Shell Model for the Positive-Parity States of N^{15}†

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Energy levels and wave functions arising from the configurations $s^4p^{10}s$, $s^4p^{10}d$, and s^3p^{12} have been calculated using a central plus single-particle spin-orbit interaction. Correlations have been made between theory and experiment for a dozen identified positive parity levels in N^{15} (including the 5.31-Mev level). For the seven levels below 9 Mev this has been done mainly by considering $N^{14}(d,p)$ l values and reduced widths. In order of increasing energy, the theoretical spin assignments for these levels are 5/2, 1/2, 7/2, 3/2, 5/2, 1/2, 3/2 (the third and fifth could be interchanged); the wave functions derived for these levels give fair agreement for level positions and surprisingly good agreement for reduced widths. For the upper levels correlations are made by means of the experimental spin assignments. The general agreement here is poor; in particular, a state which has been invoked to explain thermal neutron capture and other neutron processes is not predicted, and the C^{15} β-decay lifetime is not properly given. In general, the wave functions indicate a small interaction between configurations but, apart from this, are not consistent with the idea that the inequivalent particle is effectively coupled to only one state for $A=14$.

I. INTRODUCTION

THOUGH nuclei in the mass range $5<A<16$ have been investigated theoretically by many authors, detailed shell model claculations have usually been restricted to the s^4p^{A-4} configuration.[1] Such calculations, of course, apply only to states of parity $(-1)^A$. The interaction used in nearly all these studies has been a two-body central interaction, together with a simple one-body spin-orbit interaction which is supposed to reproduce the effects due to the noncentral part of the true interaction. This shell model has encountered considerable success.

Since many levels of parity $(-1)^{A+1}$ have been observed throughout the p-shell region, it is of interest to consider states of configurations higher than s^4p^{A-4}. In this paper we use the usual two-body central and one-body spin-orbit potentials to consider the positive-parity levels for $A=15$. These levels we regard as belonging to the configurations[2] $s^4p^{10}s+s^4p^{10}d+s^3p^{12}$. On an independent-particle harmonic oscillator model, all three configurations are degenerate and lie one quantum of excitation higher than the s^4p^{11} configuration.

One essentially new feature encountered for these excited configurations is the interaction of an unfilled shell of nucleons with an inequivalent nucleon. Lane[3] has suggested that, in a nucleus A in the p-shell region, states of parity $(-1)^{A+1}$ may be describable in terms of the weak coupling of a $2s_{\frac{1}{2}}$, $1d_{\frac{3}{2}}$ or $1d_{\frac{5}{2}}$ nucleon to a definite s^4p^{A-5} state of the nucleus $(A-1)$. In addition, Lane proposes that there may be other states of parity $(-1)^{A+1}$ corresponding to the removal of a $1s$ nucleon from a definite s^4p^{A-3} state of the nucleus $(A+1)$. In both cases a $(-1)^{A+1}$–parity state in nucleus A belongs to a single configuration and is associated with a single state of one or other of the neighboring nuclei. The lowest few such states in nucleus A would be expected to be associated with the ground state (or possibly with a low excited state) of the neighboring nucleus. It should be noted that, unlike most weak-coupling assumptions made in jj-coupling shell models, the assumptions of Lane's model are not basically opposed to the requirements of the exclusion principle. The point here is that the antisymmetry requirement makes it scarcely meaningful to discuss the polarization of an unfilled shell produced by an equivalent nucleon, because the wave functions describing such an effect are in general not antisymmetric with respect to the polarizing nucleon. But when this nucleon is inequivalent there is no *a priori* objection to the idea of weak polarization. (It could happen, for example, that the exchange integrals involving the inequivalent nucleon are small compared to the direct integrals.) It is therefore of considerable interest to determine not only the validity of Lane's very simple model but also its connection with less restrictive shell models.

There are good experimental and theoretical reasons for paying special attention to the $A=15$ case. N^{15} in particular has about a dozen identified positive-parity levels.[4-6] For the lower levels spins are unfortunately unknown but reduced neutron widths have been measured by the $N^{14}(d,p)$ experiment.[5] For the upper levels

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[1] D. Kurath, Phys. Rev. **101**, 216 (1956). Other references are given in this paper.

[2] Throughout we shall often not write principal quantum numbers. The reader will easily distinguish between 1s and 2s.

[3] A. M. Lane (unpublished).

[4] For general experimental references see F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[5] T. S. Green and R. Middleton, Proc. Phys. Soc. (London) **A69**, 28 (1956); R. D. Sharp and A. Sperduto, Massachusetts Institute of Technology Laboratory for Nuclear Science Progress Report, May, 1955 (unpublished); E. K. Warburton and J. N. McGruer, Phys. Rev. **105**, 000 (1957). The earlier experiment of W. M. Gibson and E. E. Thomas [Proc. Roy. Soc. (London) **A210**, 543 (1951)] is also useful. We are indebted to Dr. Sharp, Dr. Warburton, and Dr. McGruer for giving us access to unpublished data.

