Range of 6- to 18-Mev Protons in Be, Al, Cu, Ag, and Au*

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An experimental study has been made of the range and straggling of protons of 6 to 18 Mev in Be, Al, Cu, Ag, and Au. The following values of I have been obtained: Be: 63.4 ev, Al: 166.4 ev, Cu: 375.6 ev, Ag: 585 ev, and Au: 1037 ev. New range-energy curves are computed using these data and the low-energy data summarized by Allison and Warshaw. The mean straggling is evaluated to approximately 5%. In magnitude it is in agreement with theory but there is a lack of symmetry above and below the 50% point which remains unexplained.

I. INTRODUCTION

PRESENT theory allows calculations of the range-energy relation which are accurate to about 0.5% for light elements while for heavier elements the uncertainty of the shell corrections may cause errors of more than 1%. Few accurate measurements have been made in the energy range from 6 to 18 Mev, and the most accurate measurement in this region[1] disagrees with measurements made at very high energies.[2]

The present paper reports the results of measurements of the range and straggling of protons of 6 to 18 Mev in Be, Al, Cu, Ag, and Au. The proton source was a magnetic energy analyzer used in the external beam of the Princeton 18-Mev synchrocyclotron.

A comparison is made with calculated values of stopping power, excitation potential, range, and straggling. For the preliminary comparison with theory it was possible to use the tables of Smith,[3] and Aron, Hoffman, and Williams,[4] but for the final evaluation of the results it was necessary to compute new tables of stopping power, range, multiple scattering, and straggling. The comparison with theory is complicated by the fact that the theory gives the mean proton path length in the material, while the present experiment gives projected ranges. The calculation of the difference between these two ranges is of limited accuracy.

II. RANGE MEASUREMENT

In most experimental determinations of the range-energy relation, monoenergetic particle beams have been used. In such measurements the thickness of the absorber is increased gradually until no particles pass through. The mean range is defined as the thickness of absorber at which half of the particles are transmitted. In low-energy determinations it is difficult to change the absorber thickness accurately. The straggling region in aluminum is approximately 1%. For protons of 18 Mev the absorber thickness is about 1.8 mm, and if ten steps are desired it is necessary to increase the absorber in steps of about four microns.

In the present experiment a different approach was chosen. Only one thickness R_0 of absorber was used, and the energy E of the proton beam was varied. The energy E_0 corresponding to a mean range equal to the absorber thickness was thus obtained as the energy at which half of the protons were transmitted.

A. Counting System

Figure 1 shows the equipment used for the measurement of ranges. The analyzed proton beam entered the counting system through an entrance window of 1.5 mg/cm^2 of Al. The counting system consisted of three proportional counters in tandem with the absorber foil mounted between the second and third counters. By measuring the ratio of the coincidences of Counters 1, 2, and 3 and the coincidences of Counters 1 and 2, the ratio of the protons passing through the foil to those incident on it could be obtained (Fig. 4). Since a ratio was measured, the determination was not dependent on the incident flux of protons. Coincidences were measured to avoid background effects. To reduce the background further, the discriminator biases were set at a pulse height equal to about one-third the mean pulse height produced by a proton of the proper energy travelling through Counters 1 and 2. The energy loss in each counter was of the order of 10 kev. The gas filling in the first two counters corresponded to a thickness of about 1 mg/cm^2 of Al. The background counting rates were of the order of 0.2%.

B. Absorber Foils

The machining of the counters and foil holders assured that the foils were at right angles to the axis of the counter to better than 0.2°. The foil holder was movable thus allowing a determination of the homogeneity of the foil. With the proton energy fixed, the transmission of the foil was measured at 10–20 points. Corresponding corrections were applied to the foil

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[1] E. L. Hubbard and K. R. McKenzie, Phys. Rev. 85, 107 (1952).
[2] R. Mather and E. Segrè, Phys. Rev. 84, 191 (1951).
[3] J. H. Smith, Phys. Rev. 71, 32 (1947).
[4] Aron, Hoffman, and Williams, University of California Radiation Laboratory Report UCRL-121 (unpublished).

thickness. For all except the thin Be absorber these corrections were smaller than 0.05% and were known to about 10%. For the thin Be foil the correction was 0.38% of the foil thickness.

The foils were approximately 1 cm square and were cemented to the holder. The diagonals of these quadrangles were measured to check the rectangularity of the foils. Corrections for burrs and unevenness of edges were measured with a microscope. The actual determination of the surface densities required two measurements: (1) Weighing of the foils. This was done on an analytic balance with a sensitivity of 1/20 mg. The balance was calibrated with a set of weights checked by the National Bureau of Standards. The weights of the absorber foils reproduced to 0.1 mg in measurements six months apart. Corrections for the buoyancy of air were applied. (2) Area measurement of the foils. A microscope and stage micrometer (checked by the National Bureau of Standards) were used. The width and height of the foil were measured in ten positions on both sides and the area computed from the average values. The area measurements were repeated at least two times per foil and the difference was never greater than 0.03%. The foils were of high purity (>99.8%). Since dE/dx is approximately proportional to $Z\ \ln(\text{constant}/Z)$, small impurities will cause only very small errors in the ranges.

