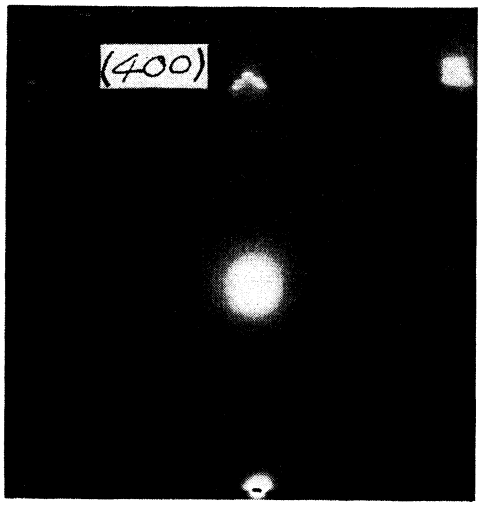


FIG. 2. After long annealing times (several days at 425°C), the crosses shown in Fig. 1 break up into discrete spots along the [111] directions.

insufficient surface roughness was obtained to produce diffraction patterns. With increasing annealing time (425°C), the diffraction patterns showed first the development of "spikes" or streaks on the Bragg spots which are finally converted to discrete satellite spots along the original spikes as seen in Fig. 2. The crosses observed with intermediate annealing time of the order of 24–36 hours are accounted for by the intersection of the Ewald sphere with spikes[3] along the eight [111] directions radiating from an integer point.

The atom form factors for Ni, Fe, and Co are indistinguishable for all practical purposes in electron diffraction. Their metallic radii are 1.24, 1.27, and 1.26 A, respectively. Superlattice reflections would be below observable intensity and local changes in lattice parameter due to segregation of solute atoms would likewise not be detected. The conclusion is that stacking faults on the close-packed {111} planes are responsible for the reciprocal lattice spikes and satellites. The formation of nodes along the octahedral spikes is presently interpreted as due to the development of a periodic or rhythmic array of {111} stacking faults. The period is estimated to be 40–75 A. The absence of satellites in the zero-order spot with increasing numbers observed in the higher orders is similar to the x-ray powder results obtained by Daniel and Lipson[4] from annealed $Cu_4FeNi_3$. The $Cu_4FeNi_3$ results were atrributed to a rhythmic segregation of solute producing a periodic error in interplanar spacing. Satellite reflections in electron diffraction patterns of CuAu are described by Ogawa and Watanabe[5] and ascribed to periodic defects. In both the $Cu_4FeNi_3$ and CuAu cases the periodic defects are along [100] directions.

The faulted structure represents the stable configuration of the crystal at room temperature. It is quite probable that cobalt and/or iron tend to segregate in the hexagonal region of the faults producing a state of lower free energy than a random solid solution. The diffraction satellites due to the faults can be smudged both by plastic deformation and by consecutive magnetic anneal in two different directions in the crystal. Prolonged annealing at elevated temperatures will greatly reduce if not eliminate the faulting. Recently it was found that a large asymmetry in the distribution of stacking faults could be produced by magnetic annealing. In this connection it was also found that Permalloy and 50:50 Co—Ni contain stacking faults and that these alloys heat-treat in a magnetic field. The results to date indicate that stacking faults in single-phase magnetic alloys may be quite important in the mechanism which causes these alloys to respond to heat treatment in a magnetic field. This work is continuing and will be reported in more detail.

A recent publication[6] illustrating the occurrence of stacking faults in cobalt-nickel alloys has just been noted. The faulting is introduced by plastic deformation and studied by means of x-ray and electrical resistivity. No mention is made of magnetic properties.

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## Solution of the Nuclear Many-Body Problem

K. A. BRUECKNER, *University of Pennsylvania, Philadelphia, Pennsylvania*

AND

J. L. GAMMEL, *Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico*

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IN a series of previous papers,[1–6] one of us (K. A. B.) has described a method for determining the properties of nuclear matter and has obtained some approximate solutions. The methods used have also been discussed by a number of other authors,[7] particularly by Bethe.[8] It is the purpose of this note to give a brief summary of the numerical solutions obtained by the authors at the computing center of the Los Alamos Scientific Laboratory, using the I. B. M. 704 fast electronic computers. A detailed description of the equations solved, the methods used, and further results will form the content of a paper now in preparation.

