Neutron Production by μ Mesons Stopped in Sodium and Magnesium*†

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Measurements of the production of neutrons by negative μ mesons which stop and interact in two materials, sodium and magnesium, are reported. The mean number of neutrons emitted per interaction was found to be 0.6 ± 0.2 for magnesium and 1.0 ± 0.4 for sodium, where the uncertainties include an estimated contribution from systematic sources as well as those of a statistical nature. The data confirm that neutron production in magnesium is less than in lead (multiplicity 1.7 ± 0.3), but show that it is not zero. Neutron production in sodium also is shown to be measurable and probably greater than in magnesium, though less than in lead. In the course of the experiment a value of $(1.18_{-0.14}^{+0.19})$ μsec (statistical) was found for the mean life of negative μ mesons in sodium.

I. INTRODUCTION

THE general characteristics of nuclear interactions of stopped negative μ mesons[1] are consistent with a charge-exchange reaction[2,3] in which most of the meson rest energy is carried away by a light neutral particle, presumably a neutrino. In this reaction a moderate amount of energy (of the order of 15 Mev) is available for the excitation of the nucleus, and the expected emission of nucleons has been experimentally verified in the case of several elements.[1,4–15] As part of a study[5,7,11–14] made at this laboratory of the μ-meson nucleon interaction through measurements on the subsequent emission of neutrons, preliminary data on the mean number of neutrons given off per interaction were obtained for the absorbers Ca and Mg.[13] The preliminary work indicated that this number, the mean multiplicity, was much smaller for Ca and Mg than for Pb; the yield of neutrons was so small as to make modification of the apparatus advisable. An improved form of the equipment has therefore been applied to measurements on the light absorbers Na and Mg, the choice of absorber having been made in the light of an argument, which will be discussed subsequently, that comparison of the multiplicities for these two elements would be of more significance. These two measurements form the subject matter of this paper. Such multiplicity measurements furnish a tool for investigating nuclear structure as well as the μ-meson nuclear interaction; studies of the neutron yield as a function of Z should prove especially valuable.

Essentially, then, this experiment measures the mean multiplicity by detecting neutrons associated with meson stoppings, the neutron detection efficiency being known. The number of interactions relative to the number of decays is determined from knowledge of the negative μ-meson mean life in the absorber; the actual number of interactions is then computed from the stopping rate.

Figure 1 furnishes a schematic view of the apparatus designating the various counter trays. Meson stoppings are selected by a coincidence-anticoincidence arrangement using magnetized lenses of the Rossi type.[13] The neutron detection efficiency is measured by recording the rate from a neutron source of known strength and

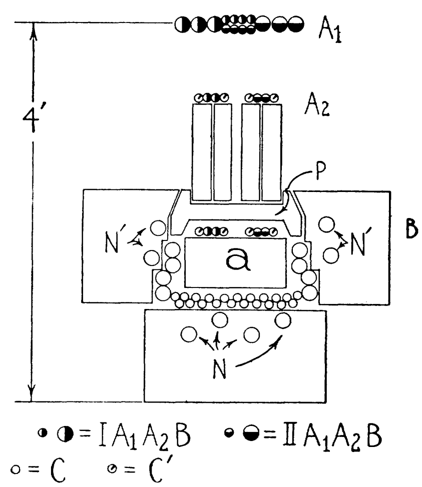

FIG. 1. Schematic representation of the apparatus.

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[1] R. D. Sard and M. F. Crouch, *Progress in Cosmic-Ray Physics* (North-Holland Publishing Company, Amsterdam, 1954), Vol. 2, p. 1.
[2] S. Sakata and T. Inoue, Progr. Theoret. Phys. Japan **1**, 143 (1946).
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[7] Sard, Conforto, and Crouch, Phys. Rev. **76**, 1134 (1949).
[8] W. B. Fowler, Sard, Fowler, and Street, Phys. Rev. **78**, 323 (1950).
[9] Groetzinger, Burger, and McClure, Phys. Rev. **81**, 969 (1950).
[10] E. J. Althaus, Phys. Rev. **81**, 647A (1951).
[11] Sard, Crouch, Jones, Conforto, and Stearns, Nuovo cimento **8**, 326 (1951).
[12] M. F. Crouch and R. D. Sard, Phys. Rev. **85**, 120 (1952).
[13] A. M. Conforto and R. D. Sard, Phys. Rev. **86**, 465 (1952).
[14] A. M. Conforto, Washington University Ph.D. thesis, 1950 (unpublished).
[15] M. Widgoff, Phys. Rev. **90**, 891 (1935).

