Superconductivity of Microscopic Tin Filaments*†

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An experimental determination has been made of the magnetic fields necessary to induce the superconducting transition in microscopic tin filaments called "whiskers." For temperatures near the zero-field transition temperature, T_c, the results are unambiguous, and in this region the critical fields are significantly higher than those of a bulk superconductor. At lower temperatures the critical field curve splits into two parts, the upper curve giving the field for destruction of superconductivity and the lower curve the field for restoration. The temperature dependence of the critical field is compared with the predictions of the London and Ginsburg-Landau theories of superconductivity. It is found that the London theory is inadequate to describe the data over the whole useful range, whereas the Ginsburg-Landau theory provides a satisfactory fit. The disappearance of hysteresis occurs at a temperature for which $2.0 \leq h_c/H_c \leq 2.4$, where h_c/H_c is the ratio of whisker to bulk critical fields. This is in reasonable agreement with the Ginsburg-Landau-Silin condition for the onset of second order transitions. An effective value of λ_0, the penetration depth at 0°K, is derived from the data for each whisker. λ_0 shows a strong dependence on the normal electrical conductivity, as estimated from the change in resistance at the transition. At long mean free path the results are in agreement with those obtained from bulk specimen measurements, but at short mean free path the present λ_0 values are higher.

I. INTRODUCTION

THE study of small superconducting specimens became of interest with the appearance of the London theory.[1] According to this theory the properties of a superconductor are characterized by the quantity λ, representing the depth to which a magnetic field penetrates at the surface of a bulk specimen. Size effects should be observable if the characteristic dimension, a, of the superconductor is comparable with λ. The first demonstration of penetration effects was that of Pontius,[2] who found enhanced critical fields in thin wires of lead. More extensive measurements, notably those of Appleyard *et al.* on thin mercury films[3] and of Shoenberg on mercury colloids,[4] indicated a strong temperature dependence of λ. The form of this dependence was suggested by the colloid results.[5] It was later confirmed for both mercury and tin by precision magnetic measurements on large cylinders.[6] The relation was as follows:

$$\lambda = \lambda_0[1-(T/T_c)^4]^{-\frac{1}{2}}, \qquad (1)$$

where T is the absolute temperature and T_c the transition temperature in the absence of a magnetic field. λ_0 is a constant interpreted as the penetration depth at 0°K and found to be of the order of 10^{-5} cm. Eq. (1) is consistent with the basic interpretation of λ in the London theory,[7] as applied in conjunction with the Gorter-Casimir two-fluid model.[8]

The early small-specimen studies, although confirming some important predictions of the London theory, also revealed a certain inadequacy. The results from the thinner mercury films gave critical fields which were systematically higher than predicted. This constituted the first serious disagreement with the theory, and later served as a prime motive in the formulation, by Ginsburg and Landau, of a new phenomenological theory.[9] The Ginsburg-Landau theory differs from that of the Londons by its explicit consideration of the relation between magnetic field and density of superconducting electrons. The critical field behavior was worked out for thin films in the original article and for spheres and cylinders by Silin.[10] An interesting result of the theory is that, for specimens smaller than a certain critical size, the isothermal transition induced by the application of a magnetic field should be one of second order. Under this condition the critical field is simply related to the ratio λ/a and is significantly higher than that predicted by the London theory. Similar predictions have been made by Pippard[11] in applying his "range of order" concept to the transitions of a small sphere. The connection between the two treatments has been discussed by Shoenberg.[12]

There has been a lack of critical field data applicable to an examination of the new theory. The agreement

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[1] F. London and H. London, Proc. Roy. Soc. (London) **A149**, 71 (1935).
[2] R. B. Pontius, Phil. Mag. **24**, 787 (1937).
[3] Appleyard, Bristow, London, and Misener, Proc. Roy. Soc. (London) **A172**, 540 (1939).
[4] D. Shoenberg, Proc. Roy. Soc. (London) **A175**, 49 (1940).
[5] Daunt, Miller, Pippard, and Shoenberg, Phys. Rev. **74**, 842 (1948).
[6] E. Laurmann and D. Shoenberg, Proc. Roy. Soc. (London) **A198**, 560 (1949).
[7] D. Shoenberg, *Superconductivity* (Cambridge University Press, Cambridge, 1952), p. 196.
[8] C. J. Gorter and H. B. G. Casimir, Physik. Z. **35**, 963 (1934).
[9] V. L. Ginsburg and L. D. Landau, Zhur. Eksptl. i Teort. Fiz. **20**, 1064 (1950).
[10] V. P. Silin, Zhur. Eksptl. i Teort. Fiz. **21**, 1330 (1951).
[11] A. B. Pippard, Phil. Mag. **43**, 273 (1952).
[12] D. Shoenberg, Suppl. Nuovo cimento **10**, 459 (1953).

of the early mercury film results was not entirely convincing. This has been attributed largely to uncertainties concerning the transition temperatures and the uniformity of the films.[9] The more recent data of Zavaritski[13] on tin films have shown much more satisfactory agreement and in particular have demonstrated a change from first order transitions, characterized by hysteresis effects, to second order transitions, in which hysteresis is absent.

