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Parity Nonconservation and a Two-Component Theory of the Neutrino

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A two-component theory of the neutrino is discussed. The theory is possible only if parity is not conserved in interactions involving the neutrino. Various experimental implications are analyzed. Some general remarks concerning nonconservation are made.

RECENTLY the question has been raised[1,2] as to whether the weak interactions are invariant under space inversion, charge conjugation, and time reversal. It was pointed out that although these invariances are generally held to be valid for all interactions, experimental proof has so far only extended to cover the strong interactions. (We group here the electromagnetic interactions with the strong interactions.) To test the possible violation of these invariance laws in the weak interactions, a number of experiments were proposed. One of these is to study the angular distribution of the β ray coming from the decay of oriented nuclei. We have been informed by Wu[3] that such an experiment is in progress. The preliminary results indicate a large asymmetry with respect to the spin direction of the oriented nuclei. Since the spin is an axial vector, its observed correlation with the β-ray momentum (which is a polar vector) can be understood only in terms of a violation of the law of space inversion invariance in β decay.

In view of this information and especially in view of the large asymmetry found, we wish to examine here a possible theory of the neutrino different from the conventionally accepted one. In this theory for a given momentum p the neutrino has only *one* spin state, the spin being always parallel to p. The spin and momentum of the neutrino together therefore automatically define the sense of the screw.

In this theory the mass of the neutrino must be zero, and its wave function need only have two components instead of the usual four. That such a relativistic theory is possible is well known.[4] It was, however, always rejected because of its intrinsic violation of space inversion invariance, a reason which is now no longer valid. (In fact, as we shall see later, in such a theory the violation of space inversion invariance attains a maximum.)

In Sec. 1 we describe this two-component theory of the neutrino. It is then shown in Sec. 2 that this theory is mathematically equivalent to a familiar four-component neutrino formalism for which all parity-conserving and parity-nonconserving Fermi couplings C and C' (as defined in the appendix of reference 1) are always related in the following manner: $C_S=C_S'$, $C_V=C_V'$, etc. or $C_S=-C_S'$, $C_V=-C_V'$, etc. Sections 3 to 8 are devoted to the physical consequences of the theory that can be put to experimental test. In the last section some general remarks about nonconservation are made.

I. NEUTRINO FIELD

1. Consider first the Dirac equation for a free spin-$\frac{1}{2}$ particle with zero mass. Because of the absence of the mass term, one needs only three anticommuting Hermitian matrices. Thus the neutrino can be represented by a spinor function φ_ν which has only two components.[4] The Dirac equation for φ_ν can be written as ($\hbar=c=1$)

$$\boldsymbol{\sigma}\cdot\mathbf{p}\varphi_\nu=i\partial\varphi_\nu/\partial t, \qquad (1)$$

where σ_1, σ_2, σ_3 are the usual 2×2 Pauli matrices. The relativistic invariance of this equation for proper Lorentz transformations (i.e., Lorentz transformations without space inversion and time inversion) is well known. In particular, for the space rotations through an angle θ around, say, the z axis, the wave function transforms in the following way:

$$\varphi\rightarrow\exp(-i\sigma_3\theta/2)\phi. \qquad (2)$$

The σ matrices are therefore the spin matrices for the neutrino. For a state with a definite momentum $\mathbf{p}$, the energy and the spin along $\mathbf{p}$ are given, respectively, by

$$H=(\boldsymbol{\sigma}\cdot\mathbf{p}),$$
$$\sigma_p=(\boldsymbol{\sigma}\cdot\mathbf{p})/|\mathbf{p}|.$$

They are therefore related by

$$H=|\mathbf{p}|\sigma_p. \qquad (3)$$

In the c-number theory, for a given momentum, the particle has therefore two states: a state with positive energy, and with $\frac{1}{2}$ as the spin component along $\mathbf{p}$, and a state with negative energy and with $-\frac{1}{2}$ as the spin component along $\mathbf{p}$.

