev. **93**, 768 (1954).
[4] Davies, Bethe, and Maximon, Phys. Rev. **93**, 788 (1954).
[5] H. Olsen, Phys. Rev. **99**, 1335 (1955).
[6] L. Eyges, Phys. Rev. **81**, 981 (1951).
[7] I. B. McDiarmid, Phil. Mag. **43**, 1003 (1952); Can. J. Phys. **30**, 670 (1952).
[8] C. D. Curtis, Phys. Rev. **89**, 123 (1953).
[9] R. Wilson, Proc. Phys. Soc. (London) **A66**, 638 (1953).
[10] J. D. Lawson, Nucleonics **10**, No. 11, 61 (1952).
[11] L. I. Schiff, Phys. Rev. **70**, 87 (1946).
[12] J. D. Lawson, Proc. Phys. Soc. (London) **A63**, 653 (1950).
[13] J. D. Lawson, Phil. Mag. **43**, 306 (1952).
[14] A. Sirlin, Phys. Rev. **101**, 1219 (1956).
[15] P. K. S. Wang and M. Wiener, Phys. Rev. **76**, 1724 (1949).
[16] H. W. Koch and R. E. Carter, Phys. Rev. **77**, 165 (1950).
[17] K. Philips, Proc. Phys. Soc. (London) **A65**, 57 (1952); **A67**, 669 (1954).
[18] V. E. Krohn and E. F. Schrader, Phys. Rev. **87**, 685 (1952).
[19] Motz, Miller, and Wyckoff, Phys. Rev. **89**, 968 (1953).
[20] R. M. Warner and E. F. Schrader, Rev. Sci. Instr. **25**, 663 (1954).
[21] E. V. Weinstock and J. Halpern, Phys. Rev. **100**, 1293 (1955).
[22] N. Starfelt and H. W. Koch, Phys. Rev. **102**, 1598 (1956).
[23] E. Hisdal and J. P. Roalsvig, Arch. Math. Naturvidenskab. **53** (1956).

