Resistance Minimum of Magnesium: Electrical and Thermal Resistivities*

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The thermal and electrical resistivities of two specimens of magnesium have been measured in the temperature range 1.5 to 25°K. One specimen was a spectrographically pure rod of 99.98+% magnesium with 0.013% iron as the major contamination. The second specimen was 99.95+% pure with 0.043% manganese as the only significant contaminant. The electrical resistivity of the purer material passed through a minimum at ~5°K and increased by 1.7% as the temperature was lowered to 1.5°K. In the same specimen, the thermal resistivity at the lowest temperature exhibited a positive deviation from normal behavior (defined by $w=\alpha T^2+\beta/T$) of the same order of magnitude. Between 5° and 15°K a negative deviation was observed in the thermal resistivity.

The electrical resistivity of the dilute manganese alloy specimen passed through a minimum at ~14°K and increased by approximately 20% as the temperature was lowered to 1.5°K. An exactly analogous effect was found in the thermal resistivity as evidenced by (1) a constant Lorenz ratio at temperatures below 4°K, and (2) the equality of the percentage deviations of the respective resistivities from normal behavior over the entire range of measurements.

INTRODUCTION

A VARIETY of investigations have established that the low-temperature electrical resistivity of numerous specimens of nonsuperconducting metals does not become independent of the temperature. A summary and bibliography of research prior to 1952 on this phenomenon appears in the publications of Mendoza and Thomas.[1,2] Some of the more recent research has been reviewed by MacDonald.[3] In most instances a simple *minimum* type anomaly is observed: the resistance decreases to a minimal value as the temperature is lowered, and then increases monotonically as the temperature is reduced further. An important exception to this type of behavior appears in the measurements of Gerritsen and Linde[4] who find that in alloys of manganese and chromium in the noble metals, the minimum of resistance is usually followed by a pronounced maximum at lower temperatures.

A considerable body of experimental evidence indicates that an ideally pure, perfect crystal of a metal would not exhibit a negative temperature coefficient of resistance at low temperature, and that such phenomena must arise in some fashion from the existence of sometimes minute chemical impurities and/or physical imperfections in the specimen being studied. Although much less dramatic in its manifestation, the resistance minimum shares with superconductivity the characteristic of having been the subject of a considerable number of proposed mechanisms and theoretical treatments.[5] These proposed explanations fall into two general categories: (1) attempts to prove a temperature dependence of the number of free electrons due, perhaps, to the existence of a small energy gap similar to that found in semiconductors, and (2) examination of interactions of the conduction electrons with grain boundaries, other electrons, phonons and impurities in the attempt to find scattering mechanisms which may be a function of temperature at the very low temperatures (~1°K) where the resistance minimum is pronounced.

To add further complexity, it is by no means evident that all of the temperature-dependent resistance phenomena found at low temperatures have the same origin. For example, it seems likely that the monotonic increase of resistance found in gold[6] down to 0.007°K and in magnesium down to 0.2°K[2,7] arises from a mechanism quite distinct from that which causes the more complex behavior found in certain noble metal alloys.[4]

One quite obvious method of narrowing the field within which a theoretical explanation must be sought is to determine experimentally the correlations between anomalies in the electrical resistance and deviations from normal behavior of other electron-properties such as the specific heat,[8] the galvanomagnetic effects,[4,9] the susceptibility,[2,10] the electron spin resonance,[10] the

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[1] E. Mendoza and J. G. Thomas, Phil. Mag. **42**, 291 (1951).

[2] J. G. Thomas and E. Mendoza, Phil. Mag. **43**, 900 (1952).

[3] D. K. C. MacDonald, *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1956), Vol. 14, pp. 188–192.

[4] A. N. Gerritsen and J. O. Linde, Physica **17**, 573 (1951) and **18**, 877 (1952); A. N. Gerritsen, Physica **19**, 61 (1953).

[5] The mechanisms and explanations proposed prior to 1951 are summarized in references 1 and 2; see also J. C. Slater, Phys. Rev. **84**, 179 (1951); J. Korringa and A. N. Gerritsen, Physica **19**, 457 (1953); J. Korringa, Physica **19**, 816 (1953); A. B. Bhatia, Phys. Rev. **95**, 914 (1954); and R. W. Schmitt, Phys. Rev. **103**, 83 (1956).

