

## Angular Correlations in $K^0$ -Decay Processes

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The reactions  $K^0 \rightarrow \{e^\pm \text{ or } \mu^\pm\} + \nu + \pi^\mp$  afford a good opportunity to study fully the dynamics of decay processes involving leptons. It is shown that, under certain rather general assumptions customarily adopted in regard to weak lepton interactions, the distribution in angle between  $\pi$  and  $\nu$  has a simple, uniquely specified form, independent of the properties of the  $K$  particle; namely, aside from a known, angle-dependent factor related to phase volume, the distribution is proportional to  $A + B \cos\theta + C \cos^2\theta$ , where  $A$ ,  $B$ , and  $C$  may depend on the  $\pi$  momentum. For any given momentum, the relative magnitudes of  $A$ ,  $B$ , and  $C$ , when determined experimentally, can be expected to illuminate a second aspect of the problem; namely, the detailed nature of the lepton coupling. The situation is especially transparent for the  $e$ - $\nu$ - $\pi$  mode in the case that the  $K$  particle has zero spin. This is discussed in some detail.

### I. INTRODUCTION

THE existence of at least one kind of long-lived neutral  $K$ -particle has recently been established.<sup>1,2</sup> The analysis of the decay modes is in its early stages. It has been found that electrons and charged pions occur as decay products, sometimes in association.<sup>3</sup> Thus by arguments of simplicity the modes

$$K^0 \rightarrow \pi^\pm + e^\mp + \nu \quad (1)$$

have been suggested.<sup>4</sup> It is the purpose of this note to show that, under rather general assumptions concerning the weak lepton interaction, useful information can be extracted<sup>5</sup> from the angular distribution of the decay products in a reaction of the type (1).

The fact that reaction (1) has two charged decay products makes it possible in principle to study fully the dynamics of individual events. As a matter of fact this is the first instance of such a favorable situation involving leptons aside from the  $\beta$  decay of the neutron. In the latter case there is so little phase space, however, that the statistical factor predominates. Reaction (1), on the other hand, with its ample phase volume, seems particularly suited for the study of the dynamics of lepton processes. In particular it should be more easily amenable experimentally<sup>6</sup> than the corresponding charged mode  $K^\pm_{e3}$ .

From this general point of view the modes

$$K^0 \rightarrow \pi^\pm + \mu^\mp + \nu \quad (2)$$

<sup>1</sup> Lande, Booth, Impeduglia, and Lederman, *Phys. Rev.* **103**, 1901 (1956).

<sup>2</sup> Fry, Schneps, and Swami, *Phys. Rev.* **103**, 1904 (1956).

<sup>3</sup> L. Lederman, private communication.

<sup>4</sup> See reference 1, footnote 9.

<sup>5</sup> It has been noted by S. B. Treiman and R. G. Sachs, *Phys. Rev.* **103**, 1545 (1956), that reaction (1) is of interest for the study of interference effects in particle mixtures. The question of such mixtures is not our concern here. In fact, the results to be discussed in this note are independent of whether particle mixtures exist or not.

<sup>6</sup> This process has been studied by S. Furuichi *et al.*, *Progr. Theoret. Phys.* **16**, 64 (1956).

would likewise be of interest. The charged counterpart  $K^\pm_{\mu 3}$  has been established with fair certainty<sup>7</sup>;  $\mu$ -mesons have been identified<sup>3</sup> among the decay products of long-lived  $K^0$ 's. It will be shown below, however, that  $K^0_{e3}$  will yield detailed information more readily than  $K^0_{\mu 3}$ .

