capacity. This anomaly arises from the energy levels associated with the paramagnetic ions. Hein and Falge[2] have shown that no resistance maximum exists above 0.22°K for either material. A comparison of their data with the calculations of Gerritsen and Korringa indicates that the predicted heat capacity anomaly would be below the sensitivity of our measurements in the temperature range we have examined.

APPENDIX

For a thermally insulated wire of length L, electrical resistivity ρ, thermal conductivity k, and zero Thomson coefficient, the steady state heat current density is determined by

$$0=j^2\rho-dq/dx, \tag{A1}$$

where q is the heat current density and $j^2\rho$ is the electrical power developed per unit volume. If ρ is constant, this equation can be integrated to give

$$0=j^2\rho x-q+q(x=0). \tag{A2}$$

Putting $q=-kdT/dx$ and performing a second integration yields

$$q(x=0)=-\tfrac{1}{2}j^2\rho L-K/L, \tag{A3}$$

where $K=\int kdT$ taken over the length of the wire. If j equals zero, q equals $-K/L$ and is constant along the wire. Equations (A2) and (A3) show that if j is not equal to zero, the heat currents at the ends of the wire are altered in that one-half of the dissipated electrical power leaves the wire at each end.

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Resistance Minimum in Magnesium: Magnetoresistance

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The magnetoresistance of two specimens of magnesium has been measured at liquid helium temperatures in transverse magnetic fields up to 25 kilogauss. One specimen, containing 0.013% iron as the predominant impurity and previously shown to possess a minimum in the electrical resistivity at 4.5°K, demonstrated a magnetoresistance which followed Kohler's rule quite exactly. The second specimen, containing 0.043% manganese and previously shown to have a rather large negative dR/dT at helium temperatures, showed a small departure from Kohler's rule. The magnetoresistance observed in these specimens is in fair agreement with the majority of results previously reported by Thomas and Mendoza and by Yntema.

INTRODUCTION

EARLIER papers have dealt with the following aspects of the resistance minimum in magnesium: the electrical and thermal resistivities in the temperature range 1.5°K to 25°K (hereafter referred to as I),[1] the electrical resistivity in the temperature range 0.2°K to 4.2°K (II),[2] and the atomic heat in the temperature range 3° to 13°K (III).[3] The general objective of this series is to report on the measurement in the same specimens of a variety of electron properties in an attempt to determine correlations which may prove useful in selecting the correct theoretical interpretation from among the many that have been proposed. A discussion and bibliography on the resistance-minimum are given in I.

Extensive research on the magnetoresistance of metals at low temperatures,[4] has demonstrated that in "normal" specimens (those which are neither ferromagnetic nor superconducting and which have a temperature-independent zero-field resistance at low temperatures) the resistance increases markedly upon the application of magnetic field. As an extreme example, a twenty million fold increase has been observed in bismuth.[5] A very useful generalization, known as Kohler's rule, states that the ratio of the resistance increase in the magnetic field, ΔR, to the zero-field resistance at that temperature, R_{0T}, is given by

$$\Delta R/R_{0T}=F(HR_\theta/R_{0T}), \tag{1}$$

where H is the magnetic field and R_θ is the resistance of the specimen at the Debye temperature. The function, F, has the important property of remaining approximately the same for any given metal without regard to the temperature of measurement or the degree of purity of the specimen. Since there is much uncertainty about the value of θ appropriate to the electrical resistance, it is common practice to replace R_θ by the ice point resistance, $R_{0°C}$ and to represent (1) by a logarithmic plot of $\Delta R/R_{0T}$ *vs* $HR_{0°C}/R_{0T}$, known as a Kohler plot.

The search for other electron properties in metals

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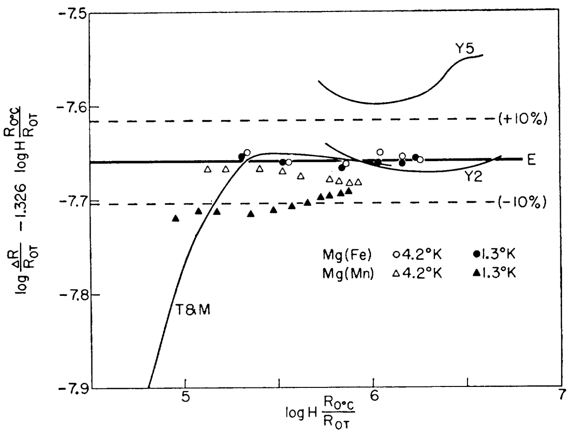