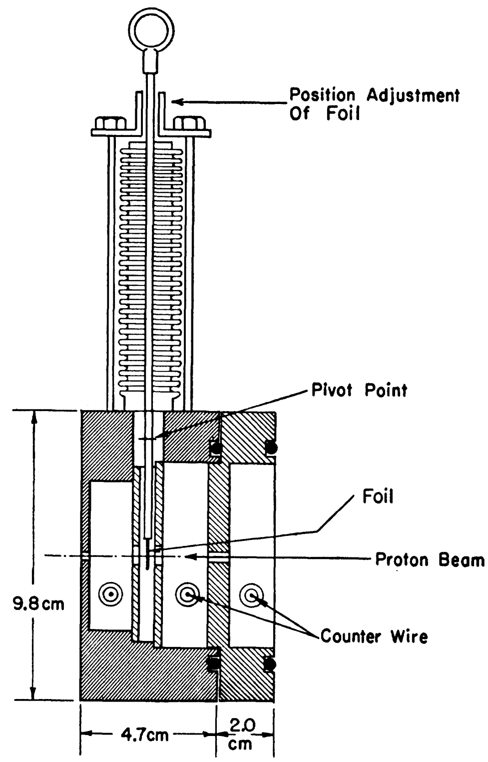

FIG. 1. Triple proportional counter.

C. Lineup of Proportional Counter System

It was important that the entrance to the counter system be as small as possible to avoid accepting protons scattered from the last collimator slit. At the same time it was necessary that the unscattered beam enter the counter system without hitting the edge of the opening. Protons scattered from the edge would introduce a lower energy component. The proportional counter was lined up optically with the collimation slits. This lineup was then checked by running the beam into a photographic plate. The final collimation slit was 0.9 mm in diameter and the entrance aperture to the counter was 2 mm. It was possible to make the lineup accurate to ~0.1 mm. Another opening was present in the back wall of the counter and it was thus possible to check that the axis of the counter was located on the line of the proton beam. Since the angle of the foil with respect to the counter was fixed, its angle with respect to the beam was known. It was perpendicular to the beam to within 0.005 radian. The error contribution from this was of the order of 0.001%.

D. Effects of the Counter Gas Filling on the Ranges

The energy loss of the protons in Counters 1 and 2 was determined experimentally. The energy loss in the third counter had to be estimated. The range in metal was then corrected to take into account this additional energy loss.

The experimental determination of the energy loss in Counters 1 and 2 was done by varying the gas-filling pressure, keeping the foil constant and measuring the energy necessary to give the mean range. Pressures of 50, 30, 15 and 10 cm Hg of a mixture of argon and 2% CO_2 were used. By extrapolating to zero pressure the energy loss in the gas could be determined. These values agreed with calculated values.

The energy loss in the third counter was small. By removing the absorbing foil and running the proton beam through all three counters the sensitivity of all three could be made equal by adjusting the counter voltages. The bias of Counter 3 was then set so that pulses corresponding to an energy loss less than 5 kev would not be detected. The range was increased by 0.05 mg/cm^2 in order to make an approximate correction for this loss.

III. ENERGY MEASUREMENT

Two different methods were used for the determination of the proton energy. In both cases the deflection of the proton beam by a stabilized electromagnet was determined, but a different geometrical measurement of the proton path was used in the two cases.

The magnet had a pole-tip 20 in. long and 8 in. wide, and at 10 000 gauss its field was uniform enough to be measured by nuclear induction within an area 1 in. from the edges. The fringing field on each side of the magnet was equivalent to about 1 in. in the gap region.

