Configurational Mixing in β-Decay Theory

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It is shown that the deviations of $\log ft$ values from those predicted by the single-particle model in the β-decay of odd-A nuclei can be accounted for in terms of simple interconfigurational mixing.

In a recent paper by Nordheim and Grayson[1] the nuclear matrix elements for allowed β decays have been evaluated on the basis of the j-j coupling independent-particle model. These results show that the use of properly antisymmetrized many-particle functions appreciably reduces the contradiction between the experimental and single-particle-model $\log ft$ values. However there is still a considerable discrepancy between the experimental results and the predictions of the independent-particle model.[2] A similar situation exists with regard to nuclear magnetic moments, and it has been shown[3,4] that here the disagreement may be attributed to interconfigurational mixing caused by the interactions between those nucleons outside closed (in the LS sense) shells. A similar calculation has been made, therefore, to estimate the effect of such mixing on the nuclear matrix elements for allowed β transitions.

As an example we consider the decays

$$\text{(I)}\quad {}_{21}\mathrm{Sc}^{43}\rightarrow{}_{20}\mathrm{Ca}^{43},$$
$$\text{(II)}\quad {}_{22}\mathrm{Ti}^{43}\rightarrow{}_{21}\mathrm{Sc}^{43}.$$

Using appropriate isotopic spin eigenfunctions of seniority one,[5] we consider the effect of those admixtures which contribute to the matrix element terms linear in their amplitudes of mixing. The internucleon interaction is taken to be of the form

$$V_{ij}=(a+b\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\delta(\mathbf{r}_i-\mathbf{r}_j)$$

(a similar interaction having been used in the magnetic moment calculations[3,4]) and we assume that the ratio of the triplet to singlet strengths is 3:2. We define $\xi=\epsilon_s/\Delta E$ (always negative) where ϵ_s is the singlet interaction energy[6] and ΔE the $f_{7/2}-f_{5/2}$ spin-orbit splitting. A straightforward first-order perturbation calculation gives

$$\begin{aligned}&\text{(I)}\quad \langle 1\rangle=0;\quad \langle\boldsymbol{\sigma}\rangle=\langle\boldsymbol{\sigma}\rangle_{\text{s.p.}}\frac{2}{3}\left[1+\frac{125}{63}\xi\right],\\&\text{(II)}\quad \langle 1\rangle=1;\quad \langle\boldsymbol{\sigma}\rangle=\langle\boldsymbol{\sigma}\rangle_{\text{s.p.}}\frac{5}{9}\left[1-\frac{148}{63}\xi\right],\end{aligned}\tag{i}$$

where $\langle\boldsymbol{\sigma}\rangle_{\text{s.p.}}$ is the single-particle-model value. Taking[2]

$$ft=5300/[|\langle 1\rangle|^2+|\langle\boldsymbol{\sigma}\rangle|^2],\tag{ii}$$

we find that for $\xi=-0.31$ this gives

	Experimental	$\log ft$ s.p.m.	j-j	Calc
I:	4.8	3.37	3.97	4.80
II:	3.4		3.58	3.38

When one uses the same value of ξ, the results of Arima and Horie[4] give for the magnetic moment of ${}_{20}\mathrm{Ca}^{43}$, $\mu=-1.15$, the measured value being -1.3. Thus it would appear that our somewhat crude approximations can in fact reproduce (at least semiquantitatively) the experimental results.

For more than three particles such a calculation is not practicable using as basic states isotopic spin eigenfunctions. We have therefore made a complete calculation, for odd-A nuclei, including the effects of all possible admixtures which contribute in first order to the matrix elements, using wave functions that are separately antisymmetric in the neutrons and protons respectively. Such functions may not be satisfactory for light nuclei so we restrict ourselves to $A>40$. Moreover for the $\Delta J=0$ transitions we must in addition assume that $|\langle 1\rangle|^2$ vanishes for the unfavored transitions, and there is every indication that it is necessary to use isotopic spin eigenfunctions (as with the two decays already described) in order to account for the experimental results. However, for the $\Delta J=1$ transitions we expect these functions to be a reasonable approximation. Again we have considered only those transitions for which the unperturbed states are assumed to be of lowest seniority, i.e., the even nucleons in the state of seniority zero and the odd of seniority one. For the $\Delta J=1$ transitions it is found that the matrix element is of the form,

$$\langle\boldsymbol{\sigma}\rangle=\langle\boldsymbol{\sigma}\rangle_{jj}[1+\textstyle\sum_n A_n\xi_n],\tag{iii}$$

where $\langle\boldsymbol{\sigma}\rangle_{jj}$ is the unperturbed element and $\xi_n=(\epsilon_s)_n/(\Delta E)_n$. Here $(\epsilon_s)_n$ and $(\Delta E)_n$ are the singlet interaction energies and energy differences, respectively, for the different possible particle excitations which lead to first-order contributions to the matrix element. The coefficients A_n are generally positive and in all cases considered are such that with reasonable values of ξ_n the reduction in $|\langle\boldsymbol{\sigma}\rangle|^2$ is sufficient to account for the large $\log ft$ values of the $\Delta J=1$ transitions.

Finally, for the l-forbidden transitions it is clear that there are possible admixtures which will contribute linearly to the matrix element, and a calculation to estimate this effect is in progress. A full account of this work will be published later.

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