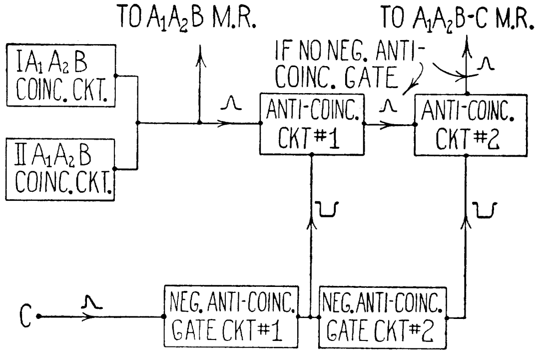


FIG. 2. Functional schematic diagram of circuits.

of an appropriate energy spectrum placed at various positions in the absorber. The meson mean life is measured by determining the time distribution of pulses (from decay electrons) detected by the anticoincidence tray (marked C in Fig. 1) with respect to the incident telescope pulse in the case of Na; for Mg, published data[16] were used.

Particles which trigger the coincidence-anticoincidence arrangement are restricted to a range interval by the requirement that they penetrate the iron magnets and paraffin barrier, P, but not the absorber, a. The magnetic deflection in the iron lens selects a momentum band the upper limit of which is set by the maximum radius which will still allow a trajectory appreciably dependent upon the sense of the magnet, since the neutron coincidence rate $(A_1A_2B-C:N)$ when the magnet is set to focus positive particles is subtracted from that for negative. A simultaneous momentum and range selection of course amounts to a restriction on the mass of the acceptable particles; in this case only μ mesons contribute appreciably to the rate. Protons of the correct energy to stop in the absorber have a radius of curvature in the magnet of at least 300 meters; their trajectory is thus insensitive to the sign of the magnet and any contribution to the delayed neutron rate will be eliminated by the subtraction. π mesons are rare at sea level[17,18] and have only a small chance of penetrating the magnet without a nuclear interaction. Thus the only appreciable contribution to the delayed coincidence neutron rate arises from μ mesons. For a more detailed discussion of the focusing properties of the magnetic lens one may refer to Conforto and Sard[13] and Conforto.[14]

The average number of neutrons per interaction, or multiplicity, m, is determined by

$$(A_1A_2B-C:N)^{+/-}{}_{\text{abs. diff.}} = \epsilon_{\text{eff}} m f (A_1A_2B-C)^{-}{}_{\text{abs. diff.}}. \quad (1)$$

[16] H. K. Ticho, Phys. Rev. **74**, 1337 (1948).
[17] B. Rossi, *High-Energy Particles* (Prentice Hall, New York, 1952), first edition.
[18] M. G. Mylroi and J. G. Wilson, Proc. Phys. Soc. (London) **A64**, 404 (1951).

Here $(A_1A_2B-C:N)^{+/-}{}_{\text{abs. diff.}}$ = the measured rate of delayed neutrons in coincidence with stoppings; the superscript $+/-$ indicates that the rate obtained when the magnet sense is such as to focus positive particles is subtracted from the corresponding negative rate; the subscript abs. diff. indicates that no-absorber rates have been subtracted for both magnet senses. ϵ_{eff} = the detection efficiency for neutrons created in the absorber; because of the finite probability that the neutron may not trigger the BF_3 counter until after the delayed coincidence gate has closed, this efficiency is less than the incoherent efficiency. f = the fraction of stopped mesons that interact; f is related to the mean life in free space (designated T^+ since it equals that of positive mesons) and the measured mean life in the absorber (T_{abs}^-) as follows: $f=1-(T_{\text{abs}}^-/T^+)$. $(A_1A_2B-C)_{\text{abs. diff.}}^-$ = the stopping rate of negative μ mesons; the no-absorber rate is again subtracted.

Equation (1) may then be solved for m, thus giving a measure of the mean multiplicity.

II. APPARATUS

As shown in Fig. 1, the main parts of the apparatus are the two magnetized iron lenses (15 000 gauss, 239 g/cm² in vertical thickness) placed side by side over the absorber a; the input telescopes IA_1A_2B and IIA_1A_2B (each telescope has its own separate coincidence circuit); the paraffin barrier P (designed to eliminate the low-energy mesons which scatter strongly and therefore are not well focused, and also to decrease the detection efficiency for interactions in the magnet); the anticoincidence tray C; the neutron counter N and N' surrounded by the paraffin moderator. The neutron counters were of course operated in the proportional region, and their output was fed first to a pre-amplifier, thence to an amplifier and discriminator. The output of the C tray was sent to a three-channel delay discriminator triggered by input coincidence A_1A_2B. Detailed descriptions of the magnets, their field measurements, and operation of the Geiger-Mueller counters and the circuits mentioned above may be found in references 13 and 14. The apparatus was modified in several respects in order to make it more suitable for measurements on light nuclei. A description of the changes made follows:

In order to reduce counts due to side showers and thus increase the purity of the apparent meson stoppings (an especially important consideration in the case of sparse neutron emitters) the A_1 trays were added, making the input coincidence triple rather than double. With the same purpose, anticoincidence guard counters (C') in parallel with C were placed on each side of the A and B trays.