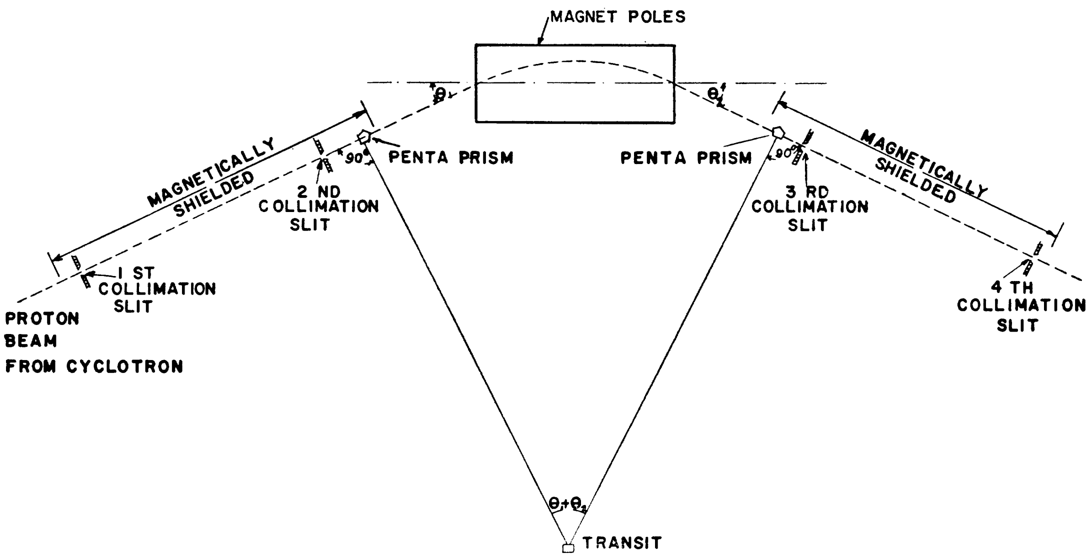

FIG. 2. Equipment for energy measurement (four-slit system). Proton path, S, - - -; X axes perpendicular to short side of magnet poles.

About one-sixth of the total field could not be measured by nuclear induction because of nonuniformity.

A. Four-Slit Method

In one method of energy determination (referred to as "the four-slit system") the angular deflection θ of the beam as it passed through the magnet was measured and used to determine the proton momentum:

$$mv=(e/\theta c)\int Bds,$$

where mv is the momentum of the proton, e its charge, c the velocity of light, and B the magnetic field along the path s (see Fig. 2). It was necessary to measure the fringing field accurately. Assuming that a negligible error was made in measuring the internal field, a precision of 0.6% would be required in the fringing-field measurement to obtain an accuracy of 0.1% in the momentum determination and 0.2% in energy. Since other errors were present, an attempt was made to measure the fringing field to about 0.1%.

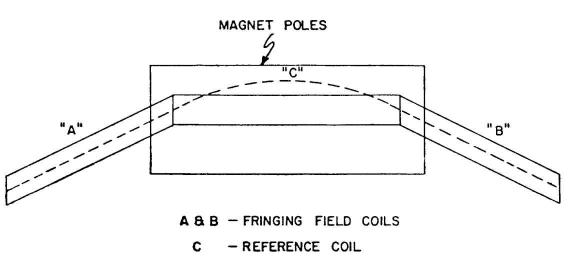

FIG. 3. Coils for fringing field measurements. Proton path, S, - - -.

The measurement of $\int Bds$ was made by a method similar to that used by Wilson and Creutz.[5] This avoided a point-by-point integration of the fringing field. The method is based on the relation

$$\int eBdx=mvc(\sin\theta_1+\sin\theta_2),$$

where x can be taken in an arbitrary direction, θ_1 and θ_2 are the angles between x and the proton path s before the proton path enters and after it leaves the magnetic field. The measurement of $\int Bdx$ becomes relatively simple if the magnet has parallel-edged pole-tips and a region of uniform field which can be used as a reference field. A coil can be inserted which will measure $\int Bdxdy$ over a region near the path of the beam. If coordinate directions are chosen with the y axis parallel to the pole-tips, B will have only a small variation in the y direction. The integral then becomes $D\int Bdx$, where D is the width of the coil. This method was used for measuring the fringing field while the interior field was measured by nuclear induction.

The coil for measuring the fringing field consisted of three parts located in the magnet as shown in Fig. 3. Coils A and B measured the fringing field and were placed so as to follow the approximate path of the beam in the fringing-field region; C was a reference coil. The magnet was cycled from negative to positive field and the voltages from coils A and B were compared with that from C by using a resistor divider and obtaining a null reading on a high-impedance fluxmeter. The fringing field $\int Bdx$ could then be compared with the known flux in the central region. The location of the

[5] E. Creutz and R. R. Wilson, Rev. Sci. Instr. 17, 385 (1946).

coils was not critical. Since they were mounted on a single form, an error in placing coil A was compensated by an opposite error in coil B.

Figure 2 shows the arrangement for measuring the beam energy. The beam from the cyclotron passed through two collimating slits which were about 1 mm wide and a meter apart. The beam was incident on the magnet at an angle θ_1, was deflected by the magnet through an angle $\theta_1+\theta_2$, and passed through two other symmetrically located slits. The sum of $\theta_1+\theta_2$ was measured by means of pentaprisms and a transit. If $\theta_1 \simeq \theta_2$ the energy was almost independent of the exact angle of incidence θ_1 and was dependent only on the sum. In practice it would be difficult to measure the angle of incidence with any precision.

A foil was inserted at the first slit to reduce the beam energy to approximately the desired value. Since the cyclotron beam had an energy width of more than 1% and the resolution of the system was about 0.2%, small variations in energy could be made by changing only the field of the analyzer magnet. In addition, the cyclotron energy could be varied about 3%.