The present article reports an investigation of the critical field behavior of microscopic tin filaments, known as "whiskers."[14] A preliminary study[15] indicated that this new class of superconducting specimens exhibited unusual properties in transverse magnetic fields. The present results were obtained with a parallel field geometry, and are examined with particular reference to the theories discussed above.

II. EXPERIMENTAL PROCEDURE

The specimens were obtained by an accelerated-growth technique devised by Fisher, Darken, and Carroll.[16] Following this method, a number of square sections of electrolytic tin plate are stacked together and clamped. An edge of the stack is polished and a pressure of several thousand psi is applied broadside. The whiskers grow from the tin on the polished edge. At full growth they are generally about 0.5 mm in length. In manipulating and mounting the whiskers it was necessary to exercise considerable caution in order to avoid breakage or deformation. A factor favorable to success was the high yield strength, reported by Herring and Galt.[17] In order to detach and mount individual filaments, a special probe assembly was constructed. This consisted, in its essential features, of a short length of bare No. 44 copper wire soldered at one end to a loop of resistance wire, which could be heated as required. The probe assembly was mounted on the platform of a laboratory-type micromanipulator, and subsequent operations were performed under a microscope. A small particle of "Apiezon" wax was melted on the end of the probe. The probe was then moved into contact with the free end of the whisker, the wax remelted and cooled, and the whisker removed from the surface. In view of the possibility of deformation which would appear to be inherent in such a technique, it should be mentioned that the whiskers almost invariably were separated at the base quite easily and very rarely fractured in the central portion.

In order to mount them as circuit elements, the whiskers were deposited on optically flat, circular, Pyrex absorption cell windows. Electrical contact was made by depositing a drop of silver paste in such a manner that it flowed just over the end of the whisker. The electrodes were subsequently enlarged by spreading additional paste on opposite sides of the Pyrex surface.

The diameters of the samples were determined by making photomicrographs at a magnification of 1000 and measuring the width of the image on the print. A wide variation in diameters was found among whiskers grown in a given operation. No detailed study of this feature was made, but it was observed that the first whiskers to appear, after clamping the plates, were extremely thin and fragile. It seems likely, therefore, that the thicker whiskers had longer induction periods. Photomicrographs of the extremities of three mounted specimens are shown in Fig. 1. In making the diameter measurements by optical means it was important, at least in the case of the smaller specimens, to consider the limitations in optical resolution of the microscope and the photographic process. In this connection it should be observed that it is possible to distinguish individual silver particles of dimensions less than one-tenth of the measured diameter of the smallest whisker. It seems reasonable to conclude that errors due to resolution are less than 10%.

An important consideration in mounting the Pyrex disk was the alignment of the whisker with the direction of the magnetic field. For this purpose the disk was inserted in a slotted brass block in such a way that it could be rotated. The whisker axis could be brought into alignment with an edge of the block by use of a cross hair in the microscope eyepiece. The disk was cemented in place by means of silver paste, the block being in contact with one of the electrodes. The block edge was aligned with the field direction by conventional means.

The whisker mounting and other features of the experimental arrangement are shown in Fig. 2. The helium Dewar rests on the bottom of a liquid air

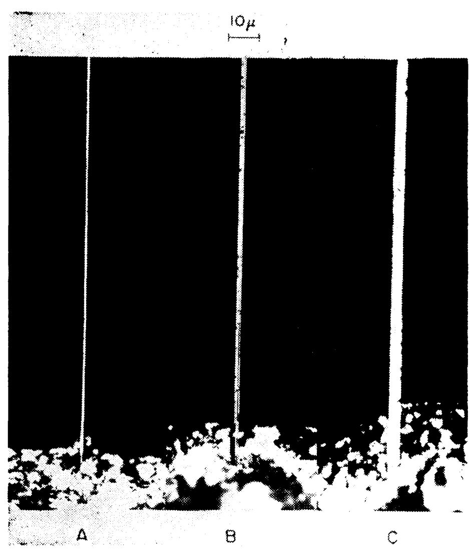

FIG. 1. Photomicrographs of tin whiskers.

[13] N. V. Zavaritski, Doklady Akad. Nauk S.S.S.R. **78**, 665 (1951).
[14] Compton, Mendizza, and Arnold, Corrosion **7**, 327 (1951).
[15] O. S. Lutes and E. Maxwell, Phys. Rev. **97**, 1718 (1955).
[16] Fisher, Darken, and Carroll, Acta Metallurgica **2**, 368 (1954).
[17] C. Herring and J. K. Galt, Phys. Rev. **85**, 1060 (1952).