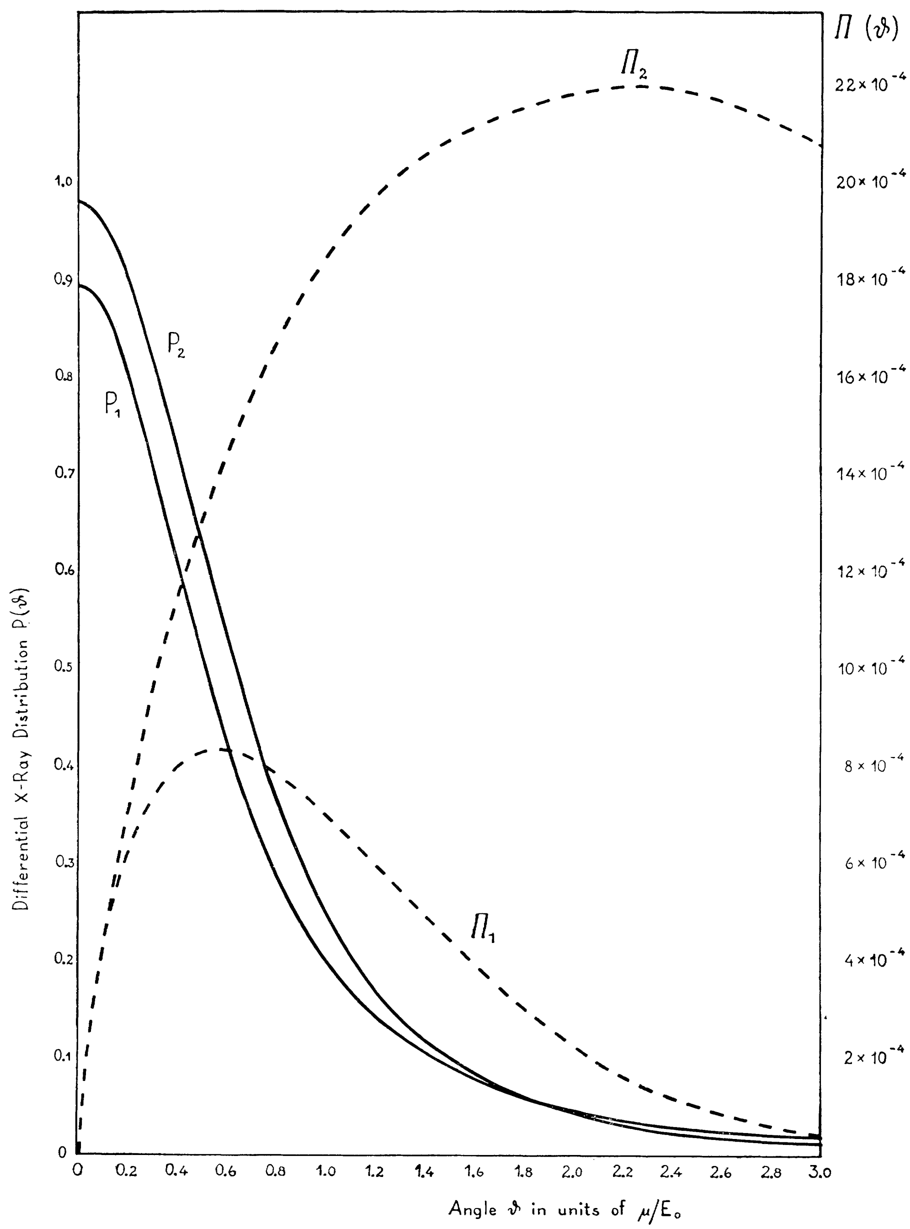

FIG. 1. P curves—Angular distribution of the intrinsic bremsstrahlung from Pt per solid angle $\pi(\mu/E_0)^2$ steradian for $E_0-\mu=30$ Mev. P_1—spectral component $k/(E_0-\mu)=0.1$; P_2—spectral component $k/(E_0-\mu)=0.9$. Both curves give the same flux through a sphere around the target. Π curves—effective multiple scattering distributions of electrons. Π_1—0.5-mil Pt target; Π_2—5-mil Pt target.

photon, $\epsilon=E/E_0$=fractional energy of scattered electron, $\kappa=k/E_0=1-\epsilon$=fractional energy of photon, μ=rest energy of electron, μ/E_0=average angle of emission of photons which is used as unit for measuring bremsstrahlung angles, θ=angle between incoming electron and emitted photon in radians, $\vartheta=\theta/(\mu/E_0)$, Z=atomic number of target material, and n=number of target atoms per cm^3.

Considering only the intrinsic bremsstrahlung distribution, one obtains the following formula for the number of photons of fractional energy between κ and $\kappa+d\kappa$ which are emitted into a solid angle $\pi(\mu/E_0)^2$ steradian around ϑ[24]:

$$\nu(\kappa,\vartheta)d\kappa=\frac{2nZ^2}{137}\left(\frac{e^2}{\mu}\right)^2\frac{d\kappa}{\kappa}i(\kappa,\vartheta). \qquad (1a)$$

Here we assume an incident beam of 1 electron per cm^2 and a target 1 cm thick.

The quantity $i(\kappa,\vartheta)$ is proportional to the intensity (number of photons multiplied by their energy) per

[24] We have here halved Schiff's numerical factor to make it agree with the factor in Eq. (2a). Therefore we must use half the solid angle, i.e., $\pi(\mu/E_0)^2$ instead of $2\pi(\mu/E_0)^2$.

unit solid angle, and is given according to Schiff[2] by:

$$i(\kappa,\vartheta)=\frac{1}{(1+\vartheta^2)^2}\left\{\frac{16\vartheta^2\epsilon}{(1+\vartheta^2)^2}-(1+\epsilon)^2+\left[1+\epsilon^2-\frac{4\vartheta^2\epsilon}{(1+\vartheta^2)^2}\right]\ln M(\vartheta)\right\}, \quad (1b)$$

where

$$\frac{1}{M(\vartheta)}=\left(\frac{\mu}{2E_0}\frac{\kappa}{\epsilon}\right)^2+\left[\frac{Z^{\frac{1}{3}}}{111(1+\vartheta^2)}\right]^2. \quad (1c)$$

