be 5 or 6 and that the parity is odd.[3] This argument implies that the spins of levels E and G should be greater than 2.[3] A comparable argument suggests that the spin of level d is greater than 2.[3] If the assignment of spin 3 to level E is correct, the spin of Ir^{192} must be 5 rather than 6.

Odd parity and a high spin for Ir^{192} can be reconciled with the shell model by assignment of the odd proton to an $h_{11/2}$ orbital and the odd neutron to an $i_{13/2}$ orbital.[13] Nearby odd-A nuclei do not indicate these assignments; however, a similar configuration for Lu^{176} has been postulated.[14]

ACKNOWLEDGMENTS

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Internal Conversion Electrons Following Coulomb Excitation of Highly Deformed Nuclei*

E. M. BERNSTEIN† AND H. W. LEWIS
Department of Physics, Duke University, Durham, North Carolina

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A study of internal conversion electrons following Coulomb excitation by alpha particles of a number of nuclei in the rare earth region has been made. In four cases, Eu^{153}, Ho^{165}, Lu^{175}, and Ta^{181}, it was possible to obtain sufficient experimental data to make a fairly complete comparison with the predictions of the simple rotational model of Bohr and Mottelson. Within the large experimental uncertainties the data were consistent with the theory.

First rotational state transitions have been studied in Gd^{155}, Gd^{157}, Re^{185}, Re^{187}, Ir^{191}, and Ir^{193}. In addition transitions of 73 and 83 kev were observed in Ir^{191} and Ir^{193}, respectively. The data are consistent with the assignment of the transitions as coming from levels of these energies.

INTRODUCTION

TRANSITIONS following the Coulomb excitation of heavy nuclei can be studied by observation of the internal-conversion electrons[1-3] in addition to the gamma rays. By using techniques described previously,[2] measurements have been made of the conversion electrons from the following elements under alpha-particle bombardment: Eu^{153}, Gd^{155}, Gd^{157}, Ho^{165}, Lu^{175}, Ta^{181}, Re^{185}, Re^{187}, Ir^{191}, Ir^{193}.

When one can excite the first two rotational states of a nucleus, there are several possibilities for testing the theoretical formalism of Bohr and Mottelson.[4] If it is found that the ratio of the energy of the second rotational state to the energy of the first rotational state is given correctly by the theory, it is expected that the simple rotational model is applicable to the nucleus. Then one can compare the Q_0 values obtained from the cross section for excitation of the two levels, and also the $E2$ to $M1$ mixing ratios for the first rotational state transition and the cascade transition. The latter information can be obtained from K/L ratios and in the case of the cascade transition from the branching ratio of the second rotational state.

In the case of Eu^{153}, Ho^{165}, Lu^{175}, and Ta^{181} we were able to obtain sufficient experimental data to make the above-mentioned comparisons with the theory. Preliminary results for these nuclei have been presented verbally[5] and showed large discrepancies with the theory. However, the results were partially based on previous measurements[6] of gamma-ray transition probabilities which have been revised recently.[7] The new values have been incorporated into the analysis presented here and within the experimental uncertainties the data are now in good agreement with the theoretical predictions of the simple rotational model.

EXPERIMENTAL PROCEDURE

A. Targets and Background Radiation

All the targets were made by vacuum evaporation onto thick aluminum or copper backings. It was found difficult in most cases to make homogeneous targets as

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† E. I. du Pont de Nemours Fellow (1955–1956).

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[5] E. M. Bernstein and H. W. Lewis, Bull. Am. Phys. Soc. Ser. II, 1, 41 (1956).

[6] N. P. Heydenburg and G. M. Temmer, Phys. Rev. 100, 150 (1955).

[7] N. P. Heydenburg and G. M. Temmer (private communication).

thick as desired. Elements of natural isotopic abundance in the form of powdered oxides were used, except for Ta, Re, and Ir which were available in metallic form. The powders were heated in a carbon boat to about 3000°C. The resulting deposits were uniform and metallic in appearance even in the case of oxides. Whether or not the targets were wholly or partly oxides is not of importance, even for target thickness measurements (see below). Usually, when the deposit was more than ~ 0.015 mg/cm^2 thick, it flaked off the backing, rendering the target useless. In general, targets of thickness ~ 0.05 to ~ 0.2 mg/cm^2 would have been preferable from yield considerations and would still have been thin enough to produce negligible absorption or scattering of the electrons. The targets were inspected after bombardment and showed no signs of deterioration.