[6] Bartholomew, Brown, Gove, Litherland, and Paul, Can J. Phys. **33**, 441 (1955). References to earlier work are given in this paper.

spin values and both neutron and proton widths have been determined by resonant reactions.[6] Besides this it is expected that before long more information will become available as a result of measurements of $C^{14}(d,p)$, $C^{14}(d,n)$, and possibly even $O^{16}(p,d)$ and $O^{16}(n,d)$ cross sections. These reduced widths would be particularly instructive because both C^{14} and O^{16} have zero spin and simple wave functions.[7] The $A=15$ polyad is in fact unique in that its levels may be investigated by reactions involving so many and such simple ground states of adjacent nuclei. Besides the particle widths for these $A=15$ levels, there is available also some pertinent information on β and γ transitions.[4,6]

Calculations involving excited configurations are in general considerably more complicated than the analogous calculations for the s^4p^{A-4} configuration. An important practical reason for studying $A=15$ in particular is that the $A=15$ case is one of the few in the p-shell region for which the problem is tractable without the use of exceedingly elaborate computing. Another feasible case is $A=16$, which has been studied by Elliott.[8]

In the next section we consider some general features of our wave function and energy level calculations. Following this we consider the evaluation of various numerical parameters and then give a brief discussion of the pertinent eigenfunctions and energy eigenvalues which emerge from the calculation. The experimental data are then considered and compared with our theoretical results.

II. FORMAL THEORETICAL SPECTROSCOPY

We do not intend to discuss in any detail the rather complicated formal problems involved in the spectroscopy of excited configurations[9]; we content ourselves with only a brief discussion of some essential features.

We work in a function space defined by all the antisymmetric states of configurations $s^4p^{10}s$, $s^4p^{10}d$, s^3p^{12}. If we disregard the trivial J_z, T_z dependence the total number of these "basic" states is 120. For a given J, T combination the number of basic states varies from 1 (for J, $T=\frac{1}{2}$, $\frac{1}{2}$) up to 25 (for J, $T=\frac{3}{2}$, $\frac{1}{2}$).

[7] The zero spin of the target reduces the ambiguity in the J determination for $A=15$. Furthermore, for reactions to a definite $A=15$ state, the l and j values of the transferred nucleon are each fixed. Thus the interpretation of results with C^{14} and O^{16} targets is much more clear-cut than for the N^{14} target, especially if we use for the wave functions a representation in which l and j are specified for the inequivalent nucleon. Moreover the unique closed-shell wave function for O^{16} allows a particularly simple interpretation of $O^{16}(p,d)$ and $O^{16}(n,d)$ reduced widths. For a C^{14} target the situation is almost as simple. The C^{14} wave function has a single degree of freedom (only two contributing multiplets); the wave function is rather well known [R. Sherr *et al.*, Phys. Rev. **100**, 945 (1955)], and can in fact be more accurately determined by the $C^{14}(d,n)$ experiment itself [J. B. French, Phys. Rev. **103**, 1391 (1956)].

[8] J. P. Elliott (unpublished). Elliott has also made calculations for $A=15$ similar to those reported here.

[9] The complete discussion is given by E. C. Halbert, thesis, University of Rochester, 1956 (unpublished). For some of these matters we owe a considerable debt to J. Hope, Ph.D. thesis, University of London, 1952 (unpublished).

We introduce these basic states by first constructing a set whose functions are not completely antisymmetrized. Consider the function

$$|(s^4)(p^{10})_\alpha\, d;\quad TSLJJ_zT_z\rangle. \tag{1}$$

For this state, particles $1\cdots4$ form a closed $1s$ shell; particles $5\cdots14$ form an antisymmetric p^{10} function with resultants T_α, S_α, L_α; particle 15 is in a $1d$ state; and the three groups are coupled together to form resultants $TSLJJ_zT_z$. For the radial dependence of the single-particle functions we use the customary harmonic oscillator forms.

From (1) we construct an antisymmetric state by operating with an antisymmetrizer and then renormalizing. In the same way we construct antisymmetric functions belonging to the other configurations. For the s^3p^{12} configuration, there is of course only one basic state.

These basic antisymmetric states supply a representation in which we evaluate matrices of the interaction Hamiltonian. Before diagonalizing these to produce the theoretical wave functions and relative energy values, we perform a transformation to eliminate certain spurious center-of-mass effects. The point here is that our basic wave functions, like all independent-particle wave functions, are functions of the center-of-mass coordinate $\mathbf{R}$ as well as the internal coordinates. When considered as functions of the internal coordinates only ($\mathbf{R}$ held constant) the members of a complete set of independent-particle wave functions are not all linearly independent. Elliott and Skyrme[10] have pointed out that this may lead to important errors. They have shown too that, when harmonic oscillator functions are used, such errors arise only in calculations for excited configurations; here the errors may be avoided by transforming to a new representation in which each function has an $\mathbf{R}$ dependence corresponding to a definite harmonic oscillator state for the mass center. Then those transformed functions with the mass center in a $1s$ state are linearly independent. The remaining ones are "spurious" and are to be discarded. The nonspurious functions may be converted into a set of internal functions simply by dividing them by the normalized $1s$ function for the mass center. In practice this last step is seldom necessary, since it will usually happen that matrix elements are invariant under this operation. When this occurs (and it does for all the operators which we shall encounter), we may calculate the matrix of the operator in question and then transform it by a rectangular matrix to the representation supplied by the nonspurious states. This is the technique which we have used.