It is easy to see that in a hole theory of such particles, *the spin of a neutrino* (defined to be a particle in the

[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[2] Lee, Oehme, and Yang, Phys. Rev. (to be published).
[3] Wu, Ambler, Hayward, Hoppes, and Hudson. We wish to thank Professor C. S. Wu for informing us of the progress of the experiment.
[4] See, e.g., W. Pauli, *Handbuch der Physik* (Verlag Julius Springer, Berlin, 1933), Vol. 24, 226–227.

positive-energy state) *is always parallel to its momentum while the spin of an antineutrino* (defined to be a hole in the negative-energy state) *is always antiparallel to its momentum* (i.e., the momentum of the antineutrino). Many of the experimental implications discussed in later sections are direct consequences of this correlation between the spin and the momentum of a neutrino. We have remarked in the introduction that such a correlation defines automatically the sense of a screw. With the usual (right-handed) conventions which we adopt throughout this paper, the spin and the velocity of the neutrino represent the spiral motion of a right-handed screw while the spin and the velocity of the antineutrino represent the spiral motion of a left-handed screw.

We shall now discuss some general properties[5] of this neutrino field:

(A) In this theory it is clear that the neutrino state and the antineutrino state cannot be the same. A Majorana theory for such a neutrino is therefore impossible.

(B) The mass of the neutrino and the antineutrino in this theory is necessarily zero. This is true for the physical mass even with the inclusion of all interactions. To see this, one need only observe that all the one-particle *physical* states consisting of one neutrino (or one antineutrino) must belong to a representation of the inhomogeneous proper Lorentz group identical with the representation to which the free neutrino states discussed above belong. For such a representation to exist at all, the mass must be zero.

(C) That the theory does not conserve parity is well known. We see it also in the following way: Under a space inversion P, one inverts the momentum of a neutrino but not its spin direction. Since in this theory the two are always parallel, the operator P applied to a neutrino state leads to a nonexisting state. Consequently the theory is not invariant under space inversion.

(D) By the same reasoning one concludes that the theory is also not invariant under charge conjugation C which changes a particle into its antiparticle but does not change its spin direction or momentum.

(E) It is possible, however, for the theory to be invariant under the operation CP, as this operation changes a neutrino into an antineutrino and simultaneously reverses its momentum while keeping the spin direction fixed. By the Lüders-Pauli theorem[6] it follows that the theory can be invariant under time reversal T.

For the free neutrino field, as described by (1), one can prove that the theory is indeed invariant under time reversal and under CP.

2. We shall in this section indicate how one can use the conventional four-component formalism of the neutrino (with violation of parity conservation) and obtain the same results as the present theory.

We start from Eq. (1) and enlarge the matrices by the following definitions (1 represents a 2×2 unit matrix):

$$\boldsymbol{\alpha}\equiv\begin{pmatrix}\boldsymbol{\sigma} & 0\\ 0 & -\boldsymbol{\sigma}\end{pmatrix},\quad \beta\equiv\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\tag{4}$$

$$\psi_\nu\equiv\begin{pmatrix}\phi\\ 0\end{pmatrix},\tag{5a}$$

$$\boldsymbol{\gamma}\equiv-i\beta\boldsymbol{\alpha},\quad \gamma_4\equiv\beta,\quad \gamma_5\equiv\gamma_1\gamma_2\gamma_3\gamma_4=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}.\tag{6}$$