Some effort was made to increase the anticoincidence efficiency, since the stopping rate for light absorbers is relatively low, and any inefficiency gives rise to a background to be subtracted from an already

TABLE I. Comparison of anticoincidence rates with and without absorbers.

Absorber	Pb	Na	Mg
Absorber stopping rate	0.574 ± 0.013	0.390 ± 0.006	0.564 ± 0.010
No-absorber stopping rate	0.0903 ± 0.005	0.0942 ± 0.0054	0.099 ± 0.004
Measured background, %	15.7%	24.1%	17.5%
Expected stopping in counter walls, %	5.6%	5.6%	5.2%

small rate. Thus, the anticoincidence (C) tray was extended so as to cover the sides as well as the bottom of the absorber.

Previously, the counting efficiency of the C tray had been kept high by frequently monitoring counter pulse heights and discarding any counter whose pulses became smaller because of age. In an effort to increase the average counter life and general reliability of the C tray, five univibrator quench circuits[19] were installed. The counters were divided into five groups, with one quench circuit per group. The univibrator, triggered by the counter pulse, applies a negative pulse of about 250 volts, 80 μsec in duration, to the counter wire, stopping the discharge before it has traveled the length of the tube. The charge transferred (and, consequently, the number of alcohol molecules dissociated) is decreased and counter life thereby increased. Such quench circuits were also placed on the A_1, A_2, and B trays during the sodium run.

False stoppings would be recorded when particles, having triggered an A_1A_2B coincidence, penetrate the absorber but traverse C (anticoincidence) counters which are dead or not sufficiently recovered from a previous count. The circuitry was modified to minimize this effect by the use of two anticoincidence gates in series (see Fig. 2). The result is that no anticoincidence can be recorded for about 180 μsec following any pulse in one of the C counters. During this interval the counter and the first anticoincidence univibrator can regain their sensitivity.

False stoppings are thus strongly discriminated against at the cost of discarding a small, known portion (about 4.5%) of true stoppings. That penetrating particles could contribute to the delayed neutron rate without regard to the sense of the magnet is shown by the previous results,[13] which indicated an appreciable rate of neutrons associated with nonstopping particles when the absorber was present. In addition, because of the high ratio of penetrating particles to stoppings, a small inefficiency in the C tray leads to a large error in the determination of the stopping rate. The measured rates (see Table I) indicate some improvement in the background compared to the unmodified form of the apparatus; for example, in the case of Mg the no-absorber:absorber ratio was changed from 65% to 17.5%.

[19] Based on a circuit described by H. Elliot, Proc. Phys. Soc. (London) **A62**, 369 (1949).

For measuring neutron production, one should maximize the neutron detection efficiency while retaining reasonable constancy of efficiency with energy. For this purpose, four more neutron counters (marked N' in Fig. 1) were added, two on each side. The paraffin immersion distance of each counter was made to vary along its length by placing it in a slanting orientation. (The opposite ends of the counters shown close to the absorber in Fig. 1 are far from it, and vice versa.) Adding the side neutron counters had the virtue of making the detection efficiency less dependent upon the vertical position of the source in the absorber. The neutron detection efficiency was determined by placing a calibrated ($\pm10\%$) Ra$-\alpha-$Be source in as many positions as practical in the absorber and comparing the N rates with the known flux of neutrons. The incoherent neutron detection efficiency averaged over three positions of the source in Pb was determined to be $3.61\pm0.04\%$ (statistical standard error only). This result may be compared with the former value of 2.20%.[13] Relative efficiencies for the source placed on the bottom and two higher plates of the four sheets forming the Pb absorber were found to be in the ratios 1.17:1.08:1.00, numbers which may be compared with the approximately corresponding ratios obtained previously,[13] namely 2.3:1.7:1.0.