Dewar (not shown). Inside the helium Dewar, and beneath the surface of the liquid helium, the brass block, A, with Pyrex disk, B, is fastened to a flat Bakelite plate, C. The latter is positioned horizontally by means of brass screws, D, and nuts, E. The screws are anchored at the top to a thicker Bakelite disk, F, which in turn is fastened to the flanged end of a straight Pyrex tube, G, by means of spring clamps (not shown). Located at the bottom, underneath the sample mount, is a carbon composition heater, H, used for "thermal stirring" of the bath. The sample and heater leads, of No. 42 enameled copper wire, are brought up from the bottom of the Dewar inside the Pyrex tube. They emerge near the top of the tube where they are soldered to heavier wires fed in through a Teflon seal fitted into the Dewar cap. The leads are mechanically anchored above the sample mount. For simplicity the details of the connection of the leads to the sample electrodes and heater are not shown.

The magnetic field was produced by a three-coil system similar to that described by Barker.[18] The field was uniform to within 0.2% over a volume included in a cylindrical region approximately 4 inches in height and 3 inches in diameter. The precise positioning of the specimen was not, therefore, a critical matter. The ratio of magnetic field to current was determined by measuring the mutual inductance between the magnet and a small auxiliary coil of accurately known area-turns.

Measurements of critical fields were made at constant temperature by observing the change in electrical resistance induced by varying the magnetic field. The resistance was measured by means of a Mueller bridge in conjunction with a high-sensitivity galvanometer. The specimen current was kept sufficiently small (of the order of a few microamperes) to insure that its magnetic field was negligible in comparison with the critical field. Temperatures were determined from measured vapor pressures by use of the 1948 scale.[19] Although recent evidence[20] indicates this scale to be in error, the deviations have not been of critical importance to the present study. In the region from 3.5° to 3.7°K, where it was important to measure small changes in temperature, the slope of the 1948 vapor pressure-temperature curve is probably correct to within 1%. The vapor pressure was regulated by means of a photoelectric device suggested by Fiske.[21] This device monitored the level in one leg of a differential oil manometer, and maintained the vapor pressure constant to within 0.2 mm Hg. In the region of principal interest, above 3.0°K, this corresponded to a temperature variation of less than 0.001°K.

The transition temperature, T_c, was determined by varying the temperature in zero field. Below the transition temperature the critical field, h_c, was determined by varying the field at constant temperature. In both cases the critical point was taken as that at which the change in resistance was half completed. In the temperature measurements primary emphasis was placed on determining the *reduced* temperature, T/T_c. For temperatures near T_c (i.e., for $T/T_c \sim 1$) this is best accomplished by measurement of the difference quantity $T_c - T$, denoted hereafter by ΔT. The reduced temperature is then given by the expression $1-[\Delta T/T_c]$, in which an uncertainty of a few percent in T_c is not serious. In practice ΔT was obtained by differential measurements in which T_c and T were determined from successive readings on an auxiliary differential oil manometer. At lower temperatures, where the uncertainty in the absolute temperature is small compared with ΔT, sufficient accuracy was obtained from absolute measurements with a mercury manometer.

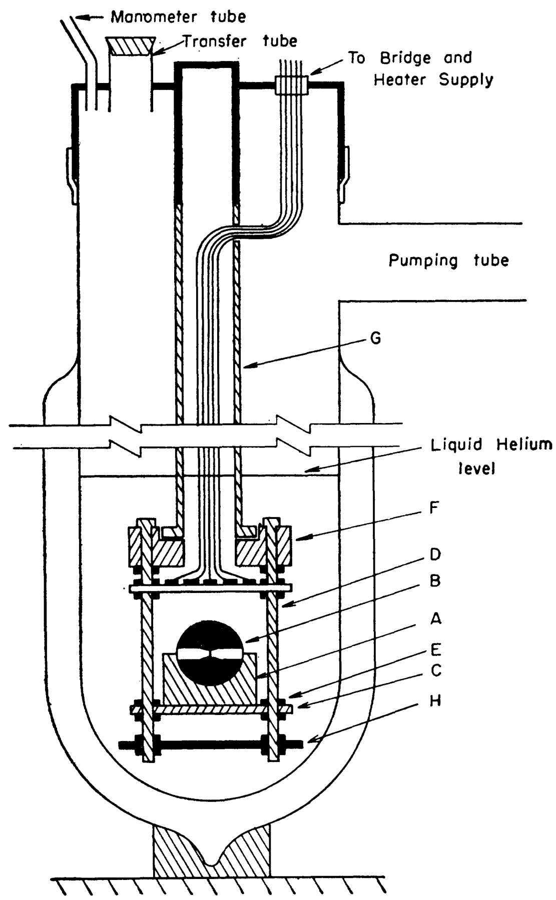

FIG. 2. Experimental apparatus.

III. RESULTS

Critical field measurements are reported for six whiskers. In order to identify the specimens, the pertinent data concerning their diameters, lengths, and resistance change are given in Table I. In this table R_N

[18] J. R. Barker, J. Sci. Instr. **27**, 197 (1950).
[19] H. Van Dijk and D. Shoenberg, Nature **164**, 151 (1949).
[20] Clement, Logan, and Gaffney, Phys. Rev. **100**, 743 (1955).
[21] M. D. Fiske (private communication).

TABLE I. Specimen properties.