An immediate consequence of these definitions is

$$\gamma_5\psi_\nu=-\psi_\nu.\tag{7a}$$

The free neutrino part of the Lagrangian is, as usual,

$$L_\nu=\psi_\nu^\dagger\gamma_4\left(\gamma_\mu\frac{\partial}{\partial x_\mu}\right)\psi_\nu,\tag{8}$$

where $\psi_\nu^\dagger$ = Hermitian conjugate of ψ_ν. The most general interaction Lagrangian not containing derivatives for the process

$$n\rightarrow p+e+\bar{\nu}\tag{9a}$$

is exactly as usual; namely, it is the sum of the usual S, V, T, A, and P couplings:

$$+L_{\text{int}}=-H_{\text{int}}=\sum[-2C_i(\psi_p^\dagger O_i\psi_n)(\psi_e^\dagger O_i\psi_\nu)],\tag{10a}$$

where i runs over S, V, T, A, and P and

$$\begin{aligned}
O_S&=\gamma_4,\\
O_V&=\gamma_4\gamma_\mu,\\
O_T&=-\frac{1}{2\sqrt{2}}i\gamma_4(\gamma_\lambda\gamma_\mu-\gamma_\mu\gamma_\lambda),\\
O_A&=-i\gamma_4\gamma_\mu\gamma_5,\\
O_P&=\gamma_4\gamma_5.
\end{aligned}\tag{11}$$

It is not difficult to prove that Eqs. (5a) and (7a) are consistent with a relativistic theory even in the presence of the interaction (10a). Another way of proving this is to start from the conventional theory of the neutrino with the interaction Hamiltonian given in (A.1) of reference 1 and observe that when

$$C_S=-C_S',\quad C_V=-C_V',\ \text{etc.}\tag{12a}$$

the neutrino field ψ_ν there always appears in interactions in the combination $(1-\gamma_5)\psi_\nu$. In the explicit representa-

[5] We have received a manuscript from Professor A. Salam on a theory of the neutrino similar to the one discussed in the present paper. He specifically discussed points (A) and (B) that we discuss here. He also gave the Michel parameter for the μ decay that agrees with the ones obtained below in Sec. 6.

[6] G. Lüders, Kgl. Dansk Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 5 (1954); W. Pauli, *Niels Bohr and the Development of Physics* (Pergamon Press, London, 1955).

tion that we have adopted above, this means that only the first two components of ψ_ν contribute to the interaction. *All calculations using the conventional theory of the neutrino with the Hamiltonian* (A.1) *of reference* 1 *concerning β decay therefore gives the same result as the present theory if we take the choice of constants* (12a). There exists, however, the possibility that in the decay of the neutron a neutrino[7] is emitted:

$$n \rightarrow p+e+\nu. \tag{9b}$$

The corresponding general form (not including derivatives of the fields) of the Hamiltonian is

$$H_{\text{int}}=\sum[2C_i(\psi_p^\dagger O_i\psi_n)(\psi_e^\dagger O_i\psi_\nu')], \tag{10b}$$

where O_i has been defined in Eq. (11). The field ψ_ν' is a four-component spinor defined in terms of the two-component neutrino field ϕ by

$$\psi_\nu'=\begin{pmatrix} 0 \\ \sigma_2\varphi^\dagger \end{pmatrix}. \tag{5b}$$

From Eq. (6), we see that

$$\gamma_5\psi_\nu'=+\psi_\nu'. \tag{7b}$$

It can be shown that (5b) and (7b) are consistent with a relativistic theory even in the presence of interaction (10b). *It can also be proved that one can use again the Hamiltonian* (A.1) *of reference* 1 *for the conventional theory of the neutrino with the choice of the coupling constants*

$$C_S=C_S', \quad C_V=C_V', \text{ etc.} \tag{12b}$$

and obtain the same result as the present theory.

The two possible choices (12a) and (12b) depend on whether, in the β decay of the neutron, process (9a) or (9b) prevails, i.e., whether a neutrino[7] or an antineutrino is emitted. We shall see in Sec. 3 that experimentally it will be easy to decide which of the two choices is appropriate (if the theory is correct). [We do not consider the possibility here of the simultaneous presence of (9a) and (9b), since the double beta decay process does not seem to be observed experimentally.]