Although the net speed of processes (1) and (2) is mainly governed by weak interactions, both  $K^0$  and  $\pi$  are strongly coupled, to baryons for example. This circumstance makes it desirable to avoid perturbation theory in calculating transition rates insofar as such strong virtual interactions are concerned. With respect to the weak interactions perturbation theory seems admissible, however. It is with regard to these weak interactions that we shall make the only dynamical assumptions on which the present work rests. They are

I. Both leptons are produced at one vertex.<sup>8</sup>

II. The coupling responsible for the production of the lepton pair does not involve derivatives of the lepton field (no "Konopinski-Uhlenbeck-interactions"). Thus, diagrammatically the situation is as shown in Fig. 1(a). This implies that the weak lepton interaction is taken to first order only and that in particular the final-state electromagnetic interaction between  $\pi$  and electron is ignored, which means an error of order  $1/137$ ; see Fig. 1(b).

In the following we always work in the rest-system of the  $K^0$ ;  $E$ ,  $\mathbf{p}$  shall denote the  $\pi$  energy and momentum. We consider first the  $K^0_{e3}$ -mode. Let  $\theta$  be the angle between  $\pi$  and electron and  $W_e(p, \theta) d\mathbf{p} d\cos\theta$  the probability distribution in terms of  $p = |\mathbf{p}|$  and  $\theta$ . It is convenient to define  $F_e(p, \theta)$  by

$$F_e(p, \theta) = (1 + x \cos\theta)^4 W_e(p, \theta), \quad (3)$$

$$x = p/(M - E), \quad (4)$$

where  $M$  is the  $K^0$  mass.  $F_e(p, \theta)$  contains essentially only the dynamical dependence of the distribution on

<sup>7</sup> Cf. Yekutieli, Kaplon, and Hoang, *Phys. Rev.* **101**, 506 (1956).

<sup>8</sup> In the sense of Feynman diagrams.

angle. In Sec. II it is shown that

$$F_e(p, \theta) = A + B \cos \theta + C \cos^2 \theta, \quad (5)$$

where  $A$ ,  $B$ , and  $C$  are functions of  $p$  only. To be precise, this result is obtained under the neglect of the electron mass  $m_e$ , which implies kinematic errors of order at most  $m_e/m$  ( $m$ =mass of the  $\pi$ ). If we would have taken  $\theta$  to be the angle between  $\pi$  and neutrino, Eq. (5) would be exact without this last slight approximation.

The main interest of Eq. (5) lies in its generality: it will be shown in Sec. II that the angular dependence given by Eq. (5) is

- (a) independent of the parity of the  $K^0$ ,
- (b) independent of the spin of the  $K^0$ ,
- (c) independent of whether parity is conserved or not in weak interactions,<sup>9</sup>
- (d) independent of whether time-reversal and/or charge conjugation invariance is valid or not in weak interactions.

Thus it will be clear that the result (5) is independent of whether we deal with one  $K^0$ -particle or with a superposition of  $K^0$ 's with very nearly the same mass. The variance of the functions  $A$ ,  $B$ ,  $C$  with the alternatives implied by (a)  $\cdots$  (d) will be discussed below.

The validity of relation (5) rests in essence only on the invariance with respect to the orthochronous Lorentz-group and on the validity of assumptions I and II. In fairness it should be said that these latter assumptions are perhaps only well-defined in the language of the usual field theories.

In discussing quantities like  $W_e(p, \theta)$ ,  $F_e(p, \theta)$  we have made no distinction between the modes  $(\pi^-, e^+, \nu)$  and  $(\pi^+, e^-, \nu)$ . If the long-lived  $K^0$  is described by a real field<sup>10</sup> (or if we deal with a superposition of  $K^0$ 's of very nearly the same mass, each of which is described by a real field) such a distinction is indeed unnecessary, for then  $F_e$  or  $W_e$  will be identically the same for both modes. In particular we will have the same  $A$ ,  $B$ , and  $C$ . It has been pointed out by Lee and Yang that we may have to envisage a situation in which the long-lived  $K^0$  (and likewise the corresponding short-lived one) is described by a complex field.<sup>11</sup> In this case the expression (5) is still separately valid for  $(\pi^-, e^+, \nu)$  and the  $(\pi^+, e^-, \nu)$  but now the  $A$ ,  $B$ , and  $C$  need no longer be the same for these two cases. A similar comment applies to the  $(\pi^-, \mu^+, \nu)$  and the  $(\pi^+, \mu^-, \nu)$  decays.