Specimen	Diameter (cm $\times 10^{-4}$)	Length (cm)	R_N (ohms)
A	0.8	0.037	1.2
B	1.6	0.046	1.2
C	3.3	0.055	0.43
D	5.0	0.102	0.59
E	2.3	0.038	1.1
F	0.7	0.037	8.1

represents the total resistance change in the transition and is assumed to equal the specimen resistance in the normal state. This point is emphasized because of the presence of contact resistance at the junction of the whisker and the silver paste. This contact resistance, which was of the order of a few ohms for most of the specimens, was stable within a few hundredths of one ohm during the course of an experiment. However it could vary by as much as fifty percent from run to run. The assumption that it was a true contact resistance, and not an important aspect of the whisker's properties, is supported by the fact that the form of the transitions and the magnitude of R_N remained the same for different runs.

Representative transitions are shown in Fig. 3. Each row contains results for a particular specimen. The transition on the left in each row is the zero-field transition. The others represent critical field determinations. The resistance change is plotted in fractional form in order to permit easier comparison of the different specimens. The choice of the critical point is designated underneath by an arrow and a symbol, T_c for the zero-field determination and h_c for the critical field case. In addition the value of the bulk critical field is indicated by the symbol H_c. The enhancement of the critical field, to be discussed later, is evident from these figures.

Certain features of the transitions deserve mention. Although the transition temperatures are quite close to the accepted value, 3.730°, for bulk natural tin,[22] the zero-field transitions show an appreciable breadth. This suggests the possibility of inhomogeneous strain. In particular, the step-like character apparent in the transitions of whisker *C* may indicate that this specimen is subdivided into monocrystalline segments with different crystal orientations. Under this condition a state of uniform stress in the whisker would result in different transition temperatures for different segments, by virtue of the anisotropic nature[23] of the stress effect in superconductors. The transition breadth will be of secondary importance except where it masks the enhancement of the critical field. From examination of Fig. 3 it is seen that the transition breadth is relatively unimportant in whiskers *A* and *B*, for which the separation of h_c and H_c is large. For *C* the uncertainty is more serious. This is to be expected, since this specimen has a larger diameter and should exhibit less enhancement.

It is apparent from Fig. 3 that at sufficiently low temperatures the character of the transitions changes. At these temperatures the transitions become sharp and exhibit hysteresis, i.e., the critical fields determined for the superconducting-normal transition and those for the normal-superconducting case form two separate threshold curves. The temperature at which the onset of hysteresis occurs and the magnitude of the hysteresis vary among the different specimens. This is apparent from Fig. 4, which shows the threshold curves of *A* and *F* for temperatures down to 2°. In the case of *A* the threshold curves join near 3.6°. For *F* the joining temperature is much lower and the extent of the hysteresis much smaller, the difference between upper and lower critical fields being contained within the dimensions of the symbols. In the temperature region where hysteresis is absent, or of small extent, the threshold curve is unambiguously determined and is seen to lie well above the curve for bulk tin. The data in this region for *A*, *B*, *C*, and *D* are shown in the upper part of Fig. 4, with temperatures plotted on the reduced basis. (In the case of *E* only one high-temperature point, at $t=0.975$, was obtained. At this temperature the critical field was 25.6 oersteds, as compared to 13.5 oersteds for bulk tin.)

In order to compute the critical field enhancement it is necessary to specify the bulk threshold curve. The curve chosen is that appropriate to the transition temperature of the whisker. In this connection we make use of the geometrical similarity of the threshold curves of bulk tin specimens having transition temperatures nearly the same.[24] For temperatures lying within about 0.2° of T_c the bulk curve is closely linear with a negative slope of about 145 oersteds/degree. This value is probably not in error by more than a few percent.[25]

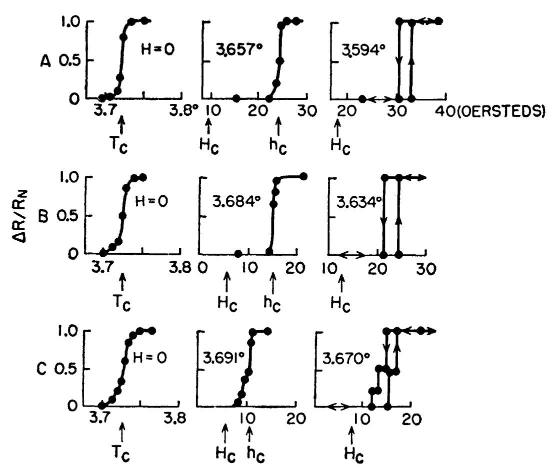

FIG. 3. Transitions of tin whiskers.

[22] D. Shoenberg, reference 7, p. 222.

[23] C. Grenier, *Proceedings of the Paris Low Temperature Conference, 1955* (Centre National de la Recherche Scientifique and UNESCO, Paris, 1956), paper 139.

[24] B. Serin, *Progress in Low Temperature Physics*, edited by C. J. Gorter (Interscience Publishers, Inc., New York, 1955), Vol. I, p. 138; P. R. Doidge, Trans. Roy. Soc. (London) **248**, 553 (1956).

