

the $l_n=1$ assignments indicated by 8-Mev data¹⁵ and our data. This difference could be due to effects similar to those seen in our Mg²⁴ data (Figs. 2, 3, and 4).

Some of the qualitative features of simple stripping theory are shown in all of our data and those of Allen *et al.*¹⁴ They are the deuteron energy independence and the angular symmetry in the (d,p) plane. The predicted azimuthal isotropy around the axis of the recoil nucleus is seen in Be⁹ but not in Si²⁸.¹⁴ The fact that proton polarization is observed both at deuteron energies of 8 to 12 Mev and of 4 Mev also implies that effects which perturb simple stripping theory must be investigated in any reaction by means of azimuthal correlation and/or polarization data. One must also investigate deuteron energy dependence in each case. Although proton angular distributions should resemble stripping,

this observation does not guarantee a concomitant stripping correlation.

Our Be⁹ data appear to be correctly described by simple correlation theory, but polarization and correlation data at higher deuteron energies would be very interesting. The Mg²⁴ correlation appears to be degraded, but extensive work on it would be desirable because there are no adjustable quantum numbers in the description of the reaction. This would simplify the detailed analysis of perturbing effects. This detailed analysis is difficult, but from it one obtains information about nucleon-nucleus interactions as well as information about nuclear energy levels.

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Splitting of 3S_1 and 1S_0 States in Li⁶

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The splitting of the 3S_1 and 1S_0 states of Li⁶ is calculated by applying Feingold's procedure of configuration mixing to the wave function which was used in our former calculation of the binding energy of the same nucleus. It is found that the usually used tensor force is strong enough to explain the big value of this splitting.

MORITA and Tamura¹ formerly calculated the binding energy of Li⁶ by using a potential chosen to fit the two-body data, and a wave function modified from the usual shell-model wave function in such a way that it depends only on the relative distance of the constituent particles. The particular two-body potential they used consisted of an almost pure Wigner-Yukawa central force and a Yukawa tensor force. The result that they obtained agreed fairly well with experiment. In that paper, the splitting δE , of the 3S_1 and 1S_0 states was also calculated, with a result $\delta E=0.16$ Mev, much too small to agree with the experimental value, 3.6 Mev. In a similar way, the theoretical value of this splitting, calculated in the lowest order perturbation theory with the usual shell-model wave function, gives too small a result, 0.21 Mev.

In a recent paper, Feingold² has shown that starting with the shell model, and taking into account a certain kind of configuration mixing, a rather big value, 1.6 Mev, can be obtained. (This value, however, is still

somewhat less than half the experimental value.³) The reason why MT obtained good agreement for the binding energy in Li⁶ appears to be that the wave function they employed had much larger positional correlations than the shell model does. Thus, it would seem that a larger value for the splitting could also be obtained by applying Feingold's idea not to the shell-model wave function, but to the more highly correlated wave function of MT.

In general, the application of Feingold's procedure to such a wave function is not easy, because some very complicated integrals must be evaluated. However, if we limit our calculation to S states, and also make the approximation that the three- and four-particle terms, in the terminology of Feingold, can be neglected compared to the two-particle terms, the whole integration can be carried through analytically, and although the final result is much more lengthy than in the case of Feingold, we can obtain the desired results fairly easily.

The two-particle interaction used in the present calculation is the following⁴:

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¹ M. Morita and T. Tamura, Progr. Theoret. Phys. (Japan) **12**, 653 (1954); referred to hereafter as MT; see also J. Irving and D. S. Schonland, Phys. Rev. **97**, 446 (1955).

² A. M. Feingold, Phys. Rev. **101**, 258 (1956).

³ It is remarked in reference 2 that Lyons obtained $\delta E=1.9$ Mev by using a Yukawa-type interaction.

⁴ P. L. Pease and H. Feshbach, Phys. Rev. **81**, 142 (1951); **88**, 945 (1952).

$$V_{ij} = V_c \frac{e^{-r_{ij}/r_c}}{(r_{ij}/r_c)} [(1-g) + gP_{\sigma}^{ij}] + S_{ij} V_t \frac{e^{-r_{ij}/r_t}}{(r_{ij}/r_t)} + \frac{e^2 (1-\tau_z^{(i)})(1-\tau_z^{(j)})}{r_{ij} 4}, \quad (1)$$

where $V_c = 46.1$ Mev, $V_t = 24.9$ Mev, $r_c = 1.184 \times 10^{-13}$ cm, $r_t = 1.67 \times 10^{-13}$ cm, P_{σ}^{ij} is the spin exchange operator, $g = 0.004$, and S_{ij} is the usual tensor operator.