[10] J. P. Elliott and T. H. R. Skyrme, Proc. Roy. Soc. (London) **A232**, 561 (1955).

III. NUMERICAL PARAMETERS

For the nucleon-nucleon interaction we use the Rosenfeld[11] two-body central interaction with a Yukawa radial dependence. This is supplemented by the usual single-particle spin-orbit interaction, and also by a configuration potential V_{conf} which we discuss below.

$$H_{\text{int}}=V_0\sum_{i<j}(0.1+0.23\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j(r_0/r_{ij})\times\exp(-r_{ij}/r_0)+\sum_i a(r_i)\mathbf{s}_i\cdot\mathbf{l}_i+V_{\text{conf}}. \quad (2)$$

We take $r_0=1.385\times10^{-13}$ cm (which corresponds to a meson mass 273 m_e) and $V_0=37$ Mev. (It was found by Elliott and Flowers[12] that for $A=18$ and 19, depths between 35 and 40 Mev were most satisfactory.)

For the radial wave functions we use harmonic oscillator forms, whose exponential factor $\exp(-\frac{1}{2}\nu r^2)$ defines the parameter ν. To fix the value of ν we use Swiatecki's criterion[13] and, taking the nuclear radius to be $R=1.4\times10^{-13}$ cm, find $\nu=0.45\times10^{26}$ cm^{-2}. (The single-particle harmonic oscillator corresponding to this would have $\hbar\omega=18.7$ Mev.) These values for V_0 and ν give, for the well-known p-shell integrals,[14] $L/K=5.9$, $K=-1.01$ Mev.

The spin-orbit parameters a_p, a_d are taken as -4.22 and -2.03 Mev, respectively, being derived[4] from the p-hole splitting in N^{15} and the d particle splitting in O^{17}.

Finally we consider V_{conf}; this operator introduces *a priori* energy differences among the three different configurations which on a harmonic oscillator model are degenerate. To explain why we include this operator, and to evaluate one of the numerical parameters involved, we consider the single-particle states of O^{17}. Figure 1 shows the level spacing for the $s_{\frac{1}{2}}$, $d_{\frac{3}{2}}$, and $d_{\frac{5}{2}}$ states[4] and shows also that our central and spin-orbit potentials do not adequately account for the observed separations. We can remove the discrepancy by adding in a diagonal operator V_{conf} such that

$$V_{\text{conf}}(s^4p^{12}d)-V_{\text{conf}}(s^4p^{12}s)=2\text{ Mev}. \quad (3)$$

We assume now that the same parameter introduced into the $A=15$ matrices would similarly improve the model; i.e., we include in H_{int} a diagonal operator

$$V_{\text{conf}}(s^4p^{10}d)-V_{\text{conf}}(s^4p^{10}s)=2\text{ Mev}. \quad (4)$$

There is unfortunately no such plausible simple method for fixing the relative position of the s^3p^{12} configuration (which of course affects only the J, $T=\frac{1}{2}$, $\frac{1}{2}$ states). We therefore quite arbitrarily adopt

$$V_{\text{conf}}(s^4p^{10}d)-V_{\text{conf}}(s^3p^{12})=0. \quad (5)$$

A less arbitrary procedure would have been to keep this as a free parameter and then fix its value by requiring an optimum fit to the $A=15$ data. This would involve more labor than appears to be justified in view of the large experimental and theoretical uncertainties.[15]

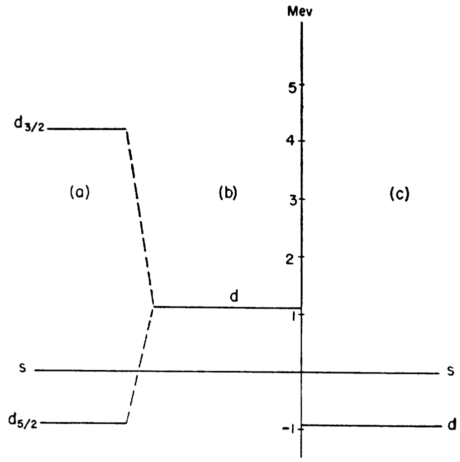

FIG. 1. Shown are (a) the single-particle levels in O^{17}, (b) the levels when corrected for the spin-orbit effect, and (c) the levels as calculated by considering the interaction of the outside particle with the closed s and p shells. To compensate for the difference between (b) and (c), we add in $V_{\text{conf}}=2$ Mev. If we consider the interaction with the s shell alone, we need $V_{\text{conf}}=-2.5$ Mev; and for the p shell alone $V_{\text{conf}}=+5.5$ Mev.

IV. EIGENVECTORS AND EIGENVALUES

With the basic states and interaction described above, the energy matrices were calculated, transformed to eliminate the spurious states, and then diagonalized to produce the eigenvalues and eigenvectors. The last two steps were carried out with the Univac at New York University.