II. EXPERIMENTAL IMPLICATIONS

3. We consider in this section the experiment of the β decay of oriented nuclei already discussed in reference 1, and currently being carried out.[3] For the present theory, according to Eqs. (12a) or (12b), Eq. (A.6) reduces to

$$\beta=\mp\left(\frac{v_e}{c}\right)\frac{|C_T|^2-|C_A|^2-(2Ze^2/\hbar cp)\,\text{Im}(C_AC_T^*)}{|C_T|^2+|C_A|^2}. \tag{13}$$

The choice of the $\mp$ sign depends on whether

$$n\rightarrow p+e+\bar{\nu} \quad (\bar{\nu}=\text{left-handed screw}),$$

or

$$n\rightarrow p+e+\nu \quad (\nu=\text{right-handed screw}). \tag{14}$$

In writing down (13) the Fierz interference terms has been set equal to zero, which is in conformity with the experimental results,[8] and which implies [see Eq. (A.5) of reference 1]:

$$\text{Real part of } C_AC_T^*=0. \tag{15}$$

By measuring the momentum dependence of the asymmetry parameter β, one can test whether the present theory is correct.

It is interesting to notice that for a positron emitter the asymmetry parameter has the opposite sign. This is a direct consequence of the fact that in positron and electron emission, the neutrino and antineutrino emitted have opposite spirality.

4. An experiment such as the one being carried out by Cowan and collaborators[9] measures the cross section for neutrino absorption, which can be calculated in both the present theory and the usual theory. Now one determines the magnitude of the β-coupling constants to give the observed lifetimes of nuclei against β decay. The calculated value of the cross section turns out then to be *twice as great* in the present theory as in the usual theory. This follows from the following simple reasoning: The neutrino flux is an experimental quantity independent of the theory. If the neutrinos in a given direction have only one spin state instead of the usual two, by a detailed balancing argument they must have twice the cross section for absorption as the usual ones.

5. In the decay of $\pi^\mp$ mesons at rest, let us consider the component of angular momentum along the direction of $\mathbf{p}_\mu$, the momentum of the μ meson. The orbital angular momentum contributes nothing to this component. The μ spin component is therefore completely determined (irrespective of its total spin) by the spin component of the ν or $\bar{\nu}$. There are then two possibilities:

(A)
$$\begin{aligned} &\pi^+\rightarrow\mu^++\nu, \quad (\mu^+ \text{ spin along } \mathbf{p}_\mu)=+\tfrac{1}{2}, \\ &\pi^-\rightarrow\mu^-+\bar{\nu}, \quad (\mu^- \text{ spin along } \mathbf{p}_\mu)=-\tfrac{1}{2}; \end{aligned} \tag{16}$$

or

(B)
$$\begin{aligned} &\pi^+\rightarrow\mu^++\bar{\nu}, \quad (\mu^+ \text{ spin along } \mathbf{p}_\mu)=-\tfrac{1}{2}, \\ &\pi^-\rightarrow\mu^-+\nu, \quad (\mu^- \text{ spin along } \mathbf{p}_\mu)=+\tfrac{1}{2}. \end{aligned} \tag{17}$$

In each case the μ mesons with fixed $\mathbf{p}_\mu$ form a polarized beam. (It was pointed out in reference 1 that if parity is not conserved in the decay of π mesons, the μ mesons would in general be polarized.) Furthermore, the polarization is now complete (i.e., in a pure state). If this theory of the neutrino is correct, then the $\pi-\mu$ decay is a perfect polarizer of the μ meson, offering a

[7] The neutrino as defined in Sec. 1 is a particle with spin parallel to its momentum representing a right-hand screw. Similarly, the antineutrino as defined there is a particle with spin antiparallel to its momentum representing a left-handed screw. We use this definition throughout the present paper.

[8] See, e.g., R. Sherr and R. H. Miller, Phys. Rev. **93**, 1076 (1954).

[9] See C. L. Cowan, Jr. *et al.*, Science **124**, 103 (1956).

natural way to measure the spin and the magnetic moment of the μ meson. (It turns out that the $\mu-e$ decay may serve as a good analyzer, as we shall discuss in the next section.)