[25] J. Eisenstein, Revs. Modern Phys. **26**, 277 (1954).

In specifying H_c at temperatures where the linear relation is no longer valid, use is made of data giving the deviations of the bulk threshold curve from a parabola.[26] The bulk threshold curve, obtained in the manner described, is shown as a broken line in Fig. 4.

IV. DISCUSSION

a. Theoretical Predictions

The transition of a superconductor in a magnetic field takes place, neglecting hysteresis, when the Gibbs free energy difference between normal and superconducting states becomes zero. This free energy difference is made up of a magnetic term, associated with the Meissner effect, and a configuration term, sometimes associated with the density of superconducting electrons. In applying the London equation to the critical field behavior of small specimens, it is usually assumed that the configuration energy density is independent of size and of magnetic field, and is therefore equal to the bulk value, $H_c^2/8\pi$. In this case the enhancement of the critical field in small specimens depends entirely upon the decrease in free energy density of the superconducting state due to the enhanced role of the field penetration. Hence the critical field is determined in a straightforward manner from the London equation, which predicts the field distribution. For a long thin cylinder in a parallel field the solution (outlined in the appendix) is given by

$$h_c/H_c = 1 - [(2\lambda/a)I_1(a/\lambda)/I_0(a/\lambda)]^{-\frac{1}{2}}, \qquad (2)$$

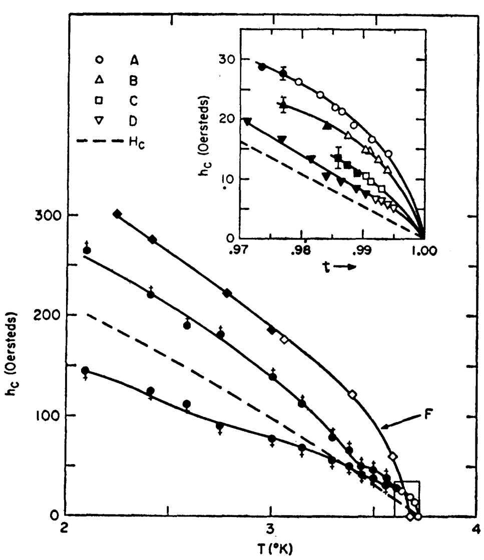

FIG. 4. Threshold curves. Hysteresis data are indicated by solid symbols. Lower part of figure: For whisker A, ♦ indicates critical field determined by increasing field, ♦ that determined by decreasing field. For whisker F the extent of hysteresis is contained within the symbol dimension. Upper part of figure shows data near T_c, with t representing the reduced temperature, T/T_c.

[26] E. Maxwell and O. S. Lutes, Phys. Rev. 95, 333 (1954).

where a is the radius and λ the bulk penetration depth. I_0 and I_1 are the hyperbolic Bessel functions of order 0 and 1, respectively.

The Ginsburg-Landau theory, as mentioned earlier, allows for an interaction between the magnetic field and the density of superconducting electrons. This leads to additional terms in the free energy. In small specimens, for which the magnetic field occupies a sizable fraction of the total volume, these terms lead to higher critical fields than predicted by Eq. (2). In the thin cylinder case[10] the approximation for small a/λ is given by the expression:

$$h_c/H_c = 4\lambda/a. \qquad (3)$$

For large a/λ it is to be expected that the predictions of the two theories should be nearly the same. This was shown to be true for films.[9] For cylinders the limiting form of Eq. (2) for large a/λ is

$$h_c/H_c = 1 + (\lambda/a). \qquad (4)$$

The predicted behavior for intermediate values of a/λ has not been derived. Consequently the range of validity of Eqs. (3) and (4) is somewhat uncertain.[27] A definite prediction is made, however, concerning the nature of the transitions of small specimens, there being a critical size below which the transitions are of second order. For cylinders the condition for second-order transitions is

$$a \leq \sqrt{3}\lambda. \qquad (5)$$

In addition to the absence of latent heat for such transitions there should also be no hysteresis.[28]

b. Critical Field Enhancement

In order to compare the critical field results with the theoretical expressions, it is appropriate to examine the temperature dependence of h_c/H_c. Comparison with Eqs. (2)–(4) is possible if we assume that λ varies with temperature according to Eq. (1).[29] In Fig. 5 the temperature dependence of h_c/H_c is shown for A and B in the temperature region for which the uncertainty due to hysteresis is small. Theoretical curves, based on Eq. (2) and Eq. (3), are also shown. It is evident that no choice of λ_0 can fit the London theory to the data, although by ignoring the data for which measurable hysteresis is present (points shown as solid symbols) the fit would be satisfactory. The latter course is unjustified, since the true critical fields for the lower temperature data should lie within the limits indicated by the symbols. On the other hand the Ginsburg-

[27] It should also be mentioned that the limiting expressions for h_c/H_c are in themselves approximations based on the assumption that a parameter κ, which is related to the field dependence of the penetration depth in a bulk specimen, is negligibly small. This assumption appears to be justified for pure superconductors.[12]

[28] V. L. Ginsburg, Doklady Akad. Nauk S.S.S.R. 83, 385 (1952).