Integrating over all angles of the photons, Schiff obtains for the total number of photons of fractional energy between κ and $\kappa+d\kappa$ from a target of thickness 1 cm:

$$\mathrm{N}(\kappa)d\kappa=\frac{2nZ^2}{137}\left(\frac{e^2}{\mu}\right)^2\frac{d\kappa}{\kappa}I(\kappa), \quad (2a)$$

where

$$I(\kappa)=(1+\epsilon^2-\tfrac{2}{3}\epsilon)\times\left[\ln M(0)+1-\frac{2}{b}\tan^{-1}b\right]+\epsilon\left[\frac{2}{b^2}\ln(1+b^2)+\frac{4(2-b^2)}{3b^3}\tan^{-1}b-\frac{8}{3b^2}+\frac{2}{9}\right] \quad (2b)$$

and

$$b=(2Z^{\frac{1}{3}}/111\mu)E_0\epsilon/\kappa.$$

I is proportional to the intensity of photons of fractional energy κ, integrated over all angles. We shall call the spectrum given by Eqs. (1) the intrinsic differential spectrum, and the one given by Eqs. (2) the intrinsic integrated spectrum.

In Fig. 1, curves P_1, P_2, we have plotted the angular distributions of the photons $i(\kappa,\vartheta)$ for two values of κ according to Eq. (1b), multiplied by the normalization factor $1/I(\kappa)$; i.e., the distributions $P=i/I$ are normalized in such a way that the photon flux through a sphere around the target is the same for all curves of a given E_0. (More exactly, the ordinates of the curves represent the fraction of all photons of energy k which is radiated into a solid angle $\pi(\mu/E_0)^2$ steradian around ϑ.) It can be seen that low-energy photons (curve P_1) are more likely to be emitted at large angles than high-energy ones. Figure 1 refers to incident electrons of 30-Mev kinetic energy.

3. MULTIPLE SCATTERING

Molière[25] has calculated the angular distribution of electrons which are scattered by a given thickness of absorber. To a first approximation the distribution per unit solid angle is Gaussian with a width proportional to $1/E_0$. It will therefore be convenient to measure angles in units of μ/E_0 as before. We shall use the notation: t_0= total thickness of target in cm, t= thickness of target in cm at which electron radiates, ϑ = multiple scattering angle at thickness t in units of μ/E_0, β= velocity of electron divided by velocity of light, ρ and A= density and atomic weight of target material; Ω_B is a measure of the average number of elastic scattering collisions in thickness t, and is given by:

$$\Omega_B=s\frac{t}{\beta^2 Z^{\frac{2}{3}}(1.13+2.003\times10^{-4}Z^2/\beta^2)},$$

where

$$s=\pi(2\times0.4685\times10^{-8}/137)^2 nZ^2=1.4696\times10^{-20}nZ^2=8854(\rho/A)Z^2.$$

B= greater solution of the equation

$$B-\ln B=\ln\Omega_B-0.1544,$$

$$C^2=\frac{4\pi e^4 nZ^2B}{\mu^2\beta^4}=0.9983\times10^{-24}\frac{nZ^2B}{\beta^4}=0.6014\frac{(\rho/A)Z^2B}{\beta^4}.$$

With this notation, Molière's theory gives the following expression for the probability that an electron which passes through a foil of thickness t is multiply scattered into an angle between ϑ and $\vartheta+d\vartheta$:

$$\frac{2}{C\sqrt{t}}\vartheta d\left(\frac{\vartheta}{C\sqrt{t}}\right)\exp\left[-\left(\frac{\vartheta}{C\sqrt{t}}\right)^2\right]=\frac{2}{C^2t}\vartheta\exp\left(-\frac{\vartheta^2}{C^2t}\right)d\vartheta. \quad (3)$$

The multiple scattering distribution per unit solid angle is thus Gaussian with a $1/e$ width of $(C\sqrt{t})\mu/E_0$ radian. For highly relativistic electrons ($\beta^2\approx1$) the parameters Ω_B, B, and C are independent of the energy of the electron, E_0.

Actually the Gaussian distribution is only the first term of a more exact expression given by Molière. The deviation from the Gaussian curve is, however, important only for large values of $\vartheta/(C\sqrt{t})$. Except for very thin targets the deviation is negligible in the case of the forward spectrum.

In Table I we have listed Ω_B, B, C^2, and $C\sqrt{t_0}$ for platinum of three thicknesses. We see that C changes only very slowly with the thickness t. Molière gives a table from which the values of B can be interpolated for all practical values of Ω_B.

TABLE I. Multiple scattering values in Pt.