Some care was taken with the lightest absorber, Na, to maximize the amount introduced into the meson beam, in order to have a reasonable counting rate. The material was cast into two galvanized iron parallelepipeds (13 in.$\times 9\frac{1}{2}$ in.$\times$7 in.) and was fitted into the apparatus with about $\frac{1}{8}$-in. clearance between the containers and the adjacent trays. In this arrangement the containers amount to about 5% of the absorber (18.5 g/cm^2 of Na compared with 0.96 g/cm^2 of iron). Identical cans devoid of Na were used for the background measurements. A minor improvement was the removal of the 1.24 g/cm^2 angle iron absorber-support, which had contributed a coherent neutron rate comparable with that of the Mg and Ca absorbers.

III. PROCEDURE

Data were taken in periods of 8–20 hours. Between each two periods a set of routine tests was performed, and the next run was made with the opposite focusing sense for the magnets, or with the alternative to the previous absorber—no-absorber arrangement, or both. The time spent in measuring the background rates is

TABLE II. Neutron source rates.

Conforto and Sard Pb		Present experiment Pb		Na		Mg	
Position of source	Relative rate	Position of source	Rate (min^{-1})	Position of source	Rate (min^{-1})	Position of source	Rate (min^{-1})
1	1	1	56.1 ± 0.6	1	61.17 ± 0.42	1	61.7 ± 0.66
2	1.7	2	60.7 ± 0.6			2	66.0 ± 0.71
3	2.3	3	65.6 ± 0.6			3	49.4 ± 0.53
						4	54.2 ± 0.50
	Incoherent efficiency		$(3.61\pm0.02)\%$		$(3.63\pm0.02)\%$		$(3.31\pm0.02)\%$
	Coherent efficiency		$(3.24\pm0.17)\%$		$(3.21\pm0.21)\%$		$(2.93\pm0.19)\%$

thus interspersed in small periods between that concerned with the raw data. This procedure was adopted to insure that the background rates (no-absorber and positive-focusing runs) are the same, on the average, when they are being measured as when they are concealed in the raw data runs.

The neutron detection efficiency was determined, as has been mentioned, by placing a calibrated Ra$-\alpha-$Be source in various positions in the absorber. Four positions of the source were used with Mg, but only one was found practicable in the case of Na because of the nature of the absorber holder. Table II gives the rates corrected for background and Fig. 3 defines the positions used.

The (about 10%) correction necessary to convert incoherent to coherent efficiency because of the finite time (328 μsec) in which the neutron may be counted involves the mean life of the neutron in the moderator. For this figure the value (152_{-35}^{+37}) μsec as determined in the unmodified apparatus[13] was employed.

The run with Pb was made as a check on the functioning of the entire apparatus, after most of the changes described above had been made. About 345 hours of running time led to a value for the mean multiplicity of 1.48 ± 0.17, which may be compared with the previous[13] value of 1.47 ± 0.13. The Pb absorber thickness of 35.7 g/cm^2 used in this run roughly corresponds to the stopping power of the light absorbers employed later, namely, 18.5 g/cm^2 for Na and 25.13 g/cm^2 for Mg. Table III summarizes a comparison of this run with the data obtained previously.[13]

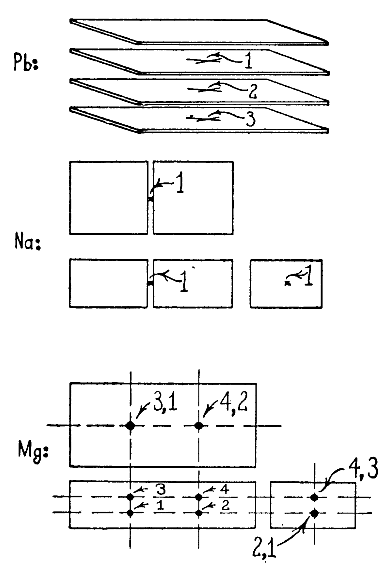


FIG. 3. Positions of source used to measure neutron detection efficiencies.

The good agreement in multiplicity values obtained in the two runs engenders confidence in the correct functioning of the apparatus after being modified as previously described. The fact that a thinner absorber leads to essentially the same result as a thick one is perhaps significant with respect to the focusing properties of the magnetic lens. The lighter absorber selects a lower momentum band (0 to 146 Mev/c instead of 0 to 215 Mev/c upon leaving the paraffin barrier) and one expects poorer focusing with lower momentum because of greater Coulomb scattering. That the change in mean momentum does not grossly increase the leakage is shown by a comparison between the two runs. Comparison of the coherent neutron rate (with Pb) with three counters in the incident particle telescope instead of two (as used, see Fig. 1) indicated that the less-restrictive geometry increased the leakage of wrong-sign particles by an appreciable amount. On the other hand a run made with better geometry, using just one counter in each tray of the telescope, gave no indication of improving the purity of the beam over the two-counter case. This result would imply that scattering of wrong-sign mesons into the beam is small for the two-counter case.