One can shield as much as desired against external background radiation such as gamma and x-rays from the target and accelerator. However, this is not the case for the background due to "stopping electrons" which are ejected from the atomic shells of the target material in the slowing-down process of the bombarding particles. Huus *et al.*[3] have calculated an approximate expression for the cross section for this process which is proportional to the fourth power of the atomic number of the target nucleus. These same authors have obtained good agreement between the theoretical expression and the experimentally observed cross section.

Using the Z^4 dependence of the cross section, one can obtain target thicknesses by comparing the "stopping electron" yield from the target with that from a foil of known thickness.[3] Owing to the Z^4 dependence of the cross section, any contamination of the target material with light elements can be neglected. Also, for the same reason it was hoped that the electron yield from the target backing material would be insignificant; however, since the targets were extremely thin this was not true. Although the "stopping electron" cross section is about a factor of 64 larger for protons than for alpha particles of the same energy, the relative yield of electrons from the target material compared to the backings was larger for alpha particles. The target thicknesses determined in this manner are believed accurate within 50%.

By considering the cross section for producing "stopping electrons" along with the cross section for Coulomb excitation, one finds an optimum bombarding condition (i.e., a kind of particle and an energy) for exciting a given level which gives a maximum "signal-to-noise" ratio. In the present experiment, it was found that alpha particles were preferred over protons for all the levels studied.

B. *K/L* Ratios and Reduced Transition Probabilities

K/L ratios were obtained from the electron spectra by measuring the areas under the *K* and *L* conversion lines and dividing each area by the $H\rho$ value at the position of the line. In order to determine mixing ratios from the experimental *K/L* ratios, it is necessary to know the theoretical values of the absolute *K* conversion coefficients and theoretical *K/L* ratios for $M1$ and $E2$ transitions. Extensive calculations of *K* and *L* conversion coefficients have been made by Rose.[8] However, there is some dispute over the validity of these *K* and *L* conversion coefficients for $M1$ transitions. Calculations of Sliv and Listengarten,[9] who have taken into account the finite nuclear size, indicate a fairly constant correction factor over the energy interval 250 kev to 1.5 Mev. The transitions investigated in this experiment are all between 70 and 170 kev. In the absence of more information, it was assumed that the correction factors for high energies could be extrapolated to give the correction factors for the energy region investigated. This is consistent with recent measurements of internal conversion coefficients.[10,11]

The angular distribution of the conversion electrons has been neglected in the interpretation of the data. The error due to this procedure is expected to be much smaller than the other uncertainties involved.

Values of the reduced $E2$ transition probabilities, $B_{\mathrm{ex}}(E2)$, were found in one of several ways. In the cases of Eu^{153}, Ho^{165}, Lu^{175}, and Ta^{181}, values of $B_{\mathrm{ex}}(E2)$ for the first rotational state were obtained by applying the correction for internal conversion to the gamma-ray measurements of Heydenburg and Temmer.[6] The total conversion coefficients were obtained from the "corrected" theoretical values of Rose, using the $E2$ to $M1$ mixing ratios obtained from the measured *K/L* ratios. For the second rotational levels of these elements, the values of $B_{\mathrm{ex}}(E2)$ were obtained in the following manner. The partial values of $B_{\mathrm{ex}}(E2)$ for excitation of the cascade were obtained from the conversion electron yield relative to that for the first rotational level. This was then added to the partial $B_{\mathrm{ex}}(E2)$ for the crossover transition measured by Heydenburg and Temmer.[6] For Re and Ir the relative target thicknesses determined by the "stopping electron" method were used. The values of $B_{\mathrm{ex}}(E2)$ were then obtained by comparison with the 114-kev transition in Lu^{175}.

THEORETICAL PREDICTIONS OF THE COLLECTIVE MODEL

Bohr and Mottelson[4] have shown that nuclei possessing large intrinsic deformations are expected to display well-developed rotational spectra, since to a good approximation one can separate the intrinsic and collective motions. The results of the theoretical calcula-

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[8] M. E. Rose, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North Holland Publishing Company, Amsterdam, 1955), Chap. 14; also M. E. Rose (private communication).