[29] It is important to emphasize that λ always refers to the penetration depth in a large specimen and that Eq. (1) should, in the ensuing discussion, be regarded as an experimental fact independent of theoretical considerations.

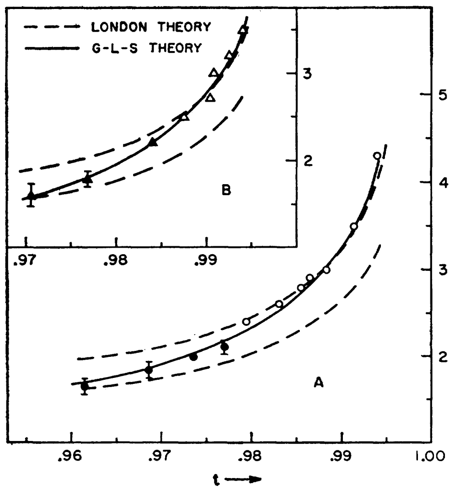

FIG. 5. Enhancement of critical field near T_c. Comparison with London and G-L-S theories is shown, with λ_0 (cm) chosen for whisker A as follows: upper London curve 8.6×10^{-6}, lower London curve 6.5×10^{-6}, G-L-S curve 6.5×10^{-6}. For whisker B: upper London curve 14.4×10^{-6}, lower London curve 11.0×10^{-6}, G-L-S curve 11.0×10^{-6}.

Landau-Silin (hereafter abbreviated to G-L-S) relation is seen to agree satisfactorily over the whole useful range.

The extent of agreement with the G-L-S theory is illustrated more completely by Fig. 6, which displays the temperature dependence of h_c/H_c in a somewhat different way. In this plot numbers on the temperature axis are proportional to λ. Thus, according to Eq. (3), the h_c/H_c data for a given whisker should lie on a straight line, and since the scales are logarithmic the slopes should be identical and equal to 1. The agreement is evidently quite good for A, B, and F. For C the useful data is too limited to be conclusive. In the case of D, however, the results seem to indicate the change in dependence of h_c/H_c upon a/λ to be expected on the basis of the G-L-S theory. At the higher temperatures (small a/λ) the dependence approaches the linear law

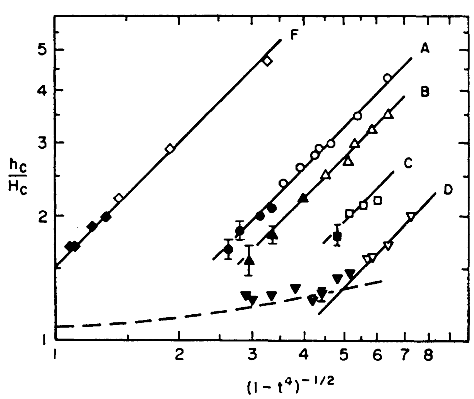

FIG. 6. Relation between critical field enhancement and penetration depth. Solid parallel lines correspond to limiting form of G-L-S theory for small a/λ. In the case of D the limiting form for large a/λ is also shown (broken line).

given by Eq. (3) while at low temperatures (large a/λ) it approaches the limiting form of Eq. (4). A similar change in behavior would presumably occur for the other whiskers, but at lower temperatures (smaller λ), since their diameters are smaller. It is preferable to consider the location of the change in terms of h_c/H_c rather than a/λ, since the former is independent of diameter measurement, and according to Eq. (3) the two ratios are equivalent. One would therefore expect the initial departure from the linear law to occur at the same h_c/H_c for all the whiskers. From the data for D the critical ratio is seen to be about 1.5. This is just below the useful experimental range for A and B, while for F it appears possible that the linear law is obeyed at all temperatures.

c. Hysteresis

The prediction of the G-L-S theory regarding the disappearance of hysteresis is embodied in Eq. (5). At

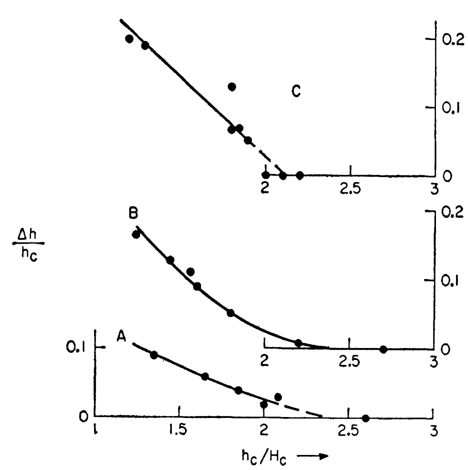

FIG. 7. Hysteresis behavior, showing disappearance of hysteresis at high temperatures (large h_c/H_c). Δh is difference between upper and lower critical fields, and h_c is average critical field. G-L-S theory predicts second order transitions for $h_c/H_c\geq 2.31$.

temperatures above that corresponding to $a/\lambda=\sqrt{3}$, one should observe second-order transitions. According to Eq. (3) this condition is equivalent to:

$$h_c/H_c\geq 2.31. \tag{6}$$

A comparison with the present results is illustrated in Fig. 7, wherein the degree of hysteresis is plotted against h_c/H_c. The intercept on the h_c/H_c axis is determined with a certainty dependent upon the steepness of the slope and the scatter of the data. For both A and B the intercept occurs at 2.4, with an uncertainty of about ± 0.1. The agreement with Eq. (6) is, therefore, excellent. For C the uncertainty is greater, but the agreement still quite good. The hysteresis data for D and F do not determine the intercept accurately, since the degree of hysteresis is small over the range of measurement. This is evident from Fig. 6. On the basis of the values of h_c/H_c for which hysteresis was first observed, the intercept for D is about 1.5 and that for F about 2.0.