For these  $K^0_{\mu 3}$  modes we first introduce the corresponding distribution  $W_\mu(p, \theta)$ . In order to get simple results it is now essential to define  $\theta$  as the angle between  $\pi$  and neutrino. Put  $F_\mu(p, \theta) = (1 + x \cos \theta)^4 W_\mu(p, \theta)$  for the purpose of defining a function  $F_\mu$  which again essen-

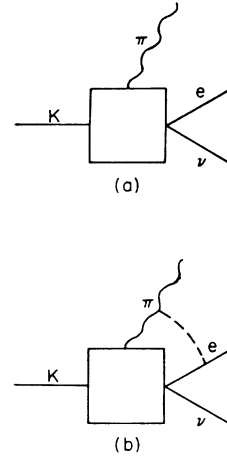


FIG. 1. (a) Schematic diagram for  $\pi\nu$ -decay with leptons emerging from one vertex. (b) An ignored diagram involving a virtual photon.

tially only depends on angle through the dynamics of the interaction. In Sec. II it is shown that the dependence of  $F_\mu$  on  $\theta$  is again given in general by an expression of the form (5).

Thus up to this point, the  $\mu 3$  and the  $e 3$  mode behave in the same way. It will be seen in Sec. II that the  $e 3$  mode is much more suited for studying further details of the lepton coupling, however.

The  $e 3$  mode, whether for neutral or for charged  $K$ 's, is of special interest in that for the first time electrons appear in boson decays. In particular, the experimental appearance of the  $K^{\pm}_{e 3}$  and  $K^{\pm}_{\mu 3}$  decay modes with comparable rates<sup>12</sup> again raises the question whether there is some sort of equivalence between  $\mu$  and  $e$  in weak processes, a suggestion which had first come up in the study of the relation of  $\mu$  capture to  $\beta$  decay.<sup>13</sup> If one takes this equivalence seriously one is, of course, faced with the task of reconciling, both for  $\pi^{\pm}$  and  $K^{\pm}$  decays, the presence of the  $\mu\nu$  mode with the spectacular absence so far of the  $e\nu$  reaction, perhaps one of the most significant pieces of information in particle physics. Some dynamic inhibition of the  $e\nu$  mode would then be necessary. In this spirit it has often been suggested, but this may be too naive, that the explanation could lie in the dominance of one particular covariant in the  $\mu\nu$  and  $e\nu$  decay; for example, in the case of the  $\pi$  decay, a pseudovector coupling. Such an assumption could not be put to a direct test thus far. It may be noted in passing that for a  $K$ -spin  $\geq 1$  it is not possible without further restrictions to inhibit dynamically the  $e\nu$  relative to the  $\mu\nu$  decay by means of the device of a single dominant covariant.

In this connection it should also be observed that, if the diagram sketched in Fig. 1. makes any sense for  $K^{\pm}$  decay, the  $e\nu$  mode cannot be absolutely forbidden for the simple reason that the  $\pi$  which is actually emitted in the  $e 3$  mode can be imagined to be virtually

<sup>9</sup> T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

<sup>10</sup> See M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

<sup>11</sup> Private communication. As noted by Lee and Yang, this situation may arise if one is forced to give up charge-conjugation invariance for weak interactions.

<sup>12</sup> See, e.g., S. Goldhaber, *Proceedings of Sixth Rochester Conference* (Interscience Publishers, Inc., New York, 1956).

<sup>13</sup> See, e.g., L. Michel, *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1952).

