the second rotational level. No previous measurements have been made of the K/L ratio for the level in Re^{185}; however, in β-decay the level in Re^{187} has been observed. The value of $K/L=4.6$ for the heavier isotope agrees with the previously measured value[13] of $K/L\sim 5$. The values of $B_{ex}(E2)$ for these two nuclei have an uncertainty of 50% since the target thickness was determined by the "stopping electron" method. However, the relative Q_0 values obtained have approximately the same ratio as the spectroscopically measured quadrupole moments. Also, they fit into the general trend of Q_0 values in this region of the odd-Z odd-A elements.

IRIDIUM

Iridium is similar to rhenium, having two stable isotopes Ir^{191} (38.5%) and Ir^{193} (61.5%). The electron spectrum from bombardment of natural Ir with 3.85-Mev alpha particles is shown in Fig. 3. One sees the K, L, and M lines from the first rotational level in each isotope. The L line from Ir^{193} was not resolvable from the M line of Ir^{191} so that no K/L ratio could be obtained for this nucleus. In addition to these lines, one finds two conversion lines labelled A and B in Fig. 3. If one assumes that these are L conversion lines, the transition energies are 73 and 83 kev. From β-decay[14] there is a known level in Ir^{191} at 82 kev. Also, in Ir^{193} a transition of 73 kev has been observed.[15] However, the suggested level scheme from β-decay for Ir^{193} has the 73-kev transition as a cascade between two higher energy levels. The yield ratio of both these lines from 3.50 to 3.85 Mev is consistent with that to be expected for excitation of levels at 73 and 83 kev. Also, since one would expect these two nuclei to have similar level schemes, it appears almost certain that the 73-kev transition comes from a level of that energy rather than a cascade between higher levels.

The K/L ratio for the first rotational state (129 kev) in Ir^{191} is somewhat higher than the previous β-decay measured value[13] of 2.1. The $B_{ex}(E2)$ value could be obtained only for the lighter isotope because of the fact that the K and L lines from Ir^{193} were not completely resolved. The target thickness was obtained by the "stopping electron" method; however, the Q_0 value obtained seems to fit very well with those of the neighboring nuclei.

[13] Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

[14] Gillon, Gopalakrishnan, de-Shalit, and Mihelich, Phys. Rev. **93**, 124 (1954).

[15] Cork, Leblanc, Nester, Martin, and Brice, Phys. Rev. **90**, 444 (1953).

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Interpretation of Regularities in Neutron and Proton Separation Energies

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Way's empirical rule on the behavior of separation energies of nucleons is analyzed. It is shown that this rule could be expected from general properties of the shell model, and the sort of information which can be obtained from its detailed analysis is discussed.

In a recent study, Way[1] has pointed out an outstanding empirical rule concerning the separation energies of neutrons and protons for nuclei in some regions of the periodic table. If $S_{n(p)}(Z,N)$ stands for the energy required to separate a neutron (proton) from a nucleus with Z protons and N neutrons, and if $Z+N$ is even, then Way's rule claims that:

$$S_n(Z,N)\cong S_n(Z+1,N), \quad (1)$$

$$S_p(Z,N)\cong S_p(Z,N+1), \quad (2)$$

for even $A=Z+N$. Stated in other words, it says that the addition of a neutron (proton) to an odd-odd or to an even-even nucleus does not change the binding energy of the last proton (neutron) of that nucleus.

[1] K. Way, *Amsterdam Conference, July, 1956* (Nederlande Natuurkundige Vereniging, Amsterdam, 1956); *Nuclear Masses and Their Determination*, Proceedings of the Conf. held in Max Planck Institut fur Chemie, Mainz; edited by H. Hintenberger, July 1956 (Pergamon Press, London, 1956).

As was noted by Way, exceptions to this rule are associated with completion of shells or with transition into the region of high deformations. It thus seems that this behavior should result from the shell model of the nucleus.

By definition, the binding energy of the last nucleon, or its separation energy, is the difference between the binding energy of the original nucleus less the binding energy of that nucleus after removal of the last nucleon. To calculate this quantity, let us assume that the nucleus can be represented by the shell model; let us further assume that there are p protons in the state characterized by the quantum numbers n_p, l_p, and j_p [in short "the state j_p"], with all lower states filled, and let there also be n neutrons in the state j_n with all lower states filled.

Since we are dealing with differences of binding energies caused by the addition of one nucleon to the

partially filled shells, we can disregard in our calculation the binding energy of the nucleons in the closed shells and treat those in the partially filled shells only. The binding energy of the nucleons in these shells can be written in the following manner:

$$E(p,n)=pE(j_p)+nE(j_n)+E_{\rm int}(j_p{}^p)+E_{\rm int}(j_n{}^n)+[p,n], \quad (3)$$

where p and n are the numbers of protons and neutrons in the partially filled shells j_p and j_n, respectively; $E(j)$ is the binding energy of a single nucleon in the state j; $E_{\rm int}(j^k)$ is the interaction energy of k identical nucleons in the state j with each other; and $[p,n]$ is the interaction energy of p protons in the state j_p with n neutrons in the state j_n. The interaction energies $E_{\rm int}(j^k)$ and $[p,n]$ are generally unambiguous only if we specify the coupling scheme of the nucleons in question. However, to avoid complications we shall not specify this scheme unless it becomes necessary.