[9] T. A. Sliv and M. A. Listengarten, Zhur. Eksptl. i Theort. Fiz. 22, 29 (1952).

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TABLE I. $E2$ to $M1$ mixing ratios from K/L ratios.

Element	I_0	Level energy (kev)	I	Transition energy (kev)	K/L	δ^2 from K/L	α_t[a]
Eu^{153}	$5/2+$	84	$7/2+$	84	1.7	0.48	3.6
		195	$9/2+$	111	5.6	0.08	1.2
Ho^{165}	$7/2-$	96	$9/2-$	96	5.4	0.044	2.6
		218	$11/2-$	122	6.9	0	1.3
Lu^{175}	$7/2+$	114	$9/2+$	114	4.3	0.11	2.3
		253	$11/2+$	139	5.5	0.08	1.4
Ta^{181}	$7/2+$	137	$9/2+$	137	6.3	0.05	1.7
		302	$11/2+$	165	7.1	0	0.96
Re^{185}	$5/2+$	125	$7/2+$	125	2.2	0.62	2.2
Re^{187}	$5/2+$	135	$7/2+$	135	4.6	0.13	2.3
Ir^{191}	$3/2+$	129	$5/2+$	129	3.9	0.23	2.4
Ir^{193}	$3/2+$	140	$5/2+$	140	3.7[b]	0.29	2.0

[a] Total conversion coefficient, calculated by using δ^2 from K/L ratio.
[b] Taken from β-decay (reference 13).

tions of the above authors (see also reference 3) which are applicable to the experiment described here are presented below.

The ratio of the energies of the rotational levels depends only on the ground state angular momentum, I_0. For the first and second rotational levels this ratio is

$$\frac{E_2}{E_1}=\frac{2I_0+3}{I_0+1}=\begin{cases}2.40 & \text{for } I_0=3/2\\ 2.29 & \text{for } I_0=5/2\\ 2.22 & \text{for } I_0=7/2.\end{cases} \tag{1}$$

In addition to level spacings, the collective model also predicts the values of transition probabilities. The relations for the reduced electric quadrupole transition probabilities for excitation are

$$\begin{aligned} B_{\mathrm{ex}}(E2)&=\frac{15}{16\pi}e^2Q_0^2\frac{I_0}{(I_0+1)(I_0+2)} \quad \text{for } I_0\rightarrow I_0+1,\\ B_{\mathrm{ex}}(E2)&=\frac{15}{8\pi}e^2Q_0^2\frac{I_0}{(2I_0+3)(I_0+2)} \quad \text{for } I_0\rightarrow I_0+2.\end{aligned} \tag{2}$$

In odd-A nuclei the decay transitions are a mixture of magnetic dipole and electric quadrupole radiation. The mixing ratio, δ^2, is defined as the ratio of the number of $E2$ to $M1$ *gamma rays*. From the theory one finds the ratio of δ^2 for the transition from the first rotational state to the ground state, denoted by subscript 1, to δ^2 for the cascade from the second to the first rotational state, denoted by the subscript 21, is given by

$$\frac{\delta_1^2}{\delta_{21}^2}=\begin{cases}1.10 & \text{for } I_0=3/2\\ 1.04 & \text{for } I_0=5/2\\ 1.02 & \text{for } I_0=7/2.\end{cases} \tag{3}$$

With the relations given here, one can make the comparisons between the theory and experiment which are stated in the introduction.

EXPERIMENTAL RESULTS

A summary of the experimental results is given in Tables I and II. The results for each element are discussed in detail below. Comparison of the data in those cases where some previous measurements are available shows good agreement in general.[‡] Discrepancies are pointed out in the text. Absolute reduced transition probabilities are believed accurate to 50%. The relative $B_{\mathrm{ex}}(E2)$ values for the first two rotational levels in the same element are believed to be accurate to better than 20%. K/L ratios for first excited state transitions are believed to be accurate to 10%, while those for the cascade transitions are less certain, the estimated error being as high as 15%.

Eu^{153}, Ho^{165}, Lu^{175}, and Ta^{181}.