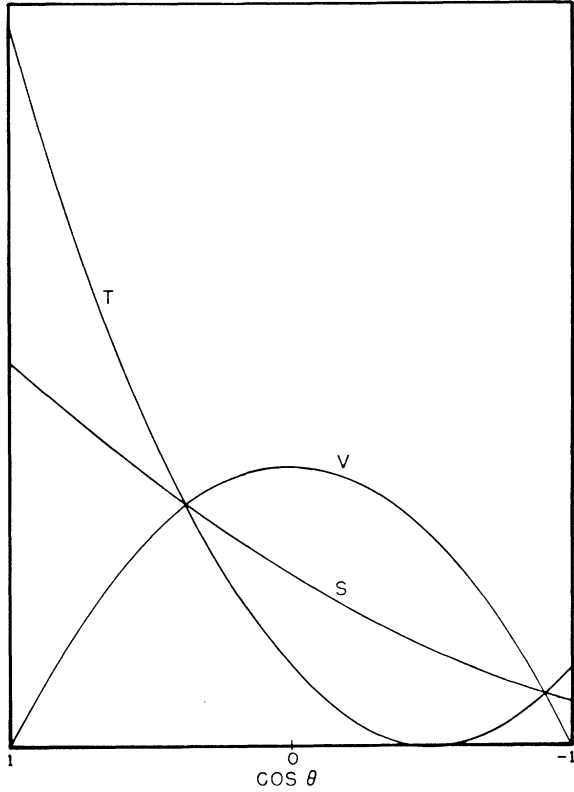


FIG. 2. Characteristic  $\pi e$  angular correlations, for zero spin  $K$  meson, for pure  $S$ ,  $V$ ,  $T$  couplings, respectively. The curves are drawn for the particular case  $x=0.5$  (corresponding to  $\pi$  momentum  $\sim 140$  Mev/c) and for convenience are normalized to give the same area.

reabsorbed via a strong interaction. This is true even if the  $e3$  mode is due to a primitive interaction, that is to say if the three vertices in Fig. 1(a) coincide.

Returning to the neutral  $K$ , we will see in Sec. II that for the case of zero spin the angular distribution  $F_e(p, \theta)$  is significantly dependent on the nature of the covariants in the lepton vertex; see Fig. 2. Thus if the spin of the  $K^0$  is actually zero it is not beyond hope that direct experimental information on the lepton coupling can be obtained. For a  $K^0$ -spin  $>0$  the theoretical situation gets much more involved and a discussion of the experimental data in terms of contributing covariants becomes more complicated.

The  $p$  dependence of the distribution cannot be studied without further assumptions involving the strong interactions. For the simplest possible  $p$  dependence, momentum plots for the  $K^\pm$  decay have been given by Furuichi *et al.*<sup>6</sup> Under the same assumptions these would, of course, apply to the present case.

Before we discuss in more detail the assertions made above, it may be appropriate to point out that all we have said for the  $e3$  and  $\mu3$  modes of a long-lived  $K^0$  actually has nothing to do with the magnitude of the lifetime of the particle and would equally well apply to anomalous  $e3$  or  $\mu3$  decays of the  $\theta^0_1$ .

## II. THE DETAILED ARGUMENT

### (a) The $K^0_{e3}$ Mode

We shall begin with a discussion of the case that the  $K^0$  has the spin-parity assignment  $0^-$  and that parity is conserved. From the analysis of this special instance it will then be easy to incorporate the alternatives (a)  $\dots$  (d) of Sec. I.