The possible sources of error were of two types, those associated with the measurement of $\int Bdx$ and those associated with the angle measurement. They are listed in Table I. It was felt that the major sources of error were random. The rms value of momentum error was 0.06% and the energy error 0.12%.

B. Three-Slit Method

Since systematic errors may have occurred in the fringing-field measurement, a second energy measurement was made in which the radius of curvature R of the beam in the homogeneous region of magnetic field was determined with three slits. Corrections were made to take into account the lack of complete field homogeneity. The accuracy of this determination was limited by the small sagitta of the curved path in the magnetic field. Moreover, the correspondingly small slit widths needed ($\simeq$0.001 in.) introduce the possibility of slit-edge penetration effects. Such effects should reduce the mean energy of the beam.

The energy W is obtained from the relativistic equation

$$\frac{W}{M_0c^2}=\left(\frac{M_0{}^2c^4+B_0{}^2R^2e^2}{M_0{}^2c^4}\right)^{\frac{1}{2}}-1,$$

where M_0 is the proton rest mass. For B_0 a suitable average of the field must be taken.

The three slits were rigidly connected to the stainless steel rods which passed through 0-ring seals to the outside and could be moved in and out of the beam completely. Their positions were checked with dial indicators. Corrections of about 0.001 in. occurred due to vacuum forces.

The three-slit system was always used in conjunction with the four-slit system (see Fig. 4). First the four-slit system was lined up, then one after another the three slits were moved into the beam until maximum transmission occurred. The slit widths were approximately 0.001 in. and when all three were in position the cutoff of the beam could be caused by moving one of them about 0.002 in. Hence the positions of the slits were defined to better than 0.001 in.

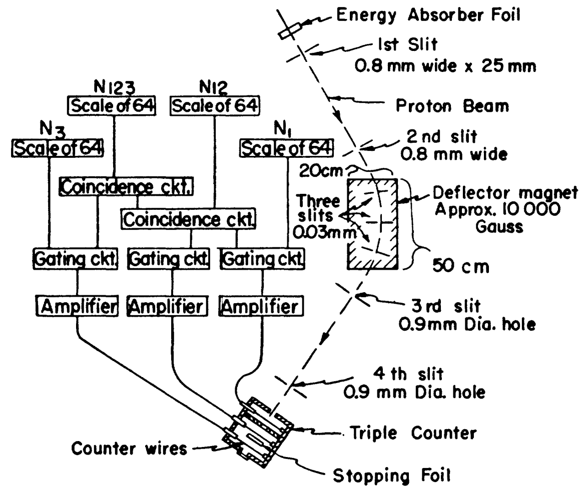

FIG. 4. Equipment for proton range energy measurement.

The coordinates of the centers of the three slits were measured by using a milling-machine table and a microscope as a two-dimensional comparator and from this the radius of curvature was computed. The screw calibrations were checked with a standard meter to 0.005 mm. Several straight edges were used to check the straightness of table movement and it was found to be better than 0.005 mm over a distance of 6 cm. The rectangularity of movement was checked with a transit and found to be accurate to one minute of angle.

The magnetic field was measured by nuclear induction and an average obtained through a suitable integration along the proton path. The shape of the field along the proton path is shown in Fig. 5. The frequency meter was checked against the WWV trans-

TABLE I. Estimates of error in "four-slit" energy determination.

(a) Interior homogeneous field measurement errors (estimated)	
Reading of the curves of transmitted protons *vs* magnetic field	0.005%
Determination of the field shape effects	0.003%
Accuracy of the frequency meter and proton moment device	0.005%
Fringing field determination (this is only a fraction of the total field and contributes an error of 0.053% to the total)	0.3%
(b) Errors associated with the angle measurement	
Transit scale reading	0.016%
Pentaprism accuracy (1.0×10^{-4})	0.010%
Vacuum force distortions (1.2×10^{-5})	0.0012%
Error due to lack of equality of incident and exit angle from magnet	0.003%

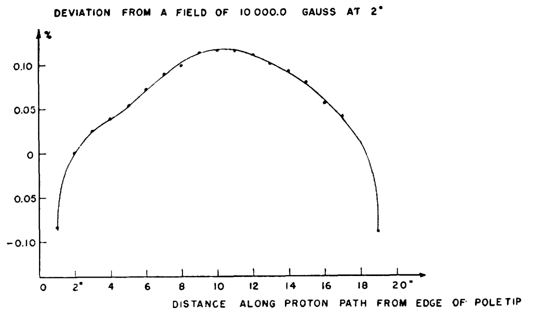

FIG. 5. Shape of magnetic field inside the magnet poles.

mitter of the National Bureau of Standards. The accuracy of field measurement was determined by the accuracy of the known value of the gyromagnetic ratio of the proton and hence accurate to about 0.003%. Estimates of the errors occurring in the measurements are given in Table II. The total root-mean-square error amounts to 0.03% in momentum or 0.06% in energy.