More direct information on this point had already been obtained[13] by looking for decay electrons after a stopping in Pb with the lens set to focus negative particles. In Pb, where interaction predominates overwhelmingly over decay, one expects no decay electrons

TABLE III. Comparison of results for lead.

Experiment	Absorber thickness g/cm^2	Hours run	Neutrons detected per stopping	Mean multiplicity, m
Conforto and Sard	86.3	273	$3.1\pm0.3\%$	1.47 ± 0.13
Present	35.7	345	$4.79\pm0.48\%$	1.48 ± 0.17

TABLE IV. Comparison of the incident telescope rates (min^{-1}).

Rate	Averaged for absorber	Pb	Mg	Na
$(A_1A_2B)^-$		23.78±0.05	27.96±0.04	23.32±0.03
$(A_1A_2B)^+$		26.09±0.06	29.86±0.05	25.16±0.03
Test run with no field gave 22.08±0.25.				

after one microsecond, and such counts may be attributed to a leakage flux of positive mesons. The conclusion was that the upper limit of the leakage factor for the Pb absorber is about 5%.

IV. RESULTS

Table IV summarizes the data relating to the incident telescope rate for the various absorbers. Comparison with calculated values indicates that, especially in the case of Na, the incident telescope was not always operating at 100% efficiency. Because of the fact that inefficiency in the A_1A_2B trays merely discards, in an unbiased manner, some of the incoming mesons, and the fact that background runs were interspersed between short periods of data taking, the results of runs made when the incident telescope rate was somewhat low were simply averaged with the rest.

Table V compares the observed stopping rate (expressed as percentage of the telescope rate and corrected for circuit dead time) with the same quantity calculated in various ways. Two estimates of the absolute sea-level μ-meson momentum spectrum are employed: one due to a survey by Rossi[20] and the other from cloud-chamber data.[21] Following the previous procedure,[13] the effect of the magnetic lens was considered to be to increase the effective width of the bottom tray.

In the sixth column these calculations are compared with a much cruder one made by ignoring the increase in lens-opening angle for the stopped particles. The lens is considered only as an absorber, and Rossi's[20] values of the differential and integral momentum spectra are used to find the relative stopping rate. As would be expected, this estimate is lower than the observed value. It appears from Table V that although the Pb stopping rate agrees satisfactorily with the calculated value, the rates for Na and Mg appear to

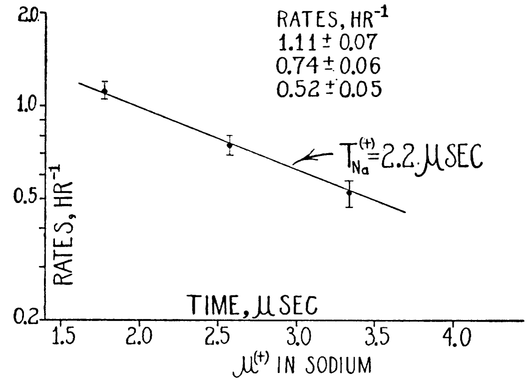


FIG. 4. Time distribution of pulses from the C tray for the sodium absorber, focusing positive particles.

be too low. One would suspect inefficiency in the C tray, but the no-absorber rates do not indicate any difficulty for these runs. The reason for the difference is not known.

Figures 4 and 5 show the time distribution of pulses from the C tray with respect to incident coincidences, for the Na absorber. The data for positives (Fig. 4) are seen to be in good agreement with the known 2.2-μsec mean life of the positive meson.[22]

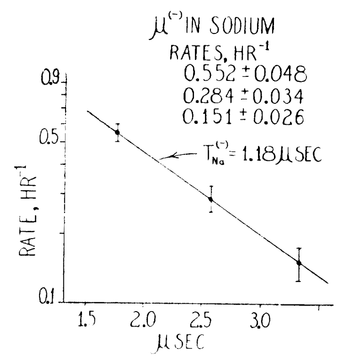
The data presented in Fig. 5 were used to determine the mean life of negative μ mesons in Na, and thereby the fraction that interact rather than decay. Correction was made for an approximately 10% overlap between the second and third channels by the following method: A straight line was fitted by eye to the data displayed in a semilogarithmic plot to give an initial mean life estimate. This estimate was then employed to compute the expected number of counts in the overlap region, which was then used to correct the third channel rate. (The correction amounted to about 9%.) A second mean life estimate was then made in the same way by using the corrected third channel rate; the process could be repeated until subsequent estimates reach a limiting value. The third estimate differed so little from the second that the process was terminated there.