For these four nuclei it was possible to observe the conversion electrons from the cascade between the second and first rotational states as well as the first rotational state transition. In all cases the ratio of the energy of the second rotational state to the energy of the first rotational state is in agreement[6] with the predictions of the simple rotational model. As can be seen in Table II, the Q_0 values calculated from the cross

TABLE II. Reduced transition probabilities, intrinsic quadrupole moments, and $E2$ to $M1$ mixing ratios.

Element	Transition energy (kev)	$B_{\mathrm{ex}}(E2)$[a] transition	Level energy (kev)	$B_{\mathrm{ex}}(E2)$[a] (level)	Q_0 (barns)	$(\delta_{21}^2)_B$[b]	$(\delta_{21}^2)_{K/L}$	$(\delta_1^2)_{K/L}$
Eu^{153}	84	2.77	84	2.77	7.6			0.48
	111	0.54	195				0.08	
	195	0.36	195	0.90	7.3	0.28		
Ho^{165}	96	2.79	96	2.79	8.1			0.044
	122	0.68	218				0	
	218	0.028	218	0.71	8.2	0.023		
Lu^{175}	114	2.86	114	2.86	8.2			0.11
	139	0.63	253				0.08	
	253	0.12	253	0.75	8.3	0.11		
Ta^{181}	137	1.9	137	1.9	6.7			0.05
	165	0.44	302				0	
	302	0.12	302	0.56	7.0	0.075		
Re^{185}			125	1.75	6.0			0.62
Re^{187}			135	1.40	5.2			0.13
Ir^{191}			129	0.97	4.0			0.23

[a] All $B_{\mathrm{ex}}(E2)$ values are given in units of 10^{-48} cm^4 e^2. The values for cascade transitions are relative to first excited state transitions given by Heydenburg and Temmer.[6] The total conversion coefficients used are given in Table I. The $B_{\mathrm{ex}}(E2)$ value for a level is given by the sum of the partial $B_{\mathrm{ex}}(E2)$ values for the various modes of decay.
[b] Mixing ratio calculated from the branching ratio of the second excited state.

[‡] *Note added in proof.*—The values for the $E2$ to $M1$ mixing ratios should be compared to those presented in a number of recent papers: G. Goldring and G. T. Paulissen, Phys. Rev. **103**, 1314 (1956); Davis, Divatia, Lind, and Moffat, Phys. Rev. **103**, 1801 (1956); N. P. Heydenburg and G. M. Temmer, Phys. Rev. **104**, 981 (1956); Wolicki, Fagg, and Geer, Phys. Rev. **105**, 238 (1957). Our values for the first four elements of Table II are in reasonable agreement with those of other workers; for the last three elements our values are considerably larger than those of the other workers. It should be noted that the mixing ratios given in the references above were obtained from the branching ratios for the gamma rays from the second excited state, assuming the simple rotational model to be correct.

section for excitation of the first and second rotational levels are in good agreement in all four cases. The $E2$ to $M1$ mixing ratios which have been determined in three ways are also given in Table II. The values of δ^2 obtained from the K/L ratio for the cascade transition are somewhat lower than the other two determinations. However, as pointed out above, these K/L ratios have a rather large experimental uncertainty. We conclude, therefore, that within the experimental error the results for all these elements are consistent with the theoretical predictions.

In addition to the transitions from the first two rotational levels of Lu^{175}, Heydenburg and Temmer[6] also observed a weak gamma ray of 180 kev when naturally abundant lutetium is bombarded with alpha particles. In the electron spectra, we observed a weak conversion line at 123 kev. Assuming this to be a K line, it represents a transition of 186 kev which is almost certainly the same as the 180-kev transition seen in the gamma measurements. A search for the L conversion line from this transition was unsuccessful, indicating an $M1$ or mostly $M1$ transition. The assignment of this transition is not certain, but it is possibly due to the odd-odd isotope Lu^{176}.