IV. EXPERIMENTAL RESULTS

An extensive discussion of the results will be given for only one element at one energy. For the other elements and energies the same considerations are valid. The results are presented in Table III.

A. Transmission Curves

Typical curves of number of transmitted protons *vs* energy for a constant foil thickness are presented in Fig. 6. The curves do not go to 100%. This can be explained by multiple scattering and nuclear reactions in the absorber. (At 18 Mev, about 1% of the protons undergo nuclear absorption or scattering.) The curves can be well approximated with the integral of a Gaussian between zero and 50%. A wider Gaussian would be required above 50% probably due to multiple scattering.

B. Energy

An Al foil of 1.8-mm thickness was used for the absolute energy calibration. Six independent energy

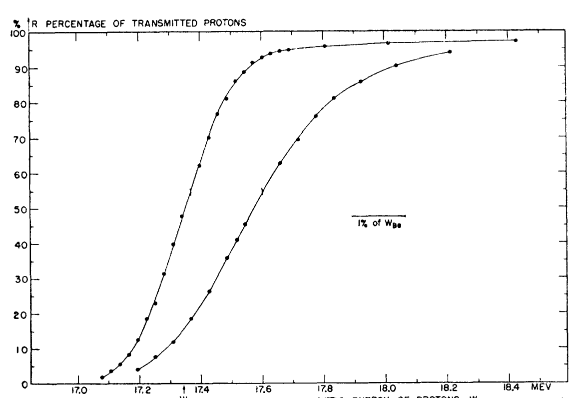

FIG. 6. Transmission curves for Be (left) and Au (right).

measurements were made with this foil by each method. As the errors in the earlier measurements were somewhat larger, only the last three measurements are considered here. The statistical weights differ slightly since different sources of error were sometimes present. The sources of energy-calibration errors are summarized in Table IV.

Combining the average energy values of the four- and three-slit systems $_4W$ and $_3W$, the energy determination becomes

$$\bar{W} = 17.836 \text{ Mev} \pm (0.05\% = 10 \text{ kev}).$$

Since $_4W$ and $_3W$ differ by an amount greater than would be expected from the error assigned, a larger error is given for the energy determination.

Therefore, the proton energy needed to penetrate a 1.8-mm Al foil is quoted as $W = 17.836 \pm 0.025$ Mev. All the other energies were obtained from the ratio of the magnetic fields with an accuracy of about $\pm 0.02\%$. The error in relative energy should be much smaller

TABLE II. Estimates of error in "three-slit" energy determination.

(a) Errors in the determination of the radius	
Determination of the geometrical positions of the slit centers	0.02%
Mechanical accuracy of milling-machine table	0.01%
Temperature (not constant)	0.01%
Distortion due to vacuum forces on the three-slit system	0.01%
(b) Errors in the determination of the magnetic field	
Reproducibility of reading the curves of the number of transmitted protons *vs* magnetic field	0.005%
Determination of the field shape effects	0.003%
Accuracy of frequency meter and proton moment device	0.005%

since exactly the same geometry and methods were used. Any systematic errors should be the same.

C. Foil Thickness

The total foil thickness was obtained from the following contributions:

(1) Entrance foil of proportional counter. The homogeneity of this foil material was checked by measuring several foils from the same roll of aluminum. All were within 0.02 mg/cm² of the same thickness. The value quoted is 1.50 ± 0.05 mg/cm². The error was due primarily to the inaccuracy of the weight determination.

(2) Gas filling of the first and second counters. The energy loss in these counters was measured and computed and is of the order of 25 kev. This value was then transformed into the corresponding thickness of the metal foil being measured. The pressure of the argon filling, the dimensions of the chamber, and the dE/dx of argon were known to about 2% and the dE/dx of the metals to better than 1%. The total rms error was estimated at $\pm 3.5\%$. The corresponding metal thicknesses were of the order of 1 mg/cm². For the 1.8-mm Al foil it was 1.12 ± 0.04 mg/cm².

(3) Absorber foil. The result for the 1.8-mm Al foil was 464.20±0.09 mg/cm². The description of the determination is given above.

(4) Gas filling in the last counter: 0.05±0.05 mg/cm².

(5) Misalignment of absorber foil: 0.05±0.05 mg/cm².

The total foil thickness was 466.92±0.30 mg/cm²; the rms sum of the individual errors is 0.13 mg/cm². Since it is questionable if it is safe to use the rms error, a value about equal to the sum of the errors is given.