TABLE V. Comparison of stopping rates.[a]

I	II	III	IV	V	VI
Pb	19	1.96%	2.18%	1.86%	1.33%
Mg	23.8	1.85%	2.73%	2.33%	1.66%
Na	17.7	1.33%	2.03%	1.73%	1.22%

[a] I: absorber. II: thickness in g/cm² air equivalent. III: observed value, %, corrected for dead time. IV: calculated stopping rate/telescope rate, %; adaptation of Sard and Conforto's method,[13] using counter values for flux. V: calculated stopping rate/telescope, %; same method as IV using cloud chamber value for flux. VI: crude calculations ignoring lens and using counter value for flux.

FIG. 5. Time distribution of pulses from the C tray for the sodium absorber, focusing negative particles.



[20] B. Rossi, Revs. Modern Phys. **20**, 587 (1948).
[21] J. G. Wilson, Nature **158**, 414 (1946).
[22] W. E. Bell and E. P. Hincks, Phys. Rev. **84**, 1243 (1951).

TABLE VI. Multiplicity data for Na and Mg.[a]

I	II $(A-C:N)^{+/-}_{\text{abs. diff.}}$ hr^{-1}	III $(A-C)^{-}_{\text{abs. diff.}}$ min^{-1}	IV II/III	V T_{abs}^{-} μsec	VI ϵ_{eff}	VII m
Na	0.271 ± 0.049	0.296 ± 0.003	$(1.52\pm0.27)\%$	$1.18_{-0.14}^{+0.19}$	$(3.21\pm0.21)\%$	$1.03_{-0.24}^{+0.31}$
Mg	0.280 ± 0.067	0.496 ± 0.008	$(0.942\pm0.23)\%$	0.96 ± 0.06	$(2.93\pm0.19)\%$	0.58 ± 0.15

[a] I: absorber. II: coherent neutron rate; positive and no-absorber rates subtracted (hr^{-1}). III: negative stopping rate; no-absorber rate subtracted (min^{-1}). IV: number of neutrons detected per stopping negative meson. V: negative μ-meson mean life in the absorber in question. VI: coherent neutron detection efficiency. VII: mean multiplicity; statistical standard error only.

The data corrected for overlap and background were analyzed by the least-squares method[23] to give an estimate of the mean life of negative μ mesons in Na, T_{Na}^{-}, of

$$T_{\text{Na}}^{-}=(1.182_{-0.144}^{+0.190})\ \mu\text{sec}.$$

Since in the least squares method the mean life is expressed as an explicit function of the channel rates, it was found possible to use the usual rule for the propagation of precision indices to compute the error to be assigned to the mean life due to the statistical standard errors of the individual channel rates. The individual channel rates were assumed to follow a Poisson distribution.

It is interesting that the same data when analyzed by the maximum likelihood method[24] lead to a mean life value of

$$T_{\text{Na}}^{-}=(1.183_{-0.113}^{+0.133})\ \mu\text{sec}.$$

The errors correspond to the $(1/e)^{\frac{1}{2}}$ points on the likelihood curve, which was computed numerically from the exact expression based on Poisson statistics.

The value $T_{\text{Na}}^{-}=(1.18_{-0.14}^{+0.19})$ μsec was adopted in the multiplicity calculations. This figure agrees well with 1.30 ± 0.12 μsec, a value obtained by extrapolating from Ticho's[16] negative μ-meson mean-life measurement in NaF along the smoothed capture-probability *versus* Z^4 curve.

For Mg, Ticho's measurement of

$$T_{\text{Mg}}^{-}=0.96\pm0.06\ \mu\text{sec}$$

was employed to find the proportion that interact.

Table VI summarizes the data obtained and shows the calculated multiplicities for Na and Mg. Included in the table for comparison are the results for Pb for both this experiment and that of Conforto and Sard.

The conclusion is that the multiplicity in Mg, while less than that of Pb, is clearly not zero. The Na multiplicity is also seen to be appreciable and probably greater than the value for Mg but less than that for Pb.

Only statistical standard errors are quoted in Table VI, but since these three elements were investigated in essentially the same apparatus, systematic errors should be irrelevant to a comparison between them.

A comparison is of some interest. Primakoff[25] has suggested that because of the high momentum of the neutrino, the meson capture process should favor relatively large angular momentum changes. This assumption allows one to draw some conclusions about the multiplicity to be expected in certain cases. Consider the situation in which the ground state of the original nucleus and the low-lying states of the nucleus resulting from the meson interaction have nearly the same angular momentum quantum number. Under these circumstances, the probability is in favor of the product nucleus' ending up in an excited state differing appreciably in angular momentum from the original nucleus, rather than in a low-energy state having the same angular momentum as the original. The product nucleus would then be more likely to emit a neutron, and one would expect a reasonably large value for the mean multiplicity for this element.