Target thickness:			
$t_0=2t$ in cm	0.00127	0.0127	0.127
in mils	0.5	5	50
in g/cm²	0.02714	0.2714	2.714
in radiation lengths	0.00441	0.0441	0.441
Ω_B	87.306	873.06	8730.6
B	6.1274	8.7908	11.3490
C^2	2454.1	3520.9	4545.5
$C\sqrt{t_0}$	1.7655	6.6890	24.0269

[25] G. Molière, Z. Naturforsch. **3a**, 78 (1948).

TABLE II. $\int P(\vartheta)\Pi(\vartheta)d\vartheta$. Values in table $\times 10^{-4}$ are the factors by which the intrinsic integrated spectrum from 1-cm Pt [Eqs. (2a,b), $Z=78$] must be multiplied in order to obtain the absolute value of the forward spectrum corrected for multiple scattering. Values refer to a Pt target of indicated thickness and to a solid angle of $\pi(\mu/E_0)^2$ steradian.

κ \ $E_0-\mu$ Mev	10	20	30	70	300
	Target thickness 0.5 mil = 0.00127 cm				
0	3.936	3.936	3.936	3.936	3.936
0.0983	4.079	4.014	3.989	3.959	3.941
0.295	4.353	4.188	4.116	4.020	3.957
0.492	4.587	4.387	4.276	4.108	3.981
0.688	4.741	4.600	4.486	4.253	4.026
0.885	4.774	4.768	4.735	4.568	4.184
	Target thickness 5 mils = 0.0127 cm				
0	8.105	8.105	8.105	8.105	8.105
0.0983	8.358	8.252	8.209	8.154	8.118
0.295	8.747	8.533	8.430	8.280	8.166
0.492	9.055	8.823	8.686	8.455	8.250
0.688	9.274	9.113	8.986	8.709	8.387
0.885	9.368	9.342	9.302	9.125	8.679
	Target thickness 50 mils = 0.127 cm				
0		11.513	11.513	11.513	11.513
0.0983		11.674	11.631	11.572	11.530
0.295		11.936	11.844	11.703	11.585
0.492		12.193	12.075	11.870	11.676
0.688		12.442	12.336	12.104	11.815
0.885		12.643	12.610	12.462	12.084

The electrons radiate with equal probability at each thickness of the target. (We are here neglecting energy loss and double radiation processes.) In order to find the effective angular distribution $\Pi(\vartheta)$, we integrate the scattering distribution (3) over the whole thickness t_0 and obtain:

$$\Pi(\vartheta)d\vartheta=\frac{2\vartheta d\vartheta}{C^2}\int_0^{t_0}\frac{1}{t}\exp\left(-\frac{\vartheta^2}{C^2t}\right)dt$$

or[26]

$$\Pi(\vartheta)d\vartheta=\frac{2\vartheta d\vartheta}{C^2}\left[-\mathrm{Ei}\left(-\frac{\vartheta^2}{C^2t_0}\right)\right]. \tag{4}$$

$\Pi(\vartheta)d\vartheta$ is the probability that the electron is scattered into an angle between ϑ and $\vartheta+d\vartheta$ (at the time it radiates)$\times$thickness of target. The dotted curves of Fig. 1 show $\Pi(\vartheta)$ for a 0.5- and 5-mil Pt target respectively. For highly relativistic electrons the curves are independent of E_0 when ϑ is measured in units of μ/E_0.

In deriving Eq. (4) we assumed that C is constant throughout the whole thickness of the target. This assumption does not introduce a large error, since we saw that when the thickness is increased by a factor of 100, C^2 is increased by a factor of 1.9 only. For each target we have listed in Table I average values of Ω_B, B, and C^2 corresponding to half the target thickness.

[26] The exponential integral function $\mathrm{Ei}(-x)$ was used by Schiff[11] in this connection. For tables of $\mathrm{Ei}(-x)$ see E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945); also *Tables of Sine Cosine and Exponential Integrals* (Mathematical Tables Project, New York, 1940), 2 volumes.

TABLE III. Factors by which the intrinsic integrated spectrum [Eqs. (2a,b)] must be multiplied in order to obtain the relative corrected spectrum in Pt. All values are normalized to unity at $\kappa=0$.