V. COMPARISON OF EXPERIMENT AND THEORY

A. Correction for Scattering

In order to compare the experimentally determined range with a calculated range, a correction must be made for multiple scattering in the absorber foil. In the usual approximate determination of the correction[2] the observed range is expressed as a projection of the mean range:

$$R_{\text{obs}}=\sum_i l_i\cos\theta_i\simeq\sum_i l_i(1-\tfrac{1}{2}\theta_i^2)$$
$$=R_{\text{mean}}-\tfrac{1}{2}\sum_i l_i\theta_i^2,$$

where l_i is the distance between the ith and the $(i+1)$th small-angle collision, and θ_i is the direction with respect to the beam axis after the ith collision. Hence, in the integral relation

$$R_{\text{mean}}-R_{\text{obs}}=\Delta R=\tfrac{1}{2}\int_0^{R_{\text{obs}}}\langle\theta^2(x)\rangle_{\text{Av}}dx,$$

where $\langle\theta^2(x)\rangle_{\text{Av}}$ is the mean square deviation in angle from the normal direction and x is the projected distance in the material.

The mean square scattering angle can be calculated from an expression given by Rossi and Greisen[6]:

$$\langle\theta^2(x)\rangle_{\text{Av}}=\int_x^{R_0}8\pi Ne^4Z^2G(pv)^{-2}dx;$$

with this, the following result is obtained:

$$\frac{\Delta R}{R}=\frac{R_{\text{mean}}-R_{\text{obs}}}{R_{\text{mean}}}=\frac{Gm_0c^2Z}{(M_0c^2)^2}\frac{\left(\int_0^{E_0}\frac{\beta^2}{B/Z}dE\int_E^{E_0}\frac{1-\beta^2}{\beta^2(B/Z)}dE\right)}{\int_0^{E_0}\frac{\beta^2}{B/Z}dE},$$

where $G=2\ln 181Z^{-\frac{1}{3}}$, B is the stopping number of the atom, and m_0 is the electron rest mass. The ratio $\Delta R/R$ is a slowly varying function of the energy and amounts to several percent for high Z. Actual values were obtained numerically by evaluating the integrals down to 1 Mev and then extrapolating to zero energy. We estimate the accuracy of these results to be about 5–10%.

[6] B. Rossi and K. Greisen, Revs. Modern Phys. **13**, 240 (1941).

TABLE III. Measured values of range and energy and multiple-scattering correction.

Element	E_p (Mev)	Exp. range (mg/cm²)	Exp. range pathlength (multiple-scattering correction)	Corrected exp. range (pathlength) mg/cm²
Be	17.340	399.59	0.9990	399.99
	9.578	137.74	0.9988	137.90
Al	17.836	466.92	0.9962	468.70
	14.971	342.63	0.9960	344.01
	11.820	226.33	0.9956	227.33
	6.150	73.01 ±0.20	0.9943	73.43
Cu	17.893	588.67	0.9900	594.62
	9.938	215.80	0.9872	218.60
Ag	17.923	691.98	0.9818	704.81
	10.022	261.66	0.9759	268.12
Au	17.549	849.59	0.9626	882.60
	9.698	323.86	0.9475	341.80
|Error|	0.14%	<0.1% except as stated	a	

[a] See Sec. V(A).

By using these corrections a new set of corrected experimental ranges can be obtained which can be compared with theoretical values. These are listed in Table III.

B. Determination of Values of I

The expression generally used for the theoretical derivation of the range-energy relation is[7]

$$-dE/dx=4\pi e^4NB/mv^2;$$

$-dE/dx$ is the rate of energy loss per g/cm² of a proton of velocity v in an absorber material containing N atoms/g; m is the mass of the electron, e its charge, and B is the stopping number, usually expressed as follows:

$$B=Z\left[\ln\left(\frac{2mv^2}{I(1-\beta^2)}\right)-\beta^2\right]-\sum_i C_i.$$

Here Z is the atomic number of the absorber, I is the mean excitation potential of an electron of the absorber atom, and C_i are shell corrections which account for reduced stopping by the electrons of the ith shell.

TABLE IV. Result of energy determination for 1.8-mm Al foil.

System	Value (Mev)	Weight	Weighted average (Mev)	Error (%)
Four-slit	$W_1=17.879_3$	1	${}_4W=17.878$	0.12
	$W_2=17.874_5$	1		
	$W_3=17.879_6$	1		
Three-slit	$W_1=17.831_9$	1	${}_3W=17.825$	0.06
	$W_2=17.816_9$	0.75		
	$W_3=17.825_0$	0.75		

[7] M. S. Livingston and H. A. Bethe, Revs. Modern Phys. **9**, 264 (1937).

TABLE V. I-value determination and initial ranges.