The theoretical level scheme for the lowest 25 states is shown in Fig. 2. (Comparison with experiment will be discussed in Sec. V.) The eigenvectors emerging from the Univac were not in a representation best suited to a discussion of their general features; in particular, because of the "spurious-state" transformation and because the J value of the $A=14$ core (s^4p^{10}) is unspecified, major transformations are required before a detailed comparison with Lane's weak coupling model[3] can be made. We hope to make this comparison at a later date; in the meantime we make some comments about the first four $T=\frac{1}{2}$ and the first two $T=\frac{3}{2}$ states, whose fractional composition is shown in Fig. 3.

[11] L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1948).

[12] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).

[13] W. J. Swiatecki, Proc. Roy. Soc. (London) **A205**, 238 (1951). Swiatecki suggests that the nuclear radius R be taken as the distance at which the probability density defined by the single-particle wave-function falls to $\frac{1}{4}$ its maximum value.

[14] See, for example, reference 1.

[15] It should be noted that the values of the V_{conf} parameters depend on the total number of particles whose interactions we take into account and also on the constants of the central interaction. The technique of adding in V_{conf} is identical with that, familiar in jj-coupling theories, of assigning single-particle level differences. In any case it is a very crude way of compensating for some of the inadequacies of the model.

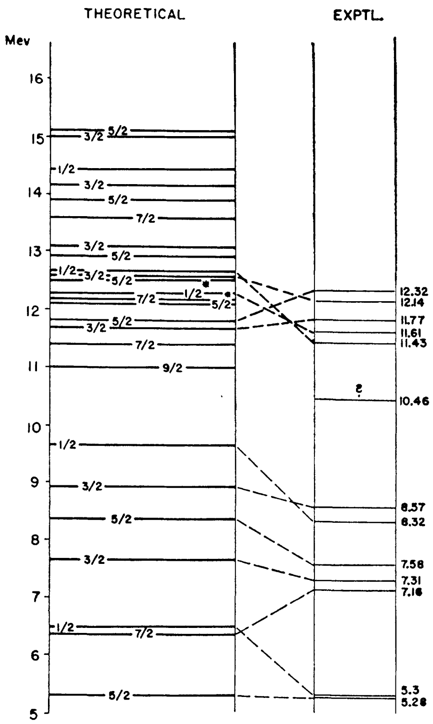

FIG. 2. The theoretical spectrum for the first 25 positive parity levels is shown and the correlations which are made with the experimental levels. As discussed in the text, there are levels of unknown parity in the 10-Mev region which we do not show. An asterisk denotes a $T=\frac{3}{2}$ level.

(a) In Lane's model each state belongs predominantly to a single configuration. The results of Fig. 3 agree well with this hypothesis (though at present we do not distinguish between the $s^4p^{10}d_{\frac{3}{2}}$ and $s^4p^{10}d_{\frac{5}{2}}$ configurations).

(b) Contrary to a demand of Lane's model, we do not find that the s^4p^{10} core is strongly associated with only one low-lying state of N^{14}. This we see from the isotopic spin decomposition; our four lowest $T=\frac{1}{2}$ states have the following $(T_\alpha=0)/(T_\alpha=1)$ ratios: 26/74, 75/25, 35/65, 76/24.

(c) Lane has suggested that the 5.31-Mev level in N^{15} is the s^3p^{12} state expected on his model. For our model the s^3p^{12} configuration is of little importance for low-lying states, and indeed we find less than a 2% contribution to each of the four lowest J, $T=\frac{1}{2}$, $\frac{1}{2}$ states.[16] [This percentage could of course be increased by taking a positive value for the parameter of Eq. (5).]

V. COMPARISON WITH EXPERIMENT: ENERGY LEVELS AND REDUCED WIDTHS

There are eleven levels in N^{15} for which there is direct evidence of positive parity and two more for which there is indirect evidence: we list all thirteen in Table I.

Consider first the seven levels below 9 Mev. Six of these lower levels have been definitely identified as positive parity; their spins are unknown though spin restrictions are supplied by the l_n values. The spin and parity of the 5.31-Mev state are undetermined by the $N^{14}(d,p)$ data since this state shows no stripping, but our analysis will favor positive parity (a second argument for this will be given in Sec. VI). In making a correlation between experimental and theoretical levels, we shall use the information on spins and reduced widths. Reduced neutron widths are available for these levels from $N^{14}(d,p)$ investigations by Green and Middleton, Sharp and Sperduto, and Warburton and McGruer,[5] at deuteron energies 8, 7 and 15 Mev,

TABLE I. Listed are the energies of the experimentally identified positive parity levels with their spins and reduced widths as determined by stripping or resonant reactions and the theoretical predictions for the same quantities.