Although detailed knowledge of the level scheme of Na is lacking, the available information is consistent with the situation described above. The spin has been measured as 3/2, and β-decay data of Ne^{23} (the product nucleus in this case) indicate that it has a spin of 5/2 [26,27]; therefore the angular momentum change upon meson interaction would be only one unit if the Ne^{23} were formed in the ground state. Thus the argument would favor a relatively large probability of neutron emission in the case of Na, if the low-lying states have approximately the same angular momentum as the ground state.

This situation may be contrasted with that of Mg^{24}, where the ground state spin is zero[28] (an even-even nucleus). The spin of the product nucleus for μ-meson interaction, Na^{24}, has been measured as four,[29] so that transitions to the product nucleus in the ground state would not be unfavorable because of a low angular momentum change. One would, from this point of view, expect a relatively low multiplicity from Mg^{24}, again assuming that the ground state and its neighboring levels have nearly the same angular momentum.

[23] A. G. Worthing and J. Geffner, *Treatment of Experimental Data* (John Wiley and Sons, New York, 1943), first edition.

[24] Following a method outlined by M. Annis (private communication), and discussed by Annis, Cheston, and Primakoff, Revs. Modern Phys. **25**, 818 (1953).

[25] H. Primakoff (private communication).

[26] R. W. King, Washington University Ph.D. thesis, 1952 (unpublished).

[27] Hollander, Perlman and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

[28] J. E. Mack, Revs. Modern Phys. **22**, 64 (1950).

[29] K. F. Smith, Nature **167**, 942 (1951).

TABLE VII. Comparison of the mean multiplicity (m) for Pb from various experiments.

	Crouch and Sard[a]	Conforto and Sard[b]	This experiment	Widgoff[c]
Statistical error only	2.16±0.15	1.47±0.13	1.48±0.17	2.14±0.13
Including estimated systematic uncertainties	2.1±0.5	1.5±0.4	1.5±0.4	
	Sard and Crouch's[d] weighted mean = 1.7±0.3.			

[a] See reference 12.
[b] See reference 13.
[c] See reference 15.
[d] See reference 1.

Mg^{24} constitutes about 78.4% of the natural isotopic mixture with the remainder being composed of about 10.2% Mg^{25} and 11.4% Mg^{26}. Of the latter two, Mg^{25} should perhaps behave like Na^{23} so far as neutron production is concerned, since Mg^{25} and its product nucleus are also related by an allowed β decay.[26] (The spin of Na^{25} is not known.) Mg^{26} would become Na^{26} which is almost certainly very unstable because of the high neutron excess. (It is not listed as a known isotope.[27]) Because of this excess one might expect it to be a fairly copious source of neutrons.

The presence of these other isotopes would tend, then, to make the multiplicity in the natural material slightly higher than in pure Mg^{24}. Since Mg^{24} constitute about $\frac{4}{5}$ of the natural mixture, the relative multiplicity values of Na and Mg expected from the original argument remain approximately the same.

Table VI indicates the degree to which this expectation is confirmed. Although the difference is not striking, it is in the sense consistent with the above point of view. To investigate this and other hypotheses concerning neutron production by stopped μ mesons it would be desirable to make multiplicity measurements on many other elements. Recent work[30,31] has indicated that only 9±5%[30] of stopped negative μ mesons interacting with C^{12} leave the product nucleus B^{12} in a bound state. The fact that C^{12} has a spin of zero[27] and that β decay data indicate that the spin of B^{12} is one[26] makes this result also consistent with the above interpretation, since the angular momentum change between the ground states would be only one unit as in the case of Na.

Widgoff[15] has determined the multiplicity in Pb, Sn, Bi, and Al by a counter experiment performed underground without a magnetic lens in an experimental arrangement essentially the same as that of Crouch and Sard.[12] The result obtained for Pb (see Table VII) is in excellent agreement with Crouch and Sard and is somewhat higher than that found with the present apparatus; a measurement with the present apparatus for Al, say, would help to show whether this difference persists for different elements. Until this is done, direct comparisons of different elements studied in the two experiments are perhaps not proper. However, Primakoff's qualitative argument would suggest a fairly high neutron multiplicity for Al, similar to Na. Widgoff's result is certainly in accord with this expectation.

Winsberg[32] has recently studied the interaction of negative μ mesons with I^{127}. In this case, radiochemical analysis showed that reactions to Te^{127} involving spin changes of three and one units were present to the extent of 5.2 and 3.0%, respectively. Hence two reactions differing in spin change value by two units appear approximately equally probable.