κ \ $E_0-\mu$ Mev	10	20	30	70	300
	Target thickness 0.5 mil = 0.00127 cm				
0	1	1	1	1	1
0.0983	1.0364	1.0198	1.0135	1.0060	1.0014
0.295	1.1061	1.0640	1.0458	1.0215	1.0055
0.492	1.1655	1.1146	1.0865	1.0436	1.0114
0.688	1.2046	1.1689	1.1398	1.0806	1.0230
0.885	1.2131	1.2115	1.2030	1.1607	1.0631
	Target thickness 5 mils = 0.0127 cm				
0	1	1	1	1	1
0.0983	1.0313	1.0181	1.0128	1.0060	1.0016
0.295	1.0793	1.0528	1.0401	1.0216	1.0075
0.492	1.1172	1.0887	1.0717	1.0432	1.0179
0.688	1.1442	1.1244	1.1087	1.0746	1.0348
0.885	1.1559	1.1526	1.1478	1.1259	1.0708
	Target thickness 50 mils = 0.127 cm				
0		1	1	1	1
0.0983		1.0140	1.0102	1.0051	1.0015
0.295		1.0367	1.0287	1.0165	1.0063
0.492		1.0591	1.0488	1.0310	1.0142
0.688		1.0807	1.0715	1.0513	1.0262
0.885		1.0982	1.0953	1.0824	1.0496

4. COMBINED EFFECT OF THE ANGULAR DISTRIBUTIONS OF MULTIPLE SCATTERING AND BREMSSTRAHLUNG

We want to find the spectral distribution of the photons emitted in the direction of the primary electron beam. If the electron is elastically scattered by an angle ϑ before radiating, the photon will have to be emitted at an angle ϑ back into the original direction. We must therefore consider the proportion of electrons $\Pi(\vartheta)d\vartheta$ [Eq. (4)] which are scattered into an angle between ϑ and $\vartheta+d\vartheta$. This we have to multiply by the proportion of electrons which emit a photon of energy k into a solid angle around the direction ϑ. In other words we must multiply the curve with the desired k value of Fig. 1 by the multiple-scattering curve belonging to the correct target thickness (dotted curve of the same figure) and integrate over all ϑ. The result will tell us what fraction of all the photons of energy k which leave the target are emitted in the forward direction into a solid angle $\pi(\mu/E_0)^2$ steradians. For each value of k, we will thus get a number by which we have to multiply the intrinsic integrated spectrum [computed for a target thickness of 1 cm, Eqs. (2)], in order to obtain the corrected spectrum for the particular target thickness. These numbers, which were obtained by numerical integration, are listed in Table II. In Table III these numbers are normalized to unity at $\kappa=0$. It can be seen from Table III that the shape of the corrected spectrum may differ considerably from the shape of the intrinsic integrated spectrum.

The solid curve I in Fig. 2 shows Schiff's intrinsic integrated spectrum in Pt [$I(\kappa)$, Eq. (2b)]. The relative corrected spectrum for a 5-mil target is shown by the