The neutron separation energy for a nucleus (p,n) thus becomes

$$S_n(p,n)=E(p,n)-E(p,n-1)=E(j_n)+[E_{\rm int}(j_n{}^n)-E_{\rm int}(j_n{}^{n-1})]+[p,n]-[p,n-1]. \quad (4)$$

Similarly for a nucleus $(p+1, n)$ we get

$$S_n(p+1,n)=E(p+1,n)-E(p+1,n-1)=E(j_n)+[E_{\rm int}(j_n{}^n)-E_{\rm int}(j_n{}^{n-1})]+[p+1,n]-[p+1,n-1]. \quad (5)$$

Finally we get, for the difference δ_n between these two separation energies, the expression

$$\delta_n=S_n(p+1,n)-S_n(p,n)=\{[p+1,n]-[p+1,n-1]\}-\{[p,n]-[p,n-1]\}. \quad (6)$$

Our only restrictions in deriving Eq. (6) were $n>0$ and $p<2j_p+1$. We shall now assume that $j_p\neq j_n$, leaving the case $(n_pl_pj_p)=(n_nl_nj_n)$ to a subsequent treatment, and we shall also assume that an even number of equivalent nucleons couple in our case (we are treating ground states!) to a zero total angular momentum.

It is shown in the appendix that whenever p or n is even and the corresponding group of nucleons is coupled to a zero total angular momentum, one has

$$[p,n]=pnF(j_p,j_n), \quad p \text{ or } n \text{ or both even}. \quad (7)$$

If both p and n are odd, Eq. (7) no longer holds but one still has

$$\frac{1}{(2J_p+1)(2J_n+1)}\sum_J(2J+1)[pJ_p,nJ_n]_J=pnF(j_p,j_n), \quad (7a)$$

where the explicit expression $[pJ_p,nJ_n]_J$ stands for the interaction between the group of p-equivalent protons coupled to angular momentum J_p with the group of n equivalent neutrons coupled similarly to J_n in a state with a total angular momentum J.

TABLE I. The difference between neutron separation energies, $\delta_n\equiv\{[p+1,n]-[p+1,n-1]\}-\{[p,n]-[p,n-1]\}$.

p	n	δ_n
even	even	$\delta_n=F(j_p,j_n)-\epsilon$
even	odd	$\delta_n=F(j_p,j_n)+\epsilon$
odd	even	$\delta_n=F(j_p,j_n)+\epsilon$
odd	odd	$\delta_n=F(j_p,j_n)-\epsilon$

Since the left-hand side of (Fa) represents an average of the interactions of the two odd groups over all possible states, it is clear that the ground state is more tightly bound than this average and we can therefore put

$$[p,n]=pnF(j_p,j_n)+\epsilon, \quad p \text{ and } n \text{ odd}, \quad (8)$$

where ϵ has the same sign as F, may depend on n, p, j_n, and j_p, and is of the order of magnitude of half the width of a configuration of an odd-odd nucleus.

It is now possible to evaluate δ_n by substituting into Eqs. (6), (7), or (8) as required. One gets the results shown in Table I. One sees immediately from this table that if $\epsilon\cong F(j_p,j_n)$, then δ_n vanishes for even-even and odd-odd nuclei, and has the value $\sim 2F(j_p,j_n)$ for even-odd and odd-even nuclei. Stated in other words it says, since δ_n is defined as the difference between two separation energies, that the neutron separation energy does not change when a proton is added to an even-A nucleus, and it changes by the amount $2F(j_p,j_n)$ when a proton is added to an odd-A nucleus. Obviously, the same rule holds for the proton separation energy.

Thus, provided $\epsilon\cong F(j_p,j_n)$, we have demonstrated not only Way's rule but we also anticipate that the differences in neutron separation energies of pairs of nuclei (Z,N) and $(Z+2,N)$ should be constant. In other words, the neutron separation energies for the nuclei (Z,N), $(Z+2,N)$, $(Z+4,N)$, etc., should lie on a straight line with a slope $F(j_p,j_n)$ Mev/proton, with a similar rule holding for the neutrons. This behavior seems to be reproduced by the experimental data.[1]

The relations between ϵ and $F(j_p,j_n)$, as well as the dependence of ϵ on p and n, are very much dependent on the nature of the proton-neutron interaction inside nuclear matter. Thus, if the interaction between neutrons and protons depends only on their distance apart it is easy to shown that $\epsilon=0$ whenever n or p represent exactly a half-filled shell. In another extreme of an interaction of the type $(\boldsymbol{\sigma}_p\cdot\boldsymbol{\sigma}_n)V(|\mathbf{r}_p-\mathbf{r}_n|)$ ϵ does not depend on n or p at all. The analysis of the experimental data can thus yield important information on the nature of the proton-neutron interaction inside nuclei.