Our experiment also gives some information about the absolute values of the multiplicities. They are of course subject to uncertainties of a nonstatistical nature, most of which have been briefly mentioned before. The three major sources of systematic uncertainty are (1) possible uncertainties in the meson stopping rate due to imperfect focusing, (2) the 10% indeterminacy in the Ra$-\alpha-$Be neutron source strength, and (3) the possibility that the neutron energy spectrum from μ-meson interactions is very different from that of the Ra$-\alpha-$Be source. The last two would of course contribute an error to the determination of the coherent neutron flux.

The item (1) has been discussed in some detail, and evidence that any leakage is reasonably small has been presented. It is estimated that the uncertainty in the stopping rate is not greater than about ±15%. It is to be borne in mind that a contamination of positive μ mesons in the negative beam would depress the apparent multiplicity, since there would be fewer neutron producing interactions than would be considered as such in the analysis of the data.

It was, of course, to guard against the possibility mentioned in item (3) that the neutron counters were rearranged to increase the flatness of the neutron detection efficiency. Evidence bearing on this question is contained in the results of a determination of the Pb multiplicity by a counter experiment performed underground (Crouch and Sard[9]). Essentially, it was found that changing the flatness of the neutron detection did not appreciably change the apparent multiplicity, so the conclusion was that the two neutron spectra in question are not grossly different. However, a ±10% uncertainty is attributed to the energy

[30] T. N. K. Godfrey, Phys. Rev. **92**, 512 (1953).
[31] E. P. Hincks (private communication).
[32] L. Winsberg, Phys. Rev. **95**, 205 (1954).

spectrum. The assigned systematic uncertainties in this case would then be

(1) uncertainty in stopping rate, ±15%;
(2) uncertainty in neutron rate from source, ±10%;
(3) uncertainty due to energy spectrum, ±10%.

The final multiplicity values become 1.0±0.4 for Na and 0.6±0.2 for Mg.

At the present time, it is possible to compare only the Pb multiplicity with other independent determinations. Since the primary interest of this experiment is not in Pb and not in the absolute values of the multiplicity, we merely quote such a comparison in Table VII.

We reiterate here the principal conclusions of the experiment, aside from the numerical values already quoted. They are that the neutron multiplicities due to μ mesons' stopping in Mg and Na are clearly nonzero and the Na value is very probably less than that for Pb but greater than that for Mg. The indicated relative multiplicity values for Na and Mg are consistent with the particular hypothesis that influenced the choice of absorber, namely, the view that the meson-nucleus interaction process favors a relatively large angular momentum change. Further experimental work is desirable to improve our understanding of this interaction.

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Pion Production in Electron-Proton Collisions*

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The close relationship between photopion and electropion production from protons allows an unambiguous first estimate (the standard value) for the ratio of these cross sections, based on assumptions very close to those of the Weizsäcker-Williams method. Deviations of the ratio from this estimate arise from pion production by the longitudinal components of the field of the scattered electron and from the variation of the off-diagonal transverse excitations from their diagonal photoproduction values. The dependence of these deviations on the physical processes contributing to the electromagnetic excitation of pions is discussed in terms of matrix elements specified in the pion-nucleon center-of-mass system, both for various phenomenological contributions and for specific meson theories. The experimental values reported are interpreted as an indication of the smallness of longitudinal production, in qualitative accord with the fixed source theory. These features may also be investigated by study of the energy spectrum of inelastically scattered electrons and of the azimuthal variation of pion production relative to the scattering plane, which are also discussed here.

1. INTRODUCTION

MEASUREMENTS on direct pion production by electrons incident on hydrogen have recently been made by Panofsky, Woodward, and Yodh.[1] Since this pion production is induced by the action of the virtual electromagnetic field of the scattered electron, it is closely related to the photoproduction of pions from hydrogen. However, in contrast to photoproduction, the energy k_0 transferred by this virtual field is not necessarily equal to the momentum transfer k. Furthermore, while the electromagnetic field in the photoproduction process is transverse, the virtual electromagnetic field in the electron-production process contains both transverse and longitudinal components. For these reasons, it is expected that the experimental results will contain new information on the electromagnetic properties of the pion-nucleon system.

The close relation between the interactions produced by a moving charged particle and those due to incident electromagnetic waves was first pointed out in 1924 by Fermi,[2] who related stopping power for α particles to the electromagnetic properties of the material. Weiz-

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[1] Panofsky, Woodward, and Yodh, Phys. Rev. **102**, 1392 (1956).

[2] E. Fermi, Z. Physik **29**, 315 (1924).