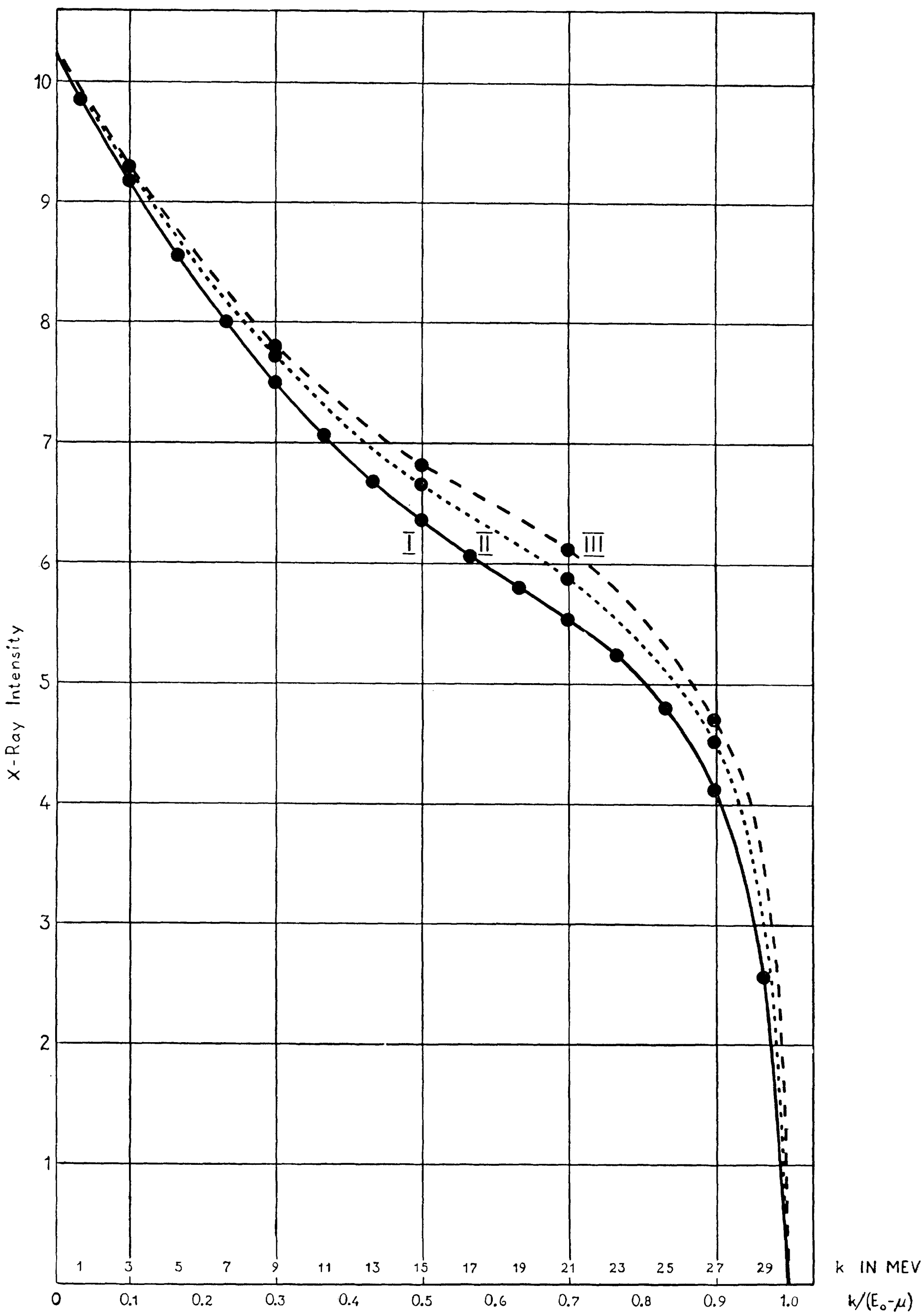

FIG. 2. Pt target, $E_0-\mu=30$ Mev. I—intrinsic integrated bremsstrahlung intensity $I(\kappa)$. II—intrinsic differential intensity in forward direction $i(\kappa,0)$. III—intensity in forward direction corrected for multiple scattering of the incident electrons in a 5-mil Pt target. Curves II and III are normalized to the same value as curve I at $k=0$. Circles show computed points.

dashed curve III. The curves refer to incident electrons of 30-Mev kinetic energy. The dotted curve II in the same figure is a plot of the intrinsic differential spectrum for $\vartheta=0(i(\kappa,0)$ from Eq. (1b) normalized to the integrated spectrum at $\kappa=0$). In this case the corrected spectrum lies nearer to the intrinsic differential spectrum than to the integrated one.

We have therefore in Table IV listed the correction factors to be applied to the intrinsic differential spectrum from a 1-cm Pt target [Eqs. (1) for $\vartheta=0$] in order to get the corrected spectrum for the indicated target thickness. In Table V these factors are normalized to unity at $\kappa=0$. Except for the 300-Mev data, the numbers in Table V are nearer to 1 than the corresponding numbers in Table III. Thus for electrons of 10–70 Mev, the differential spectrum for $\vartheta=0$ is a better approximation to the corrected spectrum than the integrated one. The formula for the differential spectrum is also easier to compute numerically.

Dividing Table IV by the corresponding target

TABLE IV. Values in table $\times 10^{-4}$ are the factors by which the intrinsic differential spectrum from a 1-cm thick target [Eqs. (1a,b) for $\vartheta=0$] must be multiplied in order to obtain the corrected spectrum in Pt for indicated target thickness.