The choice of the two possibilities (16) and (17) will be further discussed in Sec. 7.

6. For the $\mu^- - e^-$ decay the process can be

$$\mu^- \to e^- + \nu + \bar{\nu}, \tag{18}$$

or

$$\mu^- \to e^- + 2\nu, \tag{19}$$

or

$$\mu^- \to e^- + 2\bar{\nu}. \tag{20}$$

Consider process (18) first. The decay coupling can be written with the notations defined in Eq. (11). (We assume no derivitive coupling.)

$$H_{\text{int}} = \sum_{i=V,A} f_i(\psi_e^\dagger O_i \psi_\mu)(\psi_\nu^\dagger O_i \psi_\nu). \tag{21}$$

It is easy to see that in the present theory, where ψ_ν satisfies (7a), the S-, T-, and P-type couplings do not exist. We have assumed in writing down (21) that the spin of the μ meson is $\frac{1}{2}$. For a μ^- at rest with spin completely polarized, the normalized electron distribution is given by

$$dN = 2x^2[(3-2x) + \xi \cos\theta(1-2x)]dx d\Omega_e (4\pi)^{-1}, \tag{22}$$

where p = electron momentum, $x = p$/maximum electron momentum, θ = angle between electron momentum and the spin direction of the μ, Ω_e = solid angle of electron momentum, and

$$\xi = [|f_V|^2 + |f_A|^2]^{-1}[f_V f_A^* + f_A f_V^*]. \tag{23}$$

The mass of the electron is neglected in this calculation. The decay probability per unit time is ($\hbar = c = 1$):

$$\lambda = M^5[|f_A|^2 + |f_V|^2]/(3\times 2^8 \pi^3), \tag{24}$$

where M is the mass of the μ meson. The spectrum (22) for a nonpolarized μ meson,

$$dN = 2x^2[3-2x]dx d\Omega_e (4\pi)^{-1}, \tag{25}$$

is characterized[5] by a Michel[10] parameter $\rho = \frac{3}{4}$, which is consistent with known[11] experimental results.

One sees that for not too small values of ξ, the spectrum (22) is sensitive to $\cos\theta$, especially in the region of large momentum for the electrons. Therefore the $\mu-e$ decay may turn out to be a very good analyzer of the μ-meson spin.

An analysis of the so-called universality of the Fermi couplings is easier in this theory because there are fewer coupling constants, and also because $\pi-\mu-e$ decay measurements would supply information concerning the parameter ξ of (23).

If process (19) or (20) prevails, the spectrum becomes

$$dN = 12x^2(1-x)dx[1 + \eta \cos\theta]d\Omega_e (4\pi)^{-1}. \tag{26}$$

[10] L. Michel, Proc. Phys. Soc. (London) **A63**, 514 (1950).

[11] See, e.g., Sargent *et al.*, Phys. Rev. **99**, 885 (1955).

This is characterized[5] by a Michel parameter[10] $\rho = 0$ which is not consistent with experiments.[11] One therefore concludes that (18) is the correct process.

A general theorem concerning the relationship between μ^+ and μ^- decays will be stated in Sec. 9.

7. If experiments should show that in the decay of the π meson, process (16) prevails, and in the β-decay process (9a) prevails, then one would say that the ν (the right-handed screw), the μ^-, and the e^- are light particles, and there is a conservation of light particles. If processes (17) and (9b) prevail, one would say that the $\bar{\nu}$ (the left-handed screw), the μ^-, and the e^- are light particles, and there is a conservation of light particles. Similar concepts have been discussed before.[12]

We have already seen in Sec. 3 that the sign of β in Eq. (13) determines whether

$$n \to p + e + \bar{\nu} \tag{9a}$$

or

$$n \to p + e + \nu \tag{9b}$$

is the process for β decay. To decide whether

$$\pi^+ \to \mu^+ + \nu, \quad (\mu^+ \text{ spin along } \mathbf{p}_\mu) = \tfrac{1}{2} \tag{16}$$

or

$$\pi^+ \to \mu^+ + \bar{\nu}, \quad (\mu^+ \text{ spin along } \mathbf{p}_\mu) = -\tfrac{1}{2} \tag{17}$$

one will have to determine the spin of μ^+ along its direction of motion.