The energy of nuclear matter (infinite in extent with Coulomb forces neglected) as a function of density is determined by the self-consistent solution of a set of coupled equations. The basic interaction operators are determined by the integral equation[3,5,6]

$$K_{kl,\,ij}(\delta E)=v_{kl,\,ij}+\sum_{mn} v_{kl,\,mn} \times(E_i+E_j-E_m{}^*-E_n{}^*-\delta E)^{-1}K_{mn,\,ij}(\delta E), \quad (1)$$

where $v$ is the two-body interaction potential, $E_i$ and $E_j$ are the energies of particles in the states $i$ and $j$, and $E_m{}^*$ and $E_n{}^*$ are the energies of particles in the excited states $m$ and $n$. $\delta E$ determines the distance off the energy shell at which the $K$ matrix is to be evaluated[6]; $\delta E$ is zero for the ground-state $K$'s and has been taken to be equal to an average excitation in determining the excited state $K$'s. The sum over $m$ and $n$ runs over unfilled states above the Fermi surface with momentum $p_F$. The indices $i$, $j$, $m$, $n$, etc., specify the momentum, and spin, and the isotopic spin.

The energies appearing in Eq. (1) are determined from the diagonal elements of the $K$ matrices by the equations

$$E_i=[\textstyle\sum_s K_{is,\,is}(\delta E=0)-K_{is,\,si}(\delta E=0)] \quad (2)$$

for the ground state, and

$$E_m{}^*=\textstyle\sum_s[K_{ms,\,ms}(\delta E)-K_{ms,\,sm}(\delta E)] \quad (3)$$

for the excited states. In both cases the sum runs over the states of the Fermi gas. The solution to Eqs. (1), (2), and (3) completely determines the energy and equilibrium density of nuclear matter except for a correction due to the so-called "linked cluster" corrections[6] to the energy. These arise from fluctuations in the average potential acting on an interacting pair of particles. The correction has been estimated[6,8] to be of the order of 0.1 to 0.2 Mev per particle and is probably attractive.

The interaction assumed is that determined by Christian, Gammel, and Thaler.[9] This potential gives a precise fit to all low-energy two-body data and a good fit to scattering data up to 300 Mev, except for the odd-state polarization results. The interaction is characterized in even states by very short-ranged attractive central forces outside a repulsive core of radius $5\times10^{-14}$ cm and by a much longer-ranged tensor force. The odd-state potentials are a relatively weak central force and a repulsive tensor force (relative to that which binds the deuteron). The odd-state force, however, is not satisfactorily determined since the odd-state polarization predicted is incorrect. Therefore, in the preliminary results quoted here, we have omitted the odd-state contribution which an approximate estimate (based on the Christian-Gammel-Thaler potential) indicates may give a few Mev of attraction.

Equations (1), (2), and (3) have been solved essentially exactly except for one type of approximation required to allow a separation of the integral equation into states of $J$ and $l$. This approximation involves (a) a replacement of the energy denominator by its angular average over the center-of-mass momentum, and (b) a similar angular average in the otherwise exact

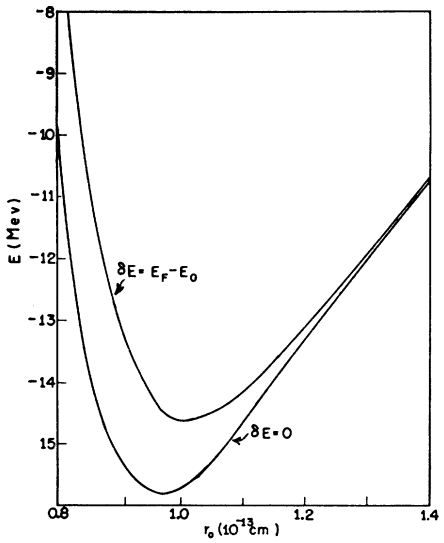


FIG. 1. Energy as a function of particle spacing. The parameter $\delta E$ is defined in Eq. (1).

treatment of the exclusion effect. The effects of off-the-energy-shell propagation have been estimated by choosing the mean excitation energy $\delta E$ of Eq. (1) to be equal in virtual states to the energy difference between the Fermi surface and the lowest state. Since this is probably somewhat of an overestimate, the sensitivity of the solution to this parameter has been checked by setting $\delta E$ equal to zero in all equations. The reduction of Eqs. (1), (2), and (3) to a form suitable for computation as well as the methods used for treating the singular core repulsion and the tensor force will be described in a separate paper.

The results are shown in Fig. 1 as a function of the average radius of the volume per particle, defined by the equation

$$\Omega=(4/3)\pi r_0{}^3N, \quad (4)$$

where $N/\Omega$ is the density. The effect of changing $\delta E$ from zero to $E_F-E_0$ (which in this case is about 90 Mev) is very small, raising the energy minimum from $-15.8$ Mev to $-14.5$ Mev and very slightly decreasing the equilibrium density. In both cases the energy minimum occurs close to $r_0=0.98\times10^{-13}$ cm. These values are to be compared with the central density of heavy nuclei which corresponds to $r_0=1.10\times10^{-13}$ cm and with the semiempirical volume energy of $-15.5$ Mev per particle.

We have also estimated the effect of the Coulomb repulsion on the density; in the case of 82 protons the minimum in the density *vs* energy curve is moved out to approximately $1.07\times10^{-13}$ cm, a decrease in density of 23%. This value is in better agreement with the empirical value.