FIG. 1. Modified Kohler diagram for the magnetoresistance of magnesium. The solid line, E, represents the magnetoresistance as given by Eq. (2). The dashed lines represent deviations of ± 10 percent in $\Delta R/R_{0T}$. The curve T & M represents the average magnetoresistance arrived at by Thomas and Mendoza (reference 11) from measurements on four specimens and the curves Y2 and Y5 give the results on two specimens measured by Yntema (reference 12).

which could be correlated with the resistance minimum received its first success in the discovery by Giauque and his associates[6] that the magnetoresistance of certain gold specimens became much smaller as the temperature was lowered below that of the resistance minimum. In one specimen the magnetoresistance passed through zero at 1.6°K and became negative for lower temperatures. Subsequent research[7] established that this anomalous behavior was associated with the presence of small amounts of iron impurity in the gold.

Recently an extensive series of investigations by Gerritsen[8] on alloys of Cr and Mn in the noble metals has shown that major deviations from Kohler's rule, as well as the occurrence of negative magnetoresistance were found in those specimens which exhibited marked anomalies in the zero-field resistance. On the basis of these studies of the dependence of the resistance on temperature and on magnetic field, Gerritsen formulated the following classification of metallic behavior at low temperatures: (1) metals having an electrical resistance independent of temperature and a magnetoresistance changing according to Kohler's rule, (2) metals with a zero-field electrical resistance showing a minimum (but no maximum), and with the magnetoresistance changing according to Kohler's rule, and (3) metals in which the zero-field resistance passes through a maximum (and consequently a minimum), accompanied by large deviations from Kohler's rule.

Theoretical treatments linking anomalies in the zero-field resistance to those in magnetoresistance have been presented by Korringa and Gerritsen[9] and by Schmitt.[10]

Measurements of the magnetoresistance of "spectrographically pure" magnesium[11,12] at helium temperatures have been reported to be in agreement with Kohler's rule. The magnetoresistance of dilute alloys of manganese in magnesium has not, to our knowledge, previously been investigated.

EXPERIMENTAL DETAILS

Both rods investigated were relatively pure magnesium. One specimen, Mg(Fe), was severely cold-worked and contained 0.013% iron and 0.002% manganese as the major impurities. The second specimen, Mg(Mn), was an annealed polycrystal containing 0.043% manganese as the only significant impurity. These specimens are the same as those discussed in I and II. Details of the origin, preparation and spectrographic analysis can be found in I.

The measurements were carried out with standard potentiometric techniques (see I) with the specimens immersed in a liquid helium bath. Transverse magnetic fields of up to 25 kilogauss were supplied by an A. D. Little electromagnet. This magnetic field was stabilized to ± 1 gauss by means of an electronic controller designed by Berlincourt.[13] The magnet was calibrated by the proton resonance technique.

RESULTS AND DISCUSSION

The resistance of both specimens was measured at 1.3°K and at 4.2°K in transverse magnetic fields of 0 to 25 kilogauss. In the Mg(Fe) specimen, the zero-field resistance increased about 1.5% on lowering the temperature from 4.2°K to 1.3°K. The magnetoresistance decreased by a corresponding amount so that Kohler's rule, Eq. (1), was verified in this specimen. In plotting the results for Mg(Fe) in the form of a logarithmic Kohler diagram, a linear relation was found:

$$\log(\Delta R/R_{0T}) = 1.326 \log(HR_{0°C}/R_{0T}) - 7.658, \quad (2)$$

where H is in gauss, or, in explicit form:

$$\Delta R/R_{0T} = 2.20\times 10^{-8}(HR_{0°C}/R_{0T})^{1.326}. \quad (3)$$

The Mg(Mn) specimen, which was shown in I to have a minimum in the zero-field resistance at about 14.5°K followed by a relatively pronounced increase of resistance at lower temperatures, showed a small but unambiguous deviation from Kohler's rule. The magnetoresistance at 4.2°K was in fair agreement with Eq. (2); that at 1.3°K was about 10% lower, except for measurements at the highest fields.