Energy		J		$(2J+1)\theta_n^2(l_n)$		$(2J+1)\theta_p^2(l_p)$		
Expt.	Th.	Expt.[a]	Th.[a]	Expt.	Th.	Expt.	Th.	Notes
5.28	5.28		5/2	0.03(2)	0.044(2)			b
5.31	6.5		1/2		0.016(0)			c
					0.006(2)			
7.16	6.4		7/2	0.33(2)	0.48(2)			d
7.31	7.7		3/2	0.48(0)	0.62(0)			e
					0.005(2)			
7.58	8.4		5/2	0.40(2)	0.41(2)			d, f
8.32	9.7		1/2	0.41(0)	0.49(0)			e
					0.001(2)			
8.57	8.9		3/2	0.018(0)	0.016(0)			g
				0.031(2)	0.11(2)			
10.46?								h
	11.0		9/2					i
	11.4		7/2					i
11.43	12.7	1/2	1/2	0.011(0)	0.021(0)	0.016(0)	0.025(0)	j, k
					0.003(2)			
11.61	12.3	1/2*	1/2*	0.007(0)	...	1.3(0)	1.4(0)	l
11.77	11.7	3/2	3/2	0.025(0)	0.005(0)	0.010(2)	0.06(2)	
					0.60(2)			
12.14	12.6	3/2	3/2	0.023(0)	0.06(0)	0.15(2)	0.005(2)	
					0.04(2)			
12.32	11.8	5/2	5/2	0.12(2)	0.01(2)	0.003(2)	0.01(2)	
	12.1		5/2*				3.8(2)	m
	12.2		7/2					

[a] The asterisk denotes $T=3/2$ states.
[b] We fix the zero of energy by placing this level at 5.28 Mev. We do not calculate either the absolute binding energy or the energy separation between + and − levels.
[c] This level shows no stripping. The small calculated reduced widths are consistent with this. There is no independent parity determination. If, however, the C^{15} ground state spin is 1/2, then the β decay demands that this level have positive parity, if the 5.28-Mev level has $J=5/2$.
[d] The assignments for the 7.16 and 7.58-Mev levels could be interchanged.
[e] Only $l_n=0$ is observed. The small calculated reduced width for $l_n=2$ is consistent with this.
[f] Experimentally there is a distinct possibility of an $l_n=0$ component here and this would invalidate the spin assignment. An experimental difficulty is that the proton group is energetically almost identical with that produced in the very strong $l_n=0$ first excited state reaction of $O^{16}(d,p)$. The oxygen contamination was least disturbing in the Warburton and McGruer experiment[5] and this group reports the $l_n=0$ component as uncertain.
[g] Green and Middleton[5] label this as $l_n=1$ but the 0+2 mixture seems much more likely.
[h] As discussed in the text, a level of spin 1/2 or 3/2 with a large $l_n=0$ reduced width is invoked in this region to explain the thermal neutron capture in N^{14} and other neutron data. A theoretical counterpart is not found.
[i] An experimental counterpart to this level has not been identified. There are however several unidentified levels in this region.
[j] The calculated reduced widths for this and higher levels make use of the resonant reaction single particle widths. The predicted stripping widths are smaller by a factor 2.2. Note also that our definition[17] of θ^2 differs by a factor from that of reference 6.
[k] For this level and the 11.61-Mev level we use revised experimental widths as given by A. J. Ferguson and H. E. Gove, Bull. Am. Phys. Soc. Ser. II, 1, 180 (1956).
[l] It is not certain that $T=3/2$ but it does seem quite probable. The experimental neutron width is given for this case in units of $3\hbar^2/2\mu a^2$ without the isotopic spin factor. The neutron transition is of course isotopic-spin forbidden.
[m] The 5/2* level which, on even the simplest arguments, is expected in this region has not been experimentally identified.

[16] Only two theoretical states have an s^3p^{12} contribution larger than 5%: a state at 25 Mev (25%) and one at 32 Mev (40%).

respectively. These widths are displayed in Table I.[17] We do not list separate values as determined by the various authors; for in cases where different investigations yield widths for the same level, these widths generally lie within 15% of the values listed. (SS have measured only relative cross sections but a single scale factor makes their widths fall within the same margin.) Various other comments about the experimental results are given also in Table I.

The isotopic spin may be taken as $\frac{1}{2}$ for all seven levels. In six cases the size of the observed neutron with (which would be zero for a $T=\frac{3}{2}$ level) justifies this assignment. For the 5.31-Mev level the low excitation itself justifies a $T=\frac{1}{2}$ assignment.

In an attempt to correlate our theoretical states with the observed one, we have calculated reduced neutron widths $\theta_n^2(l_n)$ for a dozen of the lowest theoretical levels.[18] The quantity which we calculate from the theoretical wave-functions is the reduced width in units of the corresponding single-particle reduced width (which would refer to the escape of a particle from a potential well). To evaluate the single-particle (d,p) widths, we use the measured $l_n=0$, 2 widths for the $O^{16}(d,p)$ reaction to the 0.88 Mev and ground states of O^{17}, which are single-particle states. For $E_d=7.7$ Mev the experiment of Burge *et al.*,[19] gives $\theta_n^2=0.32$, 0.11 for $l_n=0$, 2, respectively; for $E_d=19$ Mev the experiment of Freemantle *et al.*[20] gives $\theta_n^2=0.17$, 0.08. It seems improbable that the difference between these sets of values represents a real effect for there is considerable evidence[21] that (d,p) widths in this energy range do not show an energy variation of this magnitude. The value 0.17 for $l_n=0$ is suspiciously low, for in a $C^{12}(d,p)$ experiment Green and Middleton[5] have measured an $l_n=0$ width of 0.27 for the reaction to the first excited state of C^{13} and this width should supply a lower limit to the $l_n=0$ single-particle width. We therefore adopt the larger values $\theta_n^2=0.32$, 0.11 for the single-particle (d,p) widths.