For the case in hand the effective matrix element  $R$  has the form<sup>14</sup>

$$R = f_S \bar{\psi}_e \psi_\nu + (if_V/M) \bar{\psi}_e \gamma_\mu P_\mu \psi_\nu + (f_T/M^2) \bar{\psi}_e \sigma_{\mu\nu} P_\mu Q_\nu \psi_\nu, \quad (6)$$

where  $\psi_e$  and  $\psi_\nu$  are, respectively, the electron and neutrino spinors, and  $\bar{\psi} = \psi^\dagger \gamma_4$ . The Dirac matrices are taken to be Hermitian;  $\sigma_{\mu\nu} = -i[\gamma_\mu, \gamma_\nu]/2$ . The vectors  $P_\mu$ ,  $Q_\mu$  are arbitrary linear combinations of the independent four-vectors of the problem; and the functions  $f_S$ ,  $f_V$ , and  $f_T$  are scalar functions formed from these vectors. The crucial kinematic point is now this: the four-vectors we have to deal with are the energy-momentum vectors  $P^K$  for the  $K$ ,  $P$  for the  $\pi$ , and  $P^e$  and  $P^\nu$ , for the  $e$  and  $\nu$ , respectively. Because of assumptions I and II, however, these latter occur only in the combination  $P^e + P^\nu$ . This fact together with energy-momentum conservation, means that there are only two independent vectors, say  $P^K$  and  $P$ . Thus the functions  $f$  depend only on the  $\pi$  momentum [ $P^K = (0, 0, 0, iM)$ ]. Furthermore, since we neglect the electron mass, we have that

$$\bar{\psi}_e \gamma_\mu (P^e_\mu + P^\nu_\mu) \psi_\nu = 0.$$

Hence the most general form for  $R$ , evaluated in the  $K$  rest system, is

$$R = f_S \bar{\psi}_e \psi_\nu - f_V \bar{\psi}_e \gamma_4 \psi_\nu + (f_T/M) \bar{\psi}_e \gamma_4 \gamma \cdot \mathbf{p} \psi_\nu. \quad (7)$$

The transition rate is now readily found in the standard way. Summing over lepton spins, we have for the function  $F_e(p, \theta)$  of Eq. (3)

$$F_e(p, \theta) \sim (1-x^2)^2 (M-E)^2 p^2 E^{-1} \{ |f_S|^2 (1+x \cos \theta)^2 + |f_V|^2 x^2 \sin^2 \theta + (p^2/M^2) |f_T|^2 (x + \cos \theta)^2 + i(f_S f_T^* - f_S^* f_T) (p/M) (\cos \theta + x) (1+x \cos \theta) \}, \quad (8)$$

which is of the general form (5).

Observe that the value of the  $ST$  cross term depends on the relative phase of  $f_S$  and  $f_T$ . Charge conjugation and/or time-reversal invariance<sup>15</sup> require  $f_S$  and  $f_T$  to be relatively imaginary. But regardless of whether this is so or not, the form (5) persists, which proves assertion (d) for the special case:  $0^-$ , parity conserved.

Consider next the case where the  $K^0$  has positive parity, everything else unchanged. All we have to do is to insert a factor  $\gamma_5$  in each of the three covariants

<sup>14</sup> Factors  $M$  have been inserted to give  $f_S$ ,  $f_V$ ,  $f_T$  the same dimension.

<sup>15</sup> We want to thank Professor S. Furuichi for a fruitful correspondence on the subject of time-reversal.

in Eq. (6). Evidently this does not change  $F_e$ , which proves assertion (a) for spin 0, parity conserved.

If parity is not conserved we must replace in Eq. (6):  $f_S \bar{\psi}_e \psi_\nu$  by  $f_S \bar{\psi}_e \psi_\nu + f'_S \bar{\psi}_e \gamma_5 \psi_\nu$  and analogously for the  $V$  and  $T$  terms. This changes each  $|f|^2$  in Eq. (8) to  $|f|^2 + |f'|^2$  while  $(f_S f_T^* - f_S^* f_T)$  becomes the sum of this term plus a term with primes on each  $f$ . There can be no  $ff'$  cross terms: for this to be possible we should be able to construct a pseudoscalar out of the independent 3-vectors of the problem. There are only two such vectors and this is therefore impossible. Thus statement (c) has been verified for spin zero.