8. The $\pi-\mu-e$ type experiment discussed in Secs. 6 and 7 can be done with the $K_{\mu 2}-\mu-e$ decays. The analysis is dependent on the spin of $K_{\mu 2}$. If this spin is not zero, the polarization of the μ meson is not necessarily complete. The degree of polarization can be experimentally found by a comparison of the angular distribution of the electrons in $\pi-\mu-e$ decay and in $K-\mu-e$ decay.

Another interesting experiment is to measure the momentum and polarization of the electron emitted in a β decay. A polarization of the electron results only if parity is not conserved; a measurement of this polarization is a measurement of a quantity similar to the parameter β in Eq. (13). The polarization in such a case will be along the direction of the momentum of the electron. Polarization along other directions can result if the momentum of the recoil nucleus is also determined. Theoretical considerations of such possibilities are being made by Dr. R. R. Lewis.

GENERAL REMARKS

9. Some general remarks concerning the conservation and nonconservation of the parity P, the charge conjugation C, and the time reversal T will be made in this section. Except for the last paragraph, no assump-

[12] E. J. Konopinski and H. M. Mahmoud, Phys. Rev. **92**, 1045 (1953).

tion that the neutrino is a two-component wave is made.

Since the preliminary result of the oriented nucleus experiment that there is a strong asymmetry, Eq. (A.6) of reference 1 shows that not only parity, but also charge conjugation is not conserved[2] in β decay. A measurement of the velocity dependence of the asymmetry parameter could supply[2] some information concerning time reversal invariance or noninvariance. If the $\pi-\mu-e$ decay should show any forward-backward asymmetry (as discussed in reference 1, and further analyzed above in Sec. 6 for the two-component neutrino theory), it can be shown from theorem 2 of reference 2 that charge conjugation invariance must be violated in both the $\pi-\mu$ and $\mu-e$ decays.

It is, however, easy to show from the Lüders-Pauli theorem[6] that even if C, T, and P are all not conserved, a stable particle ($e^{\pm}$ or $p^{\pm}$, or a deuteron, etc.) must have *exactly* the same mass as its antiparticle.

One can also prove that even if C, T, and P are all not conserved, the e^+ angular distribution in $\pi^+-\mu^+-e^+$ decay is exactly the same as the e^- angular distribution in $\pi^--\mu^--e^-$ decay. The only difference in the two cases is that the average spin of μ^+ along $\mathbf{p}_\mu$ is the opposite of that of μ^- along $\mathbf{p}_\mu$. (The decays are here assumed to occur in free space from $\pi^{\pm}$ at rest.)

It is further obvious from the Lüders-Pauli theorem[6] that if time reversal invariance is not violated, the operation CP is conserved. This means that the left-right asymmetry that is found in a laboratory is always exactly opposite to that found in the antilaboratory.

Should it further turn out that the two-component theory of the neutrino described above is correct, one would have a natural understanding of the violation of parity conservation in processes involving the neutrino. An understanding of the $\theta-\tau$ puzzle presents now a problem on a new level because no neutrinos are involved in the decay of $K_{\pi 2}$ and $K_{\pi 3}$. Perhaps this means that a more fundamental theoretical question should be investigated: the origin of all weak interactions. Perhaps the strange particles belong to strange representations of the Lorentz group. (Nature seems to make use of simple but odd representations.) It is also interesting to note that the massless electromagnetic field is the cause of the breakdown of the conservation of isotopic spin. The similarity to the massless two-component neutrino field that introduces the nonconservation of parity may not be accidental.