Feingold's procedure is briefly to alter the expression for the energy given by the lowest order shell-model wave function ψ_0 which is proportional to $\exp(-\beta^2 r_i^2)$. He does this by modifying ψ_0 in the form $(1 + \lambda \sum_{i < j} t_{ij}) \psi_0$, where $t_{ij} = (\beta r_{ij}) S_{ij} \exp(-\nu^2 r_{ij}^2)$, and then fixing the parameter λ at each root-mean-square radius (which is inversely proportional to β) so as to minimize the energy.

We have correspondingly modified MT's wave function, which is now proportional to $\exp[-\alpha(r_{ij}^2)^{\frac{1}{2}}]$, and in the same way have variationally minimized the energy for each mean square radius which is given, if one neglects the small change caused by the mixing of the D state, by $R = r_q 6^{\frac{1}{2}} = 1.876 \alpha^{-1}$.

The results of the numerical calculations are shown in Fig. 1. In this figure the abscissa r_q is given in units of 10^{-13} cm. Feingold obtained the above-mentioned value $\delta E = 1.5$ Mev, for a value $r_q = 1.2$, the value of the nuclear radius found in high-energy electron scattering.⁵ In our calculation, this value of r_q is very near to the one for which the maximum binding energy [illustrated by curve (d) in Fig. 1] is obtained. For this value, we find a splitting which is gratifyingly large (in fact, too large): $\delta E = 5.4$ Mev.

We wish to point out, however, that various arguments may be adduced in favor of a somewhat larger radius. The high-energy electron scattering experiments⁶ indicate that light nuclei are spread out more than the heavy ones, and Kofoed-Hansen⁷ has also shown that the Coulomb energy differences between Li^6 and its mirror nucleus He^6 requires that $r_q = 1.35$. With this larger radius our value for δE is 4.6 Mev, which agrees more closely with experiment. With this radius we also obtain a binding energy, 31.4 Mev, in close agreement with the experimental value, 32.0 Mev.

Because we have made several simplifying approximations, this rather satisfactory agreement with experiment cannot be taken too literally. In particular, we have neglected the contribution from three- and four-particle terms, and we have mixed D states to the

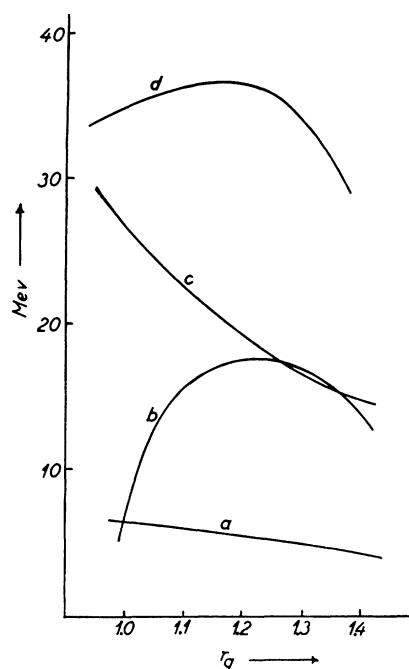


FIG. 1. Various energy values *versus* reduced nuclear radius r_q . (a) 3S_1 - 1S_0 splitting; (b) binding energy without tensor force (=algebraic sum of kinetic and central and Coulomb interaction energies); (c) contribution of the tensor interaction to the binding energy; (d) total binding energy [= (b) + (c)].

original S states in a manner which is not unique. Furthermore, the difference between our results and Feingold's shows that the results are quite sensitive to the assumed wave function.

There should also be many other ways in choosing parameters of the two-body interaction in addition to those used in this paper. It should further be mentioned that the recently developed meson theory of nuclear forces and the analysis of high-energy nucleon-nucleon scattering suggest the use of a potential with a hard core.⁸ In such a case Brueckner's theory⁹ of the optical model may be useful in obtaining the reaction matrix to be used in place of the usual short-range potential as employed here.

In spite of these ambiguities, however, it appears to be possible to conclude that the commonly used tensor force, the parameters of which are adjusted to explain the two-body data, is strong enough to explain the big splitting of the 3S_1 and 1S_0 states of Li^6 .

⁵ Hofstadter, Hahn, Knudsen, and McIntyre, Phys. Rev. **95**, 512 (1954); D. G. Ravenhall and D. R. Yennie, Phys. Rev. **96**, 239 (1954).

⁶ J. H. Fregeau and R. Hofstadter, Phys. Rev. **99**, 1503 (1955).

⁷ O. Kofoed-Hansen, Nuclear Phys. **2**, 441 (1956-1957).

⁸ In this connection see Otsuki, Sawada, and Suekane, Progr. Theoret. Phys. (Japan) **13**, 79 (1955). In their paper, however, a spin-orbit force of usual type was assumed to exist, in addition to the tensor force.

⁹ Brueckner, Levinson, and Mahmoud, Phys. Rev. **95**, 217 (1954), and many subsequent papers. See also R. F. Eden, Proc. Roy. Soc. (London) **235**, 408 (1956) and H. A. Bethe, Phys. Rev. **103**, 1353 (1956).