The peculiar behavior of the separation energies has drawn previous attention,[2] and attempts were made to

[2] A. R. Edmonds, Proc. Phys. Soc. (London) **A66**, 793 (1953) and references quoted there; S. N. Goshal and A. N. Saxena, Proc. Phys. Soc. (London) **A69**, 293 (1956).

explain this behavior on the basis of the mass formula with some shell model corrections. However, since it is not quite clear how much of the "shell model" contributions go into a semiempirical mass formula it seems that a direct shell model approach is preferable, although, to be sure, it is valid only so long as the same shells are being filled.

More detailed analyses of the experimental data will be given in a subsequent treatment of this subject.

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APPENDIX

We want to calculate the interaction energy between a group of p protons in the state j_p coupled to a zero total angular momentum with a group of n neutrons in the state j_n.[3] To do this we use the fact that it is always possible to decompose an interaction between two particles V_{pn} in the following manner:

$$V_{pn}=\sum_{k\chi}u_k(r_p,r_n)T_{k\chi}^*(p)T_{k\chi}(n), \tag{A1}$$

where $T_{k\chi}(p)$ is the χth component of an irreducible tensor operator of rank k operating on the proton coordinates only, $T_{k\chi}(n)$ has a similar meaning for the neutrons, and u_k is a function of the magnitudes only of $\mathbf{r}_p$ and $\mathbf{r}_n$. We now have

$$\langle j_p{}^pJ_pj_n{}^nJ_nJM|\sum_{i,j}V_{p_in_j}|j_p{}^pJ_pj_n{}^nJ_nJM\rangle = \sum f_kF_k, \tag{A2}$$

where

$$F_k=\int R_{n_pl_p}{}^2(r_p)R_{n_nl_n}{}^2(r_n)f_k(r_pr_n)r_p{}^2dr_p r_n{}^2dr_n \tag{A3}$$

and

$$f_k=\sum_x\langle j_p{}^pJ_pj_n{}^nJ_nJM|[\sum_iT_{kx}^*(p_i)] \times[\sum_jT_{kx}(n_j)]|j_p{}^pJ_pj_n{}^nJ_nJM\rangle. \tag{A4}$$

In these expressions, $\langle j_p{}^pJ_pj_n{}^nJ_nJM|$ stands for the wave function of p protons in the state j_p coupled to J_p and n neutrons in the state j_n coupled to J_n, both groups coupled to J; R_{nl} is the radial part of the single-nucleon wave function; and the sums over i and j are taken over all p protons and n neutrons, respectively.

The expression (A4) can be evaluated by the standard Racah algebra.[4] One obtains

$$f_k=(-1)^{J_p+J_n-J}(j_p{}^pJ_p\|\sum_iT_k(p_i)\|j_p{}^pJ_p) \times(j_n{}^nJ_n\|\sum_jT_k(n_j)\|j_n{}^nJ_n)W(J_pJ_nJ_pJ_n;Jk). \tag{A5}$$

Since we have assumed that $J_p=0$ we find, applying the triangular condition to the first term on the right of (A5), that f_k vanishes unless $k=0$. For $k=0$, one then gets

$$f_0=\frac{1}{(2J+1)^{\frac{1}{2}}}(j_p{}^p0\|\sum_iT_0(p_i)\|j_p{}^p0) \times(j_n{}^nJ\|\sum_jT_0(n_j)\|j_n{}^nJ)=\frac{1}{(2J+1)^{\frac{1}{2}}}pn \times(j_p{}^p0\|T_0(p_1)\|j_p{}^p0)(j_n{}^nJ\|T_0(n_1)\|j_n{}^nJ) =pn\frac{(j_p\|T_0\|j_p)(j_n\|T_0\|j_n)}{[(2j_p+1)(2j_n+1)]^{\frac{1}{2}}}. \tag{A6}$$

Combining these results one gets, finally

$$\langle j_p{}^pJ_pj_n{}^nJ_n;JM|\sum_{i,j}V_{p_in_j}|j_p{}^pJ_pj_n{}^nJ_n;JM\rangle =pnF(j_pj_n), \tag{A7}$$

where

$$J_p \quad \text{and/or} \quad J_n=0,$$

$$F(j_pj_n)=F_0\frac{(j_p\|T_0\|j_p)(j_n\|T_0\|j_n)}{[(2j_p+1)(2j_n+1)]^{\frac{1}{2}}}, \tag{A8}$$

and F_0 is given by (A3).

If $J_p\neq0$ and $J_n\neq0$, one can use the theorem

$$\sum_J(-1)^{J_p+J_n-J}(2J+1)W(J_pJ_nJ_pJ_n;Jk) =[(2J_p+1)(2J_n+1)]^{\frac{1}{2}}\delta_{k,0}$$

in order to prove that

$$\frac{1}{(2J_p+1)(2J_n+1)}\sum(2J+1) \times\langle j_p{}^pJ|_pj_n{}^nJ_nJM\sum_{i,j}V_{p_in_j}|j_p{}^pJ_pj_n{}^nJ_nJM\rangle=npF, \tag{A9}$$

where F is again given by (A8).

[3] See also N. Zeldes, Nuclear Phys. **2**, 1 (1956/1957).

[4] G. Racah, Phys. Rev. **62**, 438 (1942).