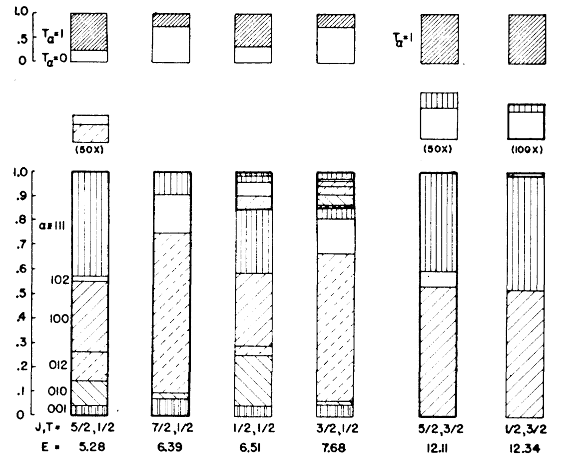

FIG. 3. The composition of six eigenvectors. $\alpha\equiv T_\alpha$, S_α, L_α refers to the s^4p^{10} multiplet and the key for the T_α, S_α, L_α composition is given at the left. The $s^4p^{10}d$ contributions are enclosed by a heavy line, the $s^4p^{10}s$ by a light line (the s^3p^{12} contribution to the $J,T=\frac{1}{2}$, $\frac{1}{2}$ state is 1%); enlarged diagrams are shown for some small contributions. The topmost diagram shows the T_α composition. The energy values are the calculated ones.

Using the (d,p) data, we have made the correlation shown in Table I for the levels below 9 Mev. The agreement in level positions is tolerable; in the worst case there is a 1.4-Mev discrepancy. The agreement could probably be improved by changing some of the parameters, but because of the experimental and theoretical uncertainties the effort involved does not seem justifiable. With respect to the (d,p) widths, we feel that only qualitative agreement can be expected. First, there are uncertainties in extracting (d,p) widths from measured cross sections when the reduced widths are small (as with 5.28-Mev level) or when more than one l_n value is important (as with the 8.58-Mev level). Second, for the theoretical widths there is little justification for the assumption that single-particle widths do not vary with excitation. In view of these uncertainties the agreement between theory and experiment is entirely satisfactory for these lower seven levels with the possible exception of the $l_n=2$ width for the 8.57-Mev level.

In making the assignments of Table I we have ignored the possibility that the 7.58-Mev level shows an $l_n=0$ component. An $l_n=0$ component would require a spin of $\frac{1}{2}$ or $\frac{3}{2}$; it is clear from Fig. 2 and Table I that such an assignment, coupled with the relative paucity of theoretical low-spin levels, would give a far less consistent relationship between observation and theory. For the 7.16- and 7.58-Mev levels, our assignments could be interchanged without seriously disturbing the fit which is obtained.

Of particular interest is the nature of the 5.31-Mev level, which shows no stripping. Butler[17] has suggested

[17] We quote all reduced widths in units of $(C^{T_0,\frac{1}{2},T})^2 3\hbar^2/2\mu a^2$, where $(C)^2$ is the appropriate isotopic spin factor, μ the reduced nucleon mass, and a the interaction radius. The (d,p) widths are deduced from the experimental curves by using the Butler formula [S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951)].

[18] In calculating the neutron widths, one also needs of course the wave function for the N^{14} ground state. We use here the wave function deduced by Sherr, Gerhart, Horie, and Hornyak [Phys. Rev. **100**, 945 (1955)] from an examination of various experimental data. A recent determination by E. K. Warburton and J. N. McGruer (private communication) of the relative reduced widths for $N^{14}(d,p)$ reactions to the two p-hole states of N^{15} is also consistent with this wave function. For calculating proton widths we use the C^{14} ground state function, also given by Sherr *et al.* (This of course is determined uniquely by the N^{14} function and the C^{14} β decay.) These $A=14$ functions, however, are not identical with those which would be deduced by using the interaction given by Eq. (2).

[19] Burge, Burrows, Gibson, and Rotblat, Proc. Roy. Soc. (London) **A210**, 534 (1951).

[20] Freemantle, Gibson, Prowse, and Rotblat, Phys. Rev. **92**, 1268 (1953). For comments on these data and those of Burge *et al.*,[19] see W. M. Fairbairn, Proc. Phys. Soc. (London) **A67**, 564 (1954).