Element	Z	I (ev)[a]	R (2 Mev) mg/cm²	Accuracy of fit	I/Z (ev)
Be	4	63.4 ± 0.5	9.1	$\pm0.05\%$	15.8
Al	13	166.45 ± 1	11.51	$\pm0.04\%$	12.8
Cu	29	375.6 ± 20	19.0	$\pm0.02\%$	12.9
Ag	47	585 ± 40	26.3	$\pm0.02\%$	12.5
Au	79	1037 ± 100	39.7	$\pm0.02\%$	13.1

[a] Changes of R (2 Mev) of about 10% cause changes of about 2.5% in I.

I is defined by

$$Z\ln I=\sum_{n,k} f_{n,k}\ln A_{n,k},$$

where $f_{n,k}$ are the oscillator strengths of the optical spectra and $A_{n,k}$ the corresponding excitation energies. It has not been possible to evaluate this formula exactly for any element used in this experiment. Bloch[8] has shown on the basis of the Fermi-Thomas model that $I=kZ$, where k is a constant.

The range is obtained through integration of the expression for $-dE/dx$:

$$R_W=R_0+\int_{W_0}^{W}(-dE/dx)^{-1}dE,$$

where R_0 is an experimental range. If the shell corrections are accurate, all experimental determinations should lead to the same values of I regardless of energy.

A value of I cannot be determined from a knowledge of the range at a single energy since the theory is not valid below about 1 Mev. Generally values are determined from a comparison of data with a theoretically determined range-energy relation. This in turn uses both an assumed value of I and an experimentally determined lower point.

Values of I were obtained from the experimental ranges in the following way: For each element a number of range-energy curves were computed with I as a parameter. For all elements, range points at 2 Mev were selected which gave theoretical curves close to the experimental points. These R (2 Mev) are in agreement within the experimental errors with ranges computed from values of dE/dx in Allison and Warshaw's review.[9] For Be and Al the C_k corrections of Walske's calculations[10] and for Al an approximation[11] $C_l=k/E$ (Mev) were used. For the heavier elements Aron's approximations for the shell corrections[12] were used. For Al it was found that $I=166.4$ ev, $k=0.685$, and R (2 Mev) $=11.51$ mg cm^{-2} give a better fit to the experimental data than $I=165.0$ ev, $k=1.37$ or $I=167.0$ ev, $k=0$. The first curve lies within $\pm0.05\%$ of the experimental path lengths.

Table V lists values of I and R (2 Mev) which give good fits to the experimental points.

A first approximation for the experimental values of I was obtained by using Aron's expression[12] for the difference between two ranges computed with two values of I:

$$[R(E_2,I_2)-R(E_1,I_2)]-[R(E_2,I_1)-R(E_1,I_1)]=\frac{A}{4\pi r_0^2mc^2N_0}\sum_{\nu=1}\left(Z\ln\frac{I_2}{I_1}\right)^{\nu}\int_{E_1}^{E_2}\frac{\beta^2dE}{B_1^{\nu+1}},$$

where $r_0=e^2/mc^2$, $A=$atomic weight of the material, $N_0=$Avogadro's number, and $B_1=$stopping number, evaluated with I_1. The values obtained with this method differ by less than 1% from the values adopted above. It should be pointed out that this method gives the best fit for a range curve through the two experimental points, but this curve would not generally go through the origin. (See Table VI.)

C. Bloch Constant

The value of I/Z should be constant if the approximations of the Bloch theory[8] apply. It is not to be expected that the theory should apply to Be. The high value obtained is explained as being in part due to polarization effects.[13] Values of I/Z are listed in Table V. It can be seen that the deviation from an average value of 12.9 ev is small and hence that the results are in good agreement with the Bloch theory.

TABLE VI. Theoretical range-energy data.

	Path length, theoretical data (mg/cm²)				
E_p (Mev)	Be	Al	Cu	Ag	Au
1	3.0	3.93	...	...	...
2	9.1	11.51	19.0	26.3	39.7
3	18.0	22.04	33.0	43.8	64.3
4	29.6	35.46	50.6	65.2	93.6
5	43.6	51.62	71.5	90.2	127.3
6	60.1	70.38	95.5	118.9	165.5
7	78.9	91.65	122.5	151.0	207.8
8	100.0	115.36	152.5	186.5	254.2
9	123.3	141.44	185.3	225.2	304.5
10	148.8	169.85	220.9	267.1	358.6
11	176.5	200.54	259.2	312.1	416.4
12	206.3	233.46	300.3	360.2	477.8
13	238.2	268.59	344.0	411.3	542.9
14	272.1	305.88	390.1	465.3	611.5
15	308.1	345.32	439.2	522.3	683.5
16	346.1	386.86	490.6	582.0	759.0
17	386.1	430.49	544.4	644.6	837.8
18	428.1	476.17	600.8	709.9	920.0
19	472.0	523.90	659.5	778.0	1005.5
20	517.6	573.64	720.8	848.8	1094.2
21	565.9	625.37	784.4	922.3	1186.2

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D. Comparison with Values Obtained by Other Experimenters

It is difficult to compare our values of I with those obtained by other experimenters since the corrections and methods of obtaining values of I are not entirely the same. Table VII gives a summary of measurements in the region above 2 Mev. In the case of the Hubbard and McKenzie measurement, we have changed their quoted value of I to correspond to the slightly larger scattering correction we have used.