κ \ $E_0-\mu$ Mev	10	20	30	70	300
		Target thickness 0.5 mil = 0.00127 cm			
0	2.232	2.232	2.232	2.232	2.232
0.0983	2.233	2.234	2.235	2.235	2.235
0.295	2.243	2.257	2.260	2.263	2.263
0.492	2.271	2.304	2.317	2.330	2.333
0.688	2.336	2.362	2.390	2.431	2.447
0.885	2.476	2.420	2.423	2.485	2.580
		Target thickness 5 mils = 0.0127 cm			
0	4.597	4.597	4.597	4.597	4.597
0.0983	4.576	4.594	4.599	4.602	4.604
0.295	4.506	4.598	4.629	4.660	4.671
0.492	4.483	4.635	4.707	4.795	4.835
0.688	4.569	4.679	4.787	4.978	5.098
0.885	4.858	4.741	4.761	4.963	5.351
		Target thickness 50 mils = 0.127 cm			
0		6.529	6.529	6.529	6.529
0.0983		6.499	6.516	6.532	6.539
0.295		6.432	6.504	6.587	6.627
0.492		6.405	6.544	6.733	6.843
0.688		6.389	6.571	6.918	7.182
0.885		6.417	6.454	6.778	7.451

TABLE V. Factors by which the intrinsic differential spectrum [Eqs. (1a,b) for $\vartheta=0$] must be multiplied in order to obtain the relative corrected spectrum in Pt. All values are normalized to unity at $\kappa=0$.

κ \ $E_0-\mu$ Mev	10	20	30	70	300
		Target thickness 0.5 mil = 0.00127 cm			
0	1	1	1	1	1
0.0983	1.0004	1.0011	1.0012	1.0013	1.0013
0.295	1.0047	1.0110	1.0126	1.0137	1.0141
0.492	1.0175	1.0324	1.0382	1.0437	1.0451
0.688	1.0464	1.0583	1.0706	1.0890	1.0965
0.885	1.1091	1.0842	1.0856	1.1131	1.1558
		Target thickness 5 mils = 0.0127 cm			
0	1	1	1	1	1
0.0983	0.9955	0.9994	1.0005	1.0013	1.0015
0.295	0.9804	1.0004	1.0072	1.0138	1.0162
0.492	0.9753	1.0084	1.0241	1.0433	1.0518
0.688	0.9940	1.0180	1.0414	1.0829	1.1091
0.885	1.0569	1.0314	1.0357	1.0797	1.1642
		Target thickness 50 mils = 0.127 cm			
0		1	1	1	1
0.0983		0.9954	0.9979	1.0004	1.0014
0.295		0.9851	0.9961	1.0088	1.0149
0.492		0.9810	1.0022	1.0311	1.0480
0.688		0.9785	1.0064	1.0595	1.0999
0.885		0.9827	0.9884	1.0380	1.1412

thickness, we get directly the reduction of the forward beam due to the multiple scattering, always assuming that our detector subtends a small angle at the target. For the three thicknesses 0.5, 5, and 50 mils the reduction factors are of the order 0.18, 0.037, and 0.0052, respectively.

5. DISCUSSION

As can be seen from Fig. 2 and Tables II and III, the corrected spectrum contains relatively more high-energy photons than the integrated one. This is because most of the photons radiated from the target in the forward direction come from electrons which are scattered by small angles ϑ (smaller than the cross-over point of P_1 and P_2 in Fig. 1). Since high-energy bremsstrahlung quanta are more likely to be emitted with small ϑ than low-energy ones, the corrected spectrum shows a relative increase of high-energy photons.

The correction to the intrinsic spectrum as displayed by the factors in Table III, depends somewhat on the atomic number of the target material. It is mainly the different variation of $M(\vartheta)$ [Eq. (1c)] for different κ which determines the correction factors. For smaller Z the relative importance of that term in $1/M(\vartheta)$ which depends on κ increases. Therefore a table equivalent to Table III for smaller Z may be expected to contain somewhat bigger correction factors. The multiple scattering works in the same direction, as smaller Z means narrower scattering distributions and is therefore equivalent to decreasing the target thickness.

In the course of this work we have computed the intrinsic differential distribution $i(\kappa,\vartheta)$ [Eqs. (1b), (1c)], the differential normalized distribution $P(\kappa,\vartheta)$, and the integral spectrum $I(\kappa)$ [Eq. (2b)] for incident electrons of 10, 20, 30, 70, and 300 Mev. Tables of these values together with a more detailed discussion will be published in *Archiv for Mathematik og Naturvidenskab.*