Because of its logarithmic form, the ordinary Kohler

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[9] J. Korringa and A. N. Gerritsen, Physica **19**, 457 (1953).
[10] R. W. Schmitt, Phys. Rev. **103**, 83 (1956).
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[13] Ted G. Berlincourt (unpublished).

plot proved to be too insensitive to represent clearly the small differences in magnetoresistive behavior found in these specimens. Accepting the apparent validity of Eq. (2) for Mg(Fe) specimen, Fig. 1 exhibits the data for both our specimens in the form of a modified Kohler plot in which $\log(\Delta R/R_{0T}) - 1.326 \log(HR_{0°\mathrm{C}}/R_{0T})$ is given as a function of $\log(HR_{0°\mathrm{C}}/R_{0T})$. In this figure, the solid horizontal line is given by Eq. (2) and the dashed lines represent deviations of $\pm 10\%$ in $\Delta R/R_{0T}$. The agreement with Kohler's rule in the measurements on the Mg(Fe) specimen, as well as the deviations from this rule found in the Mg(Mn) specimen, are quite evident.

Also shown in Fig. 1 are the results of measurements of the magnetoresistance of magnesium reported by Thomas and Mendoza[11] and by Yntema.[12] The curve "T & M" represents the average arrived at by Thomas and Mendoza from measurements on four specimens. Except for a few points taken at low magnetic fields, these data and also the results for Yntema's specimen Mg 2 are in excellent agreement with Eqs. (2) and (3). Yntema's specimen Mg 5 apparently forms an exception to the more common behavior. It should be noted that our first specimen, Mg(Fe), came from the same source (Johnson-Matthey, laboratory No. 1848) as did the specimens measured by Yntema and by Thomas. However, the mechanical preparation of these specimens was quite different in that our Mg(Fe) specimen was severely cold-worked, whereas the previously reported specimens had been drawn into wires and (except for one of the four specimens investigated by Thomas) subsequently reannealed.

CONCLUSIONS

1. The magnetoresistance at liquid helium temperatures of a magnesium specimen containing 0.013% iron, Mg(Fe), agrees both with Kohler's rule and with the majority of previous measurements on specimens of similar composition.

2. In a second specimen, Mg(Mn), containing 0.043% manganese, the magnetoresistance at 4.2°K was in fair agreement with the magnetoresistance of the Mg(Fe) specimen. At 1.3°K, however, the magnetoresistance falls about 10% below the values prescribed by the Kohler rule.

3. In terms of the Gerritsen classification (see Introduction), it seems probable that the Mg(Fe) specimen falls into group 2 (metals showing a resistance minimum, but no maximum) and that the Mg(Mn) specimen may be an extreme case of group 3 (metals showing both a resistance maximum and minimum). Since the deviation of Mg(Mn) from Kohler's rule is still quite small even at one-tenth the temperature of the resistance minimum (14.5°K), it seems reasonable that the resistance maximum, if it occurs at all, may be found at a temperature very close to 0°K. These conclusions are consistent with the fact (see II) that the electrical resistance of both the Mg(Fe) and Mg(Mn) specimens increases monotonically for temperatures down to 0.2°K.

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Ferromagnetic Relaxation by the Exchange Interaction between Ferromagnetic Electrons and Conduction Electrons*

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The Hamiltonian for the exchange interaction between the ferromagnetic d electrons and the conduction s electrons is derived. The ferromagnetic relaxation time caused by the s-d exchange interaction is calculated in a spin wave approximation. When one uses a screened value of the exchange integral ($J_{\mathrm{atomic}}/30$), the calculated relaxation time for nickel at room temperature is 5×10^{-9} sec as compared to the time 2.5×10^{-10} sec needed to account for the experimental line width. The exchange relaxation may be dominant in materials such as alloys which have narrower lines than nickel.

I. INTRODUCTION

THE line widths observed in electron spin-resonance experiments on ferromagnetic single metal crystals are of the order of several hundred oersteds,[1] corresponding to relaxation times of the order of 10^{-9} to 10^{-10} sec. The widths in several metals appear to be temperature-independent at low temperatures.[1] If we neglect all interactions except the isotropic exchange and Zeeman interactions, the Hamiltonian of the system of ferromagnetic electrons is

$$\mathcal{H}_0 = -2\sum_{i<j} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j - g\mu_B\mathbf{H}\cdot\sum_i \mathbf{S}_i. \tag{1}$$

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