VI. STRAGGLING

Straggling due to the statistics of energy loss is compared with the Bethe theory[7]:

$$\langle (R-R_0)^2 \rangle_{\mathrm{Av}} = \int_0^{E_0} \left(\frac{dE}{dx}\right)^{-3} 4\pi e^4 N Z' \times \left\{ 1 + \sum_n \left[\left(\frac{K_n I_n Z_n}{mv^2 Z'} \right) \ln\left(\frac{2mv^2}{I_n} \right) \right] \right\} dE,$$

where Z' is the total number of effective electrons, K_n is 4/3 for all shells, I_n is the average excitation potential of electrons in the nth shell, and Z_n is the number of electrons in the nth shell. The sum in brackets is between 1.04 for Be (18 Mev) and 1.4 for Au (10 Mev). The errors in this estimate are probably about 5%.

The experimental straggling is in addition determined by the following effects: (1) Straggling due to multiple scattering. This is not a symmetrical contribution. One would expect it to be between $0.2\Delta R_{\mathrm{Av}}$ and $0.5\Delta R_{\mathrm{Av}}$ (where ΔR_{Av} is the difference between the mean range along the proton path and the average projected range). (2) The contribution of the resolving power of the energy analyzer. This has been estimated to be about $\Delta E/E = \pm 0.2\%$, which causes $\Delta R/R$ to be $(0.4 \pm 0.2)\%$. (3) Unevenness of the foils. Scratches on the surfaces of the foils are of the order of 1μ in depth or less and therefore (with a foil of $\sim 500\mu$ thickness) cause an effect of about 0.2%.

TABLE VII. Comparison with other measurements.

Observer	Proton energy covered (Mev)	I values (ev) Al	Be	Cu	Ag	Au	Pb
Wilson[a]	1.5–4	150					
Hubbard *et al.*[b]	18	171					
Simmons[c]	12	155					
Bloembergen *et al.*[d]	35–50	164					
	50–75	161		375			
	70–120			365			970
Mather *et al.*[e]	340	150		310			810
Caldwell[f]				377.5	659	1136	
Present meas.	6–18	166	63.6	375	585	1037	

[a] R. R. Wilson, Phys. Rev. **60**, 749 (1941).
[b] See reference 1.
[c] D. H. Simmons, Proc. Phys. Soc. (London) **A65**, 454 (1952).
[d] N. Bloembergen and P. J. Van Heerden, Phys. Rev. **83**, 561 (1951).
[e] See reference 2.
[f] D. O. Caldwell, Phys. Rev. **100**, 291 (1955).

TABLE VIII. Straggling measurements. $\Delta R/R$ is the straggling parameter. $\Delta R/R = [\langle (R-R_0)^2 \rangle_{\mathrm{Av}}]^{\frac{1}{2}}/R_0$.

Element	Energy Mev	$\Delta R/R$ (expt) %	$\Delta R/R$ (theor) %	Percent difference
Be	10	1.44	1.41	+2.1
	18	1.34	1.34	0
Al	6	1.63	1.67	−2.4
	12	1.49	1.49	0
	15	1.42	1.44	−1.4
	18	1.42	1.42	0
Cu	10	1.71	1.72	−0.6
	18	1.55	1.60	−3.1
Ag	10	1.82	1.94	−6.2
	18	1.73	1.78	−2.8
Au	10	2.22	2.32	−4.3
	18	2.06	2.04	+1.0

Since these effects are statistically independent, the total contribution will be very small except for the heavy elements where (1) will have a large influence.

The experimental determination of the straggling was done in two ways: (a) By measuring the difference between the extrapolated and mean ranges (the straggling parameter), the standard deviation from the mean range could be determined from the relation $\langle (R-R_0)^2 \rangle_{\mathrm{Av}} = (2/\pi)(R_{\mathrm{extrap}} - R_0)^2$. (b) If it is assumed that the lower half of the transmission curve (number of transmitted protons $<50\%$) is equal to half of an integrated Gaussian, the standard deviation can be found directly. After correction of the experimental values for (1), (2), and (3), above, the two methods gave the same result to better than 5%. The points between 45 and 50% and those below 5% were of too low accuracy to be useful in the calculation. The upper half of the range curve always shows a larger straggling parameter than the lower. This difference is approximately 10% in all elements measured except Au (25%). It is possible that this effect is caused in part by the multiple scattering. The experimental values of straggling are compared with theoretical ones in Table VIII.

Since the experiment and the theory are accurate to only 5%, they are evidently in good agreement even though there seems to be some systematic deviation.

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