[6] Croft, Faulkner, Hatton, and Seymour, Phil. Mag. **44**, 289 (1953).

[7] R. A. Hein and R. L. Falge, following paper [Phys. Rev. **105**, 1433 (1957)].

[8] Logan, Clement, and Jeffers, this issue [Phys. Rev. **105**, 1435 (1957)].

[9] R. T. Webber, this issue [Phys. Rev. **105**, 1437 (1957)].

[10] Owen, Browne, Knight, and Kittel, Phys. Rev. **102**, 1501 (1956).

thermoelectric force,[3] and the thermal resistivity. Because of the strong dependence of the behavior of electrical resistance on the type and concentration of impurities, it is also evident that these correlations should be determined as far as possible on the same specimen.

In this paper are presented the results of measurements of the electrical and thermal resistance of two specimens of magnesium in the temperature range 1.5 to 25°K. The electrical resistance in the temperature range 0.2° to 4.2°K, the electronic specific heat in the temperature range 3° to 13°K, and the magnetoresistance at liquid helium temperatures are presented in the following three papers of this series.[7–9] The measurements reported in papers I, II, and IV of this series were performed on the same pair of magnesium specimens. The specific heat measurements reported in paper III were carried out on specimens taken from the same melts. It is expected that this study will be extended at a later date to include measurements of the magnetic susceptibility of these specimens.

Previous attempts to correlate measurements of the electrical and thermal resistivities in metals exhibiting a resistance-minimum have led to conflicting conclusions. Berman and MacDonald[11] reported in measurements on copper that the maximum percentage deviation of the thermal conductivity from "normal" behavior was less than the corresponding maximum percentage deviation of the electrical conductivity. White[12] noted also for copper that the percentage increase in wT (the product of the thermal resistivity and absolute temperature) below the temperature of the minimum was approximately equal to the percentage increase in the electrical resistivity over the same temperature range.

Mendelssohn and Rosenberg[13,14] reported results on magnesium specimens which exhibited values of the thermal resistivity which were lower than those which might be expected on the basis of "normal" behavior. On the other hand Kemp, Sreedhar, and White[15] found, also in magnesium, an increase of wT at the lowest temperatures, while observing a depression of wT in the intermediate temperature range. Finally, Sharkoff and Herlin[16] have reported measurements on a series of magnesium alloys in which the deviations of the thermal resistivity were qualitatively related to the magnitude of the resistance minimum.

One reason for the variety in reported results of these correlation experiments is that the resistance minimum behavior in relatively pure metals is a relatively subtle effect (~1% per degree) in the range of temperatures at which these correlations are usually attempted. The corresponding "anomalous" effect which might be expected to appear in the correlation measurement is generally of the same magnitude as the precision obtaining in the measurement itself.

One method of resolving this latter difficulty would be to extend the correlation measurements to temperatures below 1°K where the anomalous effects are usually more pronounced. Another method, which is sometimes applicable, is to alter the physical structure or the impurity content of a given metal in such a way as to enhance the effect in the higher temperature range. This latter approach has been the procedure used in the measurements reported in this paper. Two specimens of magnesium have been employed: one was a specimen of "spectrochemical" purity (the major impurity was 0.013% iron), but extensively work-hardened in an attempt to enhance the minimum effect; the other specimen was a composition (0.043% manganese) previously shown[16,17] to exhibit a marked minimum in the temperature range above 1°K.

EXPERIMENTAL DETAILS

Specimens

Both specimens were rods about 9 cm long and 3.2 mm in diameter. Thin copper bands were electrolytically deposited near the ends of the specimens by the "Dow Process"[18] to make possible soldered joints to the thermometer straps, the gas thermometer bulb and the heater bobbin. The narrow plated rings for the attachment of the gradient thermometer straps were separated from the plated ends of the specimen by a short gap.

The chemical purity of both specimens was greater than 99.9 percent magnesium. However, the resistance anomalies probably arise from the chemical impurities and physical imperfections, so these aspects are discussed in greater detail in the following sections.