The uncertainty in these results arises almost entirely from the cluster corrections and the approximate treatment of the excited state energy denomina-

tors. We estimate the cluster correction to be $0.3\pm0.3$ Mev and the energy denominator correction (compared to the result with $\delta E=0$) to be $1.0\pm0.5$ Mev, giving a final result for the energy (at the minimum) of $-14.8\pm0.6$ Mev. It is to be emphasized that this result is very much more accurate than the knowledge of the potentials, particularly since the odd-state potentials have been omitted.

Finally, it is interesting to point out that these results agree, within one or two Mev, with those which can be obtained from the same potential in the effective-mass approximation[3,4] with the effects of the exclusion principle neglected.

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## Contribution to Lamb Shift Due to Finite Proton Size

WALTER ARON AND A. J. ZUCHELLI*
*Department of Physics, University of Virginia, Charlottesville, Virginia*
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THE scattering of high-energy electrons by protons has recently been interpreted in terms of a finite spatial distribution of charge for the proton.[1] We have noticed that the resultant deviation from a pure Coulomb field is such as to reduce the existing discrepancy[2] between theoretical and experimental results for the hydrogen Lamb shift. Since the proton size is small compared to atomic dimensions, one easily finds, using nonrelativistic wave functions,

$$\Delta E=\tfrac{1}{6}|\psi(0)|^2e^2\langle R^2\rangle_{\mathrm{Av}},$$

where $\langle R^2\rangle_{\mathrm{Av}}$ is the mean square radius of the proton charge distribution and $\psi(0)$ is the amplitude of the hydrogen wave function at the origin. (A similar result was obtained by Salpeter[3] in discussing the effect of proton motion in the deuteron Lamb shift.) Taking the mean value given by Chambers and Hofstadter, $R_{\mathrm{rms}}=(0.77\pm0.10)\times10^{-13}$ cm, one finds the energy shift for the $2S_{\frac{1}{2}}$ level:

$$\Delta E=0.118\pm0.03 \text{ Mc/sec}.$$

* National Science Foundation Predoctoral Fellow.
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## Nuclear Emulsion Evidence for Parity Nonconservation in the Decay Chain $\pi^+-\mu^+-e^{+*}$†

JEROME I. FRIEDMAN AND V. L. TELEGDI
*Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois*
(Received January 17, 1957)

LEE and Yang[1] recently re-examined the problem as to whether parity is conserved in nature and emphasized the fact that one actually lacks experimental evidence in support of this most natural hypothesis in the case of weak interactions (such as $\beta$ decay). Violation of parity conservation can be inferred essentially only by measuring the probability distribution of some *pseudoscalar* quantity, e.g., of the projection of a polar vector along an axial vector, and measurements of this kind had not been reported. Lee and Yang suggested several experiments in which a spin direction is available as a suitable axial vector; in particular, they pointed out that the initial direction of motion of the muon in the process $\pi\rightarrow\mu+\nu$ can serve for this purpose, as the muon will be produced with its spin axis along its initial line of motion if the Hamiltonian responsible for this process does not have the customary invariance properties. If parity is further not conserved in the process $\mu\rightarrow e+2\nu$, then a forward-backward asymmetry in the distribution of angles $W(\theta)$ between this initial direction of motion and the momentum, $\mathbf{p}_e$, of the decay electron is predicted.

It is easy to observe the pertinent correlation by bringing $\pi^+$ mesons to rest in a nuclear emulsion in which the $\mu^+$ meson also stops. One has only to bear in mind two facts: (1) even weak magnetic fields, such as the fringing field of a cyclotron, can obliterate a real effect, as the precession frequency of a Dirac $\mu$ meson is $(2.8/207)\times10^6$ sec$^{-1}$/gauss; (2) $\mu^+$ can form "muonium," i.e., $(\mu^+e^-)$, and the formation of this atom can be an additional source of depolarization, both through its internal hyperfine splitting and the precession of its total magnetic moment around the external field. In the absence of specific experiments on muonium formation, one can perhaps be guided by analogous data on positronium in solids.[2,3]

With these facts in mind, we exposed (in early October, 1956) nuclear emulsion pellicles (1 mm thick) to a $\pi^+$ beam of the University of Chicago synchro-cyclotron. The pellicles were contained inside three concentric tubular magnetic shields and subject to $\leq4\times10^{-3}$ gauss. Over 1300 complete $\pi-\mu-e$ decays have been recorded to date, and the space angle $\theta$ defined above has been calculated for each. From these preliminary data we find[4]

$$\left\{\int_{90^\circ}^{180^\circ}|W(\theta)|d\Omega-\int_0^{90^\circ}|W(\theta)|d\Omega\right\}\Big/\int_0^{180^\circ}W(\theta)d\Omega$$
$$=0.062\pm0.027,$$