It is generally apparent from Fig. 6 that the linear relation between h_c/H_c and λ extends into the temperature region for which hysteresis is observed. In the case of D the indicated correspondence between onset of hysteresis and departure from linearity is probably misleading, owing to the uncertainty of the former.

d. Mean Free Path Effect

In taking the temperature dependence of the penetration depth to be that given by Eq. (1), we have not restricted the choice of λ_0. There is some justification for assuming the form of the temperature dependence to be more generally applicable than a particular value of λ_0. High-frequency measurements of the penetration depth in impure bulk tin have shown that λ_0 depends upon the electron mean free path in the normal metal, whereas the temperature dependence of λ/λ_0 appears to be unaffected.[30]

The agreement of the results shown in Fig. 6 with the predictions of the G-L-S theory leads naturally to

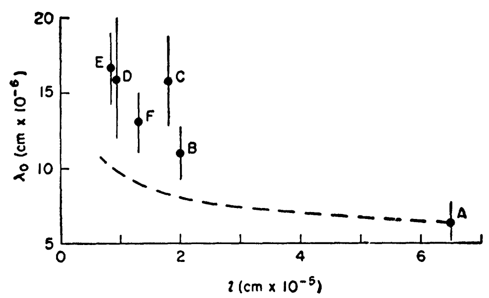

FIG. 8. Dependence of penetration depth on normal electrical conductivity. λ_0 is effective penetration depth at 0°K; l is estimated normal electron mean free path. Broken line taken from large-specimen measurements of Pippard.[30]

an evaluation of λ_0. The latter is obtained, according to Eq. (3), by extrapolation of the high temperature linear relation to $t=0$. Designating this intercept as $(h_c/H_c)_0$, we have:

$$\lambda_0=\tfrac{1}{4}a(h_c/H_c)_0. \tag{7}$$

We may now compare λ_0 with the electronic mean free path by use of the data contained in Table I. For this purpose the normal conductivity, σ, is calculated in the usual way and is converted to mean free path, l, through the relation $\sigma/l=9.5\times10^{10}$ mho cm^{-2}, taken from the work of Chambers.[31] The resulting plot of λ_0 *vs* l is shown in Fig. 8. The rather large uncertainties assigned to the data have two sources, (a) estimated errors in diameter measurement and (b) errors related to the finite breadth of the transition. The latter are most severe for the larger whiskers, as mentioned earlier. On the other hand, the diameter measurements are more critical for small diameters.[32] Comparison of the results (see Fig. 8) with those of Pippard[30] indicates qualitative agreement to the extent of showing an increase of λ_0 as l becomes small. Quantitatively the agreement is good for large l. For small l, however, the present values of λ_0 are consistently higher.

With regard to the variation in mean free path among the whiskers, we observe that, except for specimen A, the mean free path is quite small in comparison with the diameter. Thus boundary scattering from the whisker surface is probably unimportant. For these whiskers, therefore, it seems likely that the reduction in mean free path is due to chemical impurities. A spectrographic analysis of some of the tin plate used in the clamp indicated the total impurity content to be in the range 0.03 to 0.3%, the predominant impurities being silicon and chromium. With the exception of A, it appears that impurities participated in the whisker growth. For A the mean free path is much greater, being of the same order as the whisker diameter. It is conceivable that the variation in mean free path is related to differences in impurity content at the different growth sites.

V. CONCLUSIONS

We may summarize the major results of the critical field measurements on tin whiskers as follows:

1. The threshold curve exhibits hysteresis at low temperatures. This hysteresis disappears near the transition temperature, T_c.
2. Near T_c the threshold curve lies well above that of bulk tin. In this region the critical field enhancement, h_c/H_c, is directly proportional to the bulk penetration depth, λ.
3. Where it can be determined with certainty, the onset of hysteresis occurs at a temperature for which $2.0\leq h_c/H_c\leq 2.4$. The proportionality between h_c/H_c and λ appears to break down at a lower temperature.
4. The parameter λ_0, determined by application of the Ginsburg-Landau-Silin theory to the data, shows a strong dependence upon normal state electrical conductivity.

The temperature dependence of the critical field and the hysteresis behavior are both in substantial agreement with specific predictions of the Ginsburg-Landau theory, as applied to small cylinders by Silin.