[21] From an examination of the few p-shell cases where absolute reduced widths have been measured at different energies and from the many cases in the p shell and heavier nuclei where relative widths have been measured.

that this state has a high spin value; Lane[3] has suggested that it is the $\frac{1}{2}^+$ s^3p^{12} state expected on his model. (An s^3p^{12} state would have zero reduced width for nucleon emission to the s^4p^{10} ground state of N^{14}.) Our calculations do favor a $\frac{1}{2}^+$ assignment for this level, but as discussed in Sec. IV the theoretical state belongs almost entirely to the $s^4p^{10}s$ configuration and the small theoretical $l_n=0$ width is due to a partial cancellation among contributing terms.

We turn now to the higher levels. There are several levels of unknown parity between 9 and 11 Mev; two of these could correspond to the predicted $7/2^+$ and $9/2^+$ levels shown in Table I. There is indirect evidence that a $\frac{1}{2}^+$ or $\frac{3}{2}^+$ state with large reduced neutron width for emission to the N^{14} ground state should exist in this region below the neutron threshold at 10.8 Mev. This evidence which comes largely from the $N^{14}+n$ thermal neutron reaction is discussed in detail by Bartholomew *et al.*[22] who suggest that the 10.46-Mev level may be $\frac{1}{2}^+$ and be the desired state. In Table I we follow this suggestion. Among our theoretical states below 13 Mev, there are, however, only two which have $l_n=0$ widths comparable with the single-particle width and these two states have already been correlated with observed levels at 7.31 and 8.32 Mev

Though there is here a strong possibility of a serious discrepancy, we have nonetheless thought it worthwhile to compare our results with the experimental data for the states above 11 Mev. Bartholomew *et al.*[6] have used their own $C^{14}+p$ data as well as the results of other authors to give J, T assignments to five positive-parity levels between 11.43 and 12.32 Mev and to determine ground-state neutron and proton widths for these levels. The results are shown in Table I. We show also the level positions and reduced widths which are predicted when we ignore the absence of a theoretical partner for the + state which may be at 10.46 Mev.

In calculating the widths we have not used the same single-particle values as for the lower levels since much experimental evidence indicates that stripping and resonant widths are not identical.[23] Instead we take for both $l_n=2$ and $l_p=2$ the single-particle width 0.26 (as given[4] by the 1.00-Mev resonance in $O^{16}+n$, involving the $d_{\frac{5}{2}}$ state of O^{17}); while for $l_n=0$ and $l_p=0$ we take the value 0.7 based on the arbitrary assumption that the large observed proton width of the 11.61-Mev level in N^{15} is near the theoretical maximum.[24] The errors introduced by this high-handed procedure should be unimportant compared with the intrinsic errors of the entire calculation.

The agreement between theoretical and observed reduced widths is excellent for the 11.61-Mev level, tolerable for the 11.43-Mev level and poor for the other high levels. One feature is shared by theoretical and observed $T=\frac{1}{2}$ states: the ground state reduced neutron widths are much smaller for higher levels than for lower ones. An exception to this trend occurs for the theoretical state which we have correlated with the observed 11.77-Mev level. This exception supplies the worst discrepancy in Table I between theory and experiment.

VI. COMPARISON WITH EXPERIMENT: γ AND β TRANSITIONS

Kinsey *et al.*[25] have examined the γ rays of energy greater than 3–4 Mev which follow capture of thermal neutrons in N^{14}; Bent *et al.*[26] have examined those of energy greater than 5 Mev resulting from $N^{14}(d,p)$ reactions with 4-Mev deuterons. The yield of a given γ ray depends in part on the cross section for formation of the radiating state, either directly or by cascade, and in part on the particular radiation width involved. A detailed discussion of the entire process is not feasible because of the many uncertainties. Instead we refer only to a few pertinent features. We have already commented on the difficulty connected with the state responsible for the thermal neutron capture.

Bent *et al.*[26] do not observe the 7.16-, 7.58-, or 8.57-Mev ground state rays; Kinsey *et al.*[25] observe rather weak 7.16-Mev radiation but do not observe the other two.[27] The absence or weakness of the first two would be easy to understand with the spin assignments given in Table I, for then these transitions to the $\frac{1}{2}^-$ ground state would be *a priori* unfavored compared with the competing ($M1$, $E2$) transitions to the $5/2^+$ level at 5.28 Mev. For the 8.57-Mev level we have used our theoretical wave functions to calculate the $E1$ width for the ground state transition. We find the very small[28] value 0.28 ev; and this, coupled with the rather small (d,p) cross section for direct formation of this level, would satisfactorily explain the absence of this γ ray.

Thompson[29] has observed low-energy radiation following the $N^{14}(d,p)$ reaction. The most prominent peak observed was at 1.88 Mev, and this could be associated with the 7.16→5.28 Mev transition, expected to be strong according to the spin assignments of Table I.

[22] See reference 6. Also E. Melkonian, Phys. Rev. **76**, 1750 (1949).

[23] A good example is supplied by the O^{17} d-state widths given in the text. Two recent measurements by E. K. Warburton and J. N. McGruer (private communication) of the single-particle (d,p) $l=1$ widths in He^5 and N^{15} give values an order of magnitude smaller than the resonant width[4] as measured by $\alpha+n$. Other cases in the p shell are considered by J. B. French and A. Fujii, **105**, 652 (1957). How much of the difference is caused by a real failure of the Butler theory is not clear. There seems to be no compelling reason why, even if the Butler theory does not fail, stripping and resonant widths should be identical.