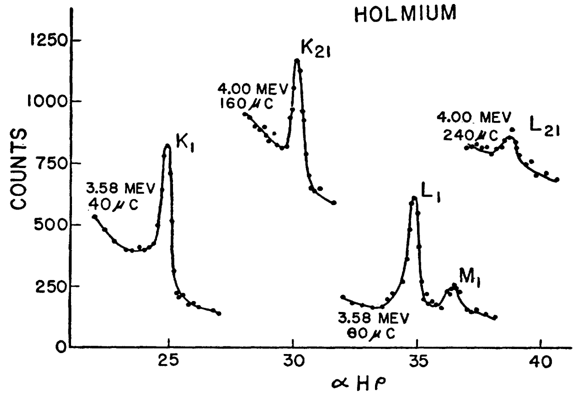

FIG. 1. Conversion lines following Coulomb excitation of holmium. The alpha-particle bombarding energy and integrated charge are indicated. Subscripts 1 and 21 denote first rotational state transition and cascade transition from the second state, respectively.

A typical conversion spectrum is shown in Fig. 1. This is for holmium which is isotopically pure with a mass of 165. One sees the K, L, and M conversion lines from the first rotational level. These are labeled with a subscript 1. Also shown are the K and L conversion lines from the cascade transition between the first and second levels denoted by the subscript 21.

GADOLINIUM

Natural Gd consists of several even-even isotopes plus two even-odd isotopes, Gd^{155}, and Gd^{157}. Figure 2 shows the electron spectrum from a natural Gd target bombarded with 3.25-Mev alpha particles. Most of the lines are from the even-even isotopes. The two lines marked L^{157} and L^{155} have been assigned as the L conversion lines from the first rotational level of these isotopes. The existence of these levels was suggested by Heydenburg and Temmer[6] on the basis of gamma-ray measurements of the second rotational states. The gamma rays from the lower levels could not be observed by them owing to the large background from the strong K x-rays. The energies of the levels are 61 kev for Gd^{155} and 56 kev for Gd^{157}. Recently, Bjerregaard and Meyer-Berkhout[12] have confirmed these assignments using separated isotopes. These authors have made an extensive study of the conversion electrons from both isotopes.

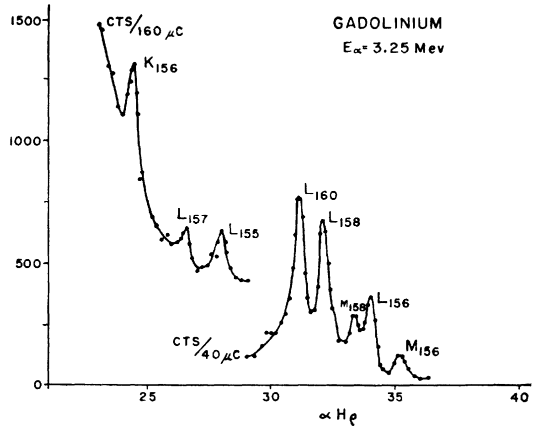

FIG. 2. Spectrum of conversion electrons from natural gadolinium. Lines from the various isotopes are distinguished by subscripts.

RHENIUM

Rhenium has two stable isotopes Re^{185} (37.1%) and Re^{187} (62.9%). K, L, and M conversion lines from the first rotational level of each isotope were measured. It was not possible to observe the cascade transition from

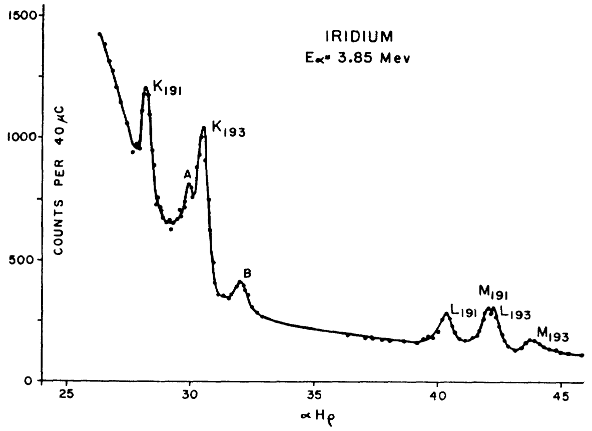

FIG. 3. Electron spectrum from natural iridium. Except for lines A and B, which are explained in the text, the lines originate from the first rotational state transitions in the two isotopes.