It is clear from Eq. (8) that the three cases of pure  $S$ ,  $V$ , or  $T$ , respectively, give quite distinct angular distributions. Figure 2 gives a sketch of the situation. We repeat that these characteristic plots are independent of parity-conservation, and are separately valid for both  $(\pi^-, e^+, \nu)$  and  $(\pi^+, e^-, \nu)$ . It is not necessarily true, however, that the  $f$ 's in Eq. (8) are the same for both modes.<sup>11</sup> Note in particular the characteristic pure  $V$  case which corresponds to the dominant covariant in the sense discussed in Sec. I.

It remains to discuss the case of higher spin. There are now more free vectors in the problem: for spin  $N$  we have  $N$  polarization 3-vectors, which occur in the combinations of the appropriate irreducible tensors. However, due to assumptions I and II the occurrence of these vectors cannot bring with them the occurrence of  $\mathbf{p}^{(e)}$  separately to a higher power than before, and therefore the angular distribution (5) remains unchanged in form after one sums over the polarization of the  $K^0$ . Of course, different spins will give in general different  $A$ ,  $B$ , and  $C$ . Note in this connection that  $f_S$ ,  $f_T$ ,  $f_V$  may depend on  $\mathbf{e} \cdot \mathbf{p}$ , where  $\mathbf{e}$  is some polarization vector. Upon averaging over polarizations one will find again that  $A$ ,  $B$ , and  $C$  depend on  $p$  only.

The discussion of the independence of parity and of

time-reversal invariance is the same as before. The remaining point is parity conservation: for spin  $\geq 1$  there are enough independent 3-vectors in the problem to ensure the possibility of constructing a pseudoscalar. Hence for polarized  $K^0$ 's there will exist the possibility of a parity interference. However, for unpolarized  $K^0$ 's the interference effect clearly vanishes.

### (b) The $K_{\mu 3}^0$ Mode

The general trend of the argument showing that  $F_\mu(p, \theta)$  is of the form (5) is identical with the one given for the  $e3$  case. We will conclude by showing that even for spin zero it will be much less easy to disentangle the various covariants.

Call the mass of the meson  $\mu$ . The expression for  $R$  corresponding to Eq. (7) for the electron case now becomes

$$R = \left( f_S - g_V \frac{\mu}{M} \right) \bar{\psi}_\mu \psi_\nu - f_V \bar{\psi}_\mu \gamma_4 \psi_\nu + \frac{f_T}{M} \bar{\psi}_\mu \gamma_4 \boldsymbol{\gamma} \cdot \mathbf{p} \psi_\nu,$$

where the  $g_V \mu/M$  term is due to a separate new vector interaction  $\bar{\psi}_\mu \gamma_\rho \cdot (P_\rho^\mu + P_\rho^\nu) \psi_\nu$ . This by itself already shows that it will be hard to disentangle the  $S$  from the  $V$ -covariant. Put  $f_S - g_V \mu/M = f_S''$ . Then

$$\begin{aligned} F_\mu(p, \theta) \sim & (1 - x^2 - y^2)^2 (M - E)^2 p^2 E^{-1} \\ & \cdot [ |f_S''|^2 (1 + x \cos \theta)^2 + |f_V|^2 (x^2 \sin^2 \theta + y^2) \\ & + p^2 M^{-2} |f_T|^2 \{ (\cos \theta + x)^2 + y^2 \sin^2 \theta \} \\ & + (f_S''^* f_V + f_S'' f_V^*) y (1 + x \cos \theta) \\ & - i (f_S''^* f_T^* - f_S'' f_T) p M^{-1} (1 + x \cos \theta) (x + \cos \theta) \\ & + i (f_V f_T^* - f_V^* f_T) \mu M^{-1} x \cos \theta (1 + x \cos \theta) ], \end{aligned}$$

where  $y = \mu/(M - E)$ . Note that the range of  $x/y$  is from zero to  $\sim 2$ . Thus all angular effects for special covariants lose their transparency as compared with the  $e3$  case.