It has been argued by Ginsburg[33] that a dependence of λ_0 upon the normal electrical properties is permissible within the framework of the Ginsburg-Landau theory. On the other hand, this dependence led Pippard[30] to formulate an integral relation between current density and magnetic field to replace the London point relation. Since for a weak (much less than critical) magnetic

[30] A. B. Pippard, Proc. Roy. Soc. (London) **A216**, 547 (1953).
[31] R. G. Chambers, Nature **165**, 239 (1950).
[32] It may be noted that, in Fig. 6, the linear character of the relation between h_c/H_c and λ is independent of diameter measurement and that the scatter of data for the smaller whiskers is of a random nature.
[33] V. L. Ginsburg, J. Exptl. Theoret. Phys. U.S.S.R. **29**, 748 (1955) [Soviet Phys. JETP **2**, 589 (1956)].

field the Ginsburg-Landau relation reduces to the London equation, the two new theories appear incompatible.[30,33] The relation of the Pippard equation to the critical field behavior of small specimens has not yet been established. A comparison of the two theories on the basis of the present data is, therefore, not possible. The results show, however, that whiskers with widely different effective values of λ_0 exhibit the temperature dependence and hysteresis properties predicted by the Ginsburg-Landau theory. To be in complete accord with the theory the effective λ_0 should agree with results from large-specimen measurements.[29] As we have seen, the agreement is satisfactory only for long mean free path, i.e., for the relatively pure metal. The importance of the discrepancy at small l is hard to judge, and more experimental data are probably required.

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APPENDIX

The origin of Eq. (2) will now be outlined. According to the London theory the supercurrent density is related to the magnetic field by the following equation:

$$(4\pi\lambda^2/c)\ \mathrm{curl}\mathbf{j}+\mathbf{H}=0. \tag{8}$$

In the static case, this is combined with the relation

$$\mathrm{curl}\mathbf{H}=4\pi\mathbf{j}/c$$

to give the differential equation which describes the field:

$$\nabla^2\mathbf{H}=\mathbf{H}/\lambda^2. \tag{9}$$

If Eq. (9) is solved in cylindrical coordinates, with the conditions $H_r=H_\theta=0$, $H_z=H_z(r)$, and $H_z(a)=H_0$, the solution corresponds to the problem of a long thin cylinder of radius a, placed with its axis parallel to a uniform magnetic field of intensity H_0. The solution for H is:

$$H=H_0\frac{J_0(ir/\lambda)}{J_0(ia/\lambda)}, \tag{10}$$

where J_0 is the Bessel function of order zero. From Eq. (10) is obtained the average magnetization of the superconducting cylinder:

$$M_s=-(H_0/4\pi)\left[1+(2i\lambda/a)\frac{J_1(ia/\lambda)}{J_0(ia/\lambda)}\right], \tag{11}$$

J_1 being the Bessel function of first order.[34]

The critical field behavior is derived by the use of Eq. (11). The Gibbs function of the superconducting state in a magnetic field, h, is given by

$$g_s(h)=g_s(0)-\int_0^h M_s dH_0.$$

For the normal state the magnetization is negligible, and the Gibbs function is simply

$$g_n(h)=g_n(0).$$

At the critical field, h_c, the Gibbs functions are equal. This gives

$$g_n(0)-g_s(0)=-\int_0^{h_c} M_s dH_0. \tag{12}$$

The left side of Eq. (12) represents the difference in Gibbs functions in zero field and is assumed to be independent of size. It is, therefore, equal to $H_c^2/8\pi$, where H_c is the bulk critical field. [This is easily seen by applying the condition $\lambda/a\rightarrow 0$ to Eq. (11).] By performing the integration, we may now solve Eq. (12) for h_c, obtaining:

$$h_c=H_c\left[1+(2i\lambda/a)\frac{J_1(ia/\lambda)}{J_0(ia/\lambda)}\right]^{-\frac{1}{2}},$$

or, in terms of hyperbolic Bessel functions,[35]

$$h_c/H_c=\left[1-(2\lambda/a)\frac{I_1(a/\lambda)}{I_0(a/\lambda)}\right]^{-\frac{1}{2}}. \tag{2}$$

[34] Equations (10) and (11) are taken from reference 7, Appendix II.

[35] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), Part II, p. 1323

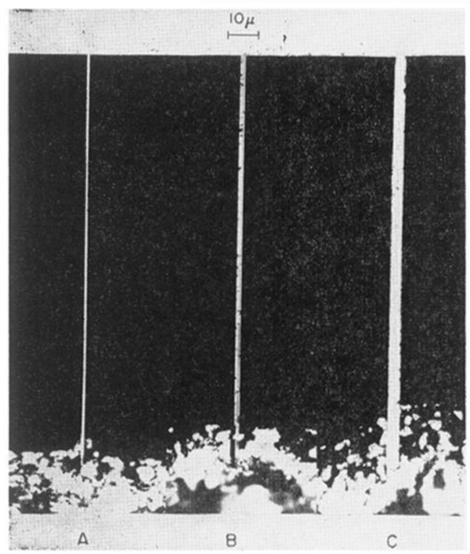

FIG. 1. Photomicrographs of tin whiskers.