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[12] J. H. Bjerregaard and U. Meyer-Berkhout, Z. Naturforsch. **11a**, 273 (1956).

the second rotational level. No previous measurements have been made of the K/L ratio for the level in Re^{185}; however, in β-decay the level in Re^{187} has been observed. The value of $K/L=4.6$ for the heavier isotope agrees with the previously measured value[13] of $K/L\sim 5$. The values of $B_{ex}(E2)$ for these two nuclei have an uncertainty of 50% since the target thickness was determined by the "stopping electron" method. However, the relative Q_0 values obtained have approximately the same ratio as the spectroscopically measured quadrupole moments. Also, they fit into the general trend of Q_0 values in this region of the odd-Z odd-A elements.

IRIDIUM

Iridium is similar to rhenium, having two stable isotopes Ir^{191} (38.5%) and Ir^{193} (61.5%). The electron spectrum from bombardment of natural Ir with 3.85-Mev alpha particles is shown in Fig. 3. One sees the K, L, and M lines from the first rotational level in each isotope. The L line from Ir^{193} was not resolvable from the M line of Ir^{191} so that no K/L ratio could be obtained for this nucleus. In addition to these lines, one finds two conversion lines labelled A and B in Fig. 3. If one assumes that these are L conversion lines, the transition energies are 73 and 83 kev. From β-decay[14] there is a known level in Ir^{191} at 82 kev. Also, in Ir^{193} a transition of 73 kev has been observed.[15] However, the suggested level scheme from β-decay for Ir^{193} has the 73-kev transition as a cascade between two higher energy levels. The yield ratio of both these lines from 3.50 to 3.85 Mev is consistent with that to be expected for excitation of levels at 73 and 83 kev. Also, since one would expect these two nuclei to have similar level schemes, it appears almost certain that the 73-kev transition comes from a level of that energy rather than a cascade between higher levels.

The K/L ratio for the first rotational state (129 kev) in Ir^{191} is somewhat higher than the previous β-decay measured value[13] of 2.1. The $B_{ex}(E2)$ value could be obtained only for the lighter isotope because of the fact that the K and L lines from Ir^{193} were not completely resolved. The target thickness was obtained by the "stopping electron" method; however, the Q_0 value obtained seems to fit very well with those of the neighboring nuclei.

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[13] Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

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Interpretation of Regularities in Neutron and Proton Separation Energies

A. DE-SHALIT
Department of Physics, The Weizmann Institute of Science, Rehovoth, Israel

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Way's empirical rule on the behavior of separation energies of nucleons is analyzed. It is shown that this rule could be expected from general properties of the shell model, and the sort of information which can be obtained from its detailed analysis is discussed.

IN a recent study, Way[1] has pointed out an outstanding empirical rule concerning the separation energies of neutrons and protons for nuclei in some regions of the periodic table. If $S_{n(p)}(Z,N)$ stands for the energy required to separate a neutron (proton) from a nucleus with Z protons and N neutrons, and if $Z+N$ is even, then Way's rule claims that:

$$S_n(Z,N)\cong S_n(Z+1,N), \tag{1}$$

$$S_p(Z,N)\cong S_p(Z,N+1), \tag{2}$$

for even $A=Z+N$. Stated in other words, it says that the addition of a neutron (proton) to an odd-odd or to an even-even nucleus does not change the binding energy of the last proton (neutron) of that nucleus.

As was noted by Way, exceptions to this rule are associated with completion of shells or with transition into the region of high deformations. It thus seems that this behavior should result from the shell model of the nucleus.

By definition, the binding energy of the last nucleon, or its separation energy, is the difference between the binding energy of the original nucleus less the binding energy of that nucleus after removal of the last nucleon. To calculate this quantity, let us assume that the nucleus can be represented by the shell model; let us further assume that there are p protons in the state characterized by the quantum numbers n_p, l_p, and j_p [in short "the state j_p"], with all lower states filled, and let there also be n neutrons in the state j_n with all lower states filled.

Since we are dealing with differences of binding energies caused by the addition of one nucleon to the

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[1] K. Way, *Amsterdam Conference, July, 1956* (Nederlande Natuurkundige Vereniging, Amsterdam, 1956); *Nuclear Masses and Their Determination*, Proceedings of the Conf. held in Max Planck Institut fur Chemie, Mainz; edited by H. Hintenberger, July 1956 (Pergamon Press, London, 1956).