[24] If, however, the 11.61-Mev level has $T=\frac{1}{2}$ instead of $T=\frac{3}{2}$, its "observed" reduced width would be halved. An independent lower limit for the $l=0$ single-particle width is supplied by the 0.46-Mev resonance in $C^{12}+p$ for which a width 0.5 is found.[4]

[25] Kinsey, Bartholomew, and Walker, Can. J. Phys. **29**, 1 (1951).

[26] Bent, Bonner, McCrary, Ranken, and Sipple, Phys. Rev. **99**, 710 (1955).

[27] A 7.58-Mev γ ray could have been obscured by the 7.72-Mev γ ray from Al^{28} in the survey measurement made by Kinsey *et al.*

[28] See D. H. Wilkinson, Phil. Mag. **1**, 127 (1956).

[29] L. C. Thompson, Phys. Rev. **96**, 369 (1954).

Among the higher N^{15} states, the ground state radiation widths for the two $\frac{1}{2}^+$ states at 11.43 and 11.61 Mev have been measured[6] to be 2.4 and 26.3 ev, respectively. The theoretical $E1$ widths are found to be 30 and 33 ev. For the 11.43-Mev level the agreement is quite unsatisfactory and remains so if we instead correlate this level with our state J, $T=\frac{1}{2}$, $\frac{1}{2}$ state, which is computed to lie at 14.4 Mev and have an $E1$ ground state width of 0.005 ev. On the other hand the agreement is quite satisfactory for the 11.61-Mev level identified as the lowest J, $T=\frac{3}{2}$, $\frac{1}{2}$ level. These results are, at least, consistent with our expectation that the higher theoretical states with given J, T are less trustworthy than the lower ones.

C^{15} undergoes β decay mainly to one or both members of the 5.3-Mev doublet,[30] with $\log ft \sim 3.8$. The C^{15} ground state spin is unknown but has usually been considered as $\frac{1}{2}^+$ or $5/2^+$ on the supposition that it belongs principally to the configuration $(p_{\frac{1}{2}}^2)_0 s_{\frac{1}{2}}$ or $(p_{\frac{1}{2}}^2)_0 d_{\frac{5}{2}}$. In our model the lowest two $T=\frac{3}{2}$ states are $J=\frac{1}{2}$ and $5/2$ and are almost degenerate near 12.2 Mev. Then no matter which is lower, one would expect β decay mainly to the 5.3-Mev doublet. Quantitatively however the agreement is poor. The calculated $\log ft$ values [using[31] $G_{GT}^2=(1.37/6550)$ sec^{-1}] are about 4.8 in each case, which compares badly with the experimental value 3.8.

Bartholomew *et al.*[32] have calculated that a level mirroring the C^{15} ground state should occur in N^{15} at about 11.8 Mev and they therefore suggest that the level at 11.61 Mev (J, $T=\frac{1}{2}$, $\frac{3}{2}$) plays this role. In this case β decay would, according to Table I, occur to the $\frac{1}{2}^+$ member of the 5.3-Mev doublet; this would supply another argument against the idea that this state is essentially pure s^3p^{12} since β decay would then be forbidden.

VII. CONCLUSIONS

The single spin-orbit model which we have used makes predictions which, at least for the levels below 9 Mev, appear to be in rather good agreement with the actual facts. We find approximate agreement in certain gross features, in particular the level spacing. But beyond this there are several cases where the agreement suggests that, even in some of their more subtle features, the theoretical wave functions appear to describe the actual physical situation. This agreement in detail is somewhat surprising, for the single-particle spin-orbit model has little *a priori* justification when there are several particles outside a closed shell.[33] On the other hand, the spins of the various levels have not been independently determined; when they are, the entire agreement could turn out to be illusory.

For the higher states, the quantitative agreement is poor for the reduced widths and there is quite possibly a gross disagreement with regard to the density of low-spin levels. The quantitative failure of the C^{15} β-decay calculation is disappointing; for this involves the lowest states for given J, T and these we might expect to be treated more accurately than the higher ones.

The effort involved in calculations of this type is such that it seems at present not worthwhile to consider in such the $(-1)^{A+1}$ parity states for the more complicated lighter p-shell nuclei. However, as discussed briefly in the introduction, further $A=15$ calculations would be in order when more experimental data are available for N^{15} and C^{15}, and also for O^{15} (which we have ignored in the present work). Certain "academic" features of the calculation may also deserve further study; as discussed above, these include some further aspects of the angular momentum coupling scheme. The available wave functions are also well suited for a study of the N^{15} photodisintegration.

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[30] Douglas, Gasten, Downey, and Mukerji, Bull. Am. Phys. Soc. Ser. II, **1**, 21 (1956).
[31] J. B. Gerhart, Phys. Rev. **95**, 288 (1954).
[32] Bartholomew, Litherland, Paul, and Gove, Can. J. Phys. **34**, 147 (1956).
[33] A recent paper by A. M. Feingold (to be published) makes a very interesting study of this question.