First Specimen—Mg(Fe)

This specimen was a Johnson-Matthey spectrographic rod whose predominant impurities (see Table I) were 0.013% iron and 0.002% manganese. Although it is not certain whether the resistance anomalies have their origin in the predominant impurity, iron, or in

[11] R. Berman and D. K. C. MacDonald, Proc. Roy. Soc. (London) **A211**, 122 (1952).
[12] G. K. White, Australian J. Phys. **6**, 397 (1953).
[13] K. Mendelssohn and H. M. Rosenberg, Proc. Roy. Soc. (London) **A65**, 385 (1952).
[14] H. M. Rosenberg, Phil. Mag. **45**, 73 (1954).
[15] Kemp, Sreedhar, and White, Proc. Phys. Soc. (London) **A266**, 1077 (1953). These authors and P. G. Klemens (private communication) have further analyzed the data and find depressed values of wT at intermediate temperatures (~5 to 11°K) as well as the increase noted in the publication. The recalculated data appear in *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1956), Vol. 14, p. 247.
[16] E. G. Sharkoff and M. A. Herlin, Quarterly Progress Report Research Laboratory of Electronics, Massachusetts Institute of Technology, July 15, 1953 (unpublished), p. 18.
[17] H. E. Rorschach, Jr., and M. E. Herlin, Phys. Rev. **81**, 467 (1951); also *Proceedings of the Schenectady Cryogenics Conference*, General Electric Company Report, NP-4840, 1952 (unpublished), p. 51; also, H. E. Rorschach, Jr., thesis, Massachusetts Institute of Technology, 1952 (unpublished).
[18] *Magnesium Finishing* (Dow Chemical Company, Midland, Michigan, 1952), pp. 94–8.

the much smaller contamination of manganese, this specimen will henceforth be referred to as Mg(Fe).

The Mg(Fe) specimen had been extensively cold-worked by swaging before the complete electrical and thermal resistivity measurements were made. This swaging, carried out in the vain hope of enhancing the magnitude of the resistance minimum, caused a 14% reduction in diameter and raised the electrical resistivity at 4.2°K from 1.84×10^{-8} ohm-cm to 6.48×10^{-8} ohm-cm. Comparison spectrograms of the specimen taken before and after the swaging operation demonstrated no change in specimen purity, particularly in the Fe impurity.

A photomicrograph of the Mg(Fe) specimen, taken after the swaging, showed no crystalline development or other pronounced structure.

Second Specimen—Mg(Mn)

This specimen was prepared by the Dow Chemical Company and consisted of a very pure magnesium matrix to which 0.043% manganese had been deliberately added. Since the manganese is the only impurity present in significant quantities (see Table I), this specimen will be referred to as Mg(Mn). The rod had been hot-extruded by the supplier, then annealed for 24 hours at 300°C and rapidly quenched in water at 40°C. It is believed that this procedure should keep the manganese in solid solution with the matrix.

A photomicrograph of the Mg(Mn) specimen showed that it was polycrystalline with a few of the crystallites ranging up to 0.1 mm in size.

TABLE I. Characteristics of magnesium specimens employed in this and other research.

Specimen designation		Mg(Fe)	Mg(Mn)	Rosenberg[a]
A. Physical characteristics:				
Supplier		J. M.[b] No. 1848	Dow[c] No. 72767	J. M.[b] No. 1703
Analysis[d]	Mg	99.98+	99.95+	99.95+
	Mn	0.0023	0.043	0.03
	Fe	0.013	0.0010	0.0075
	Pb	0.0013	0.0011	
	Al	VFV[e]	0.0002	0.004
	Cu	FV[f]	0.0001	
	Ni	···	0.0001	
	Si	FV	0.001	
	Sn	···	0.0011	
	Zn	···	0.0048	
	Ca	FV	0.0012	
	Ag	FV		
	Na	FV		
Physical state		cold-worked polycrystal	annealed polycrystal	annealed polycrystal
B. Parameters of electrical resistivity:				
Approximate temperature of minimum (°K)		5	14.5	6
Resistivity at minimum (ohm-cm)		6.458×10^{-8}	11.87×10^{-8}	2.69×10^{-8}
Resistivity at 1°K (ohm-cm)		6.624×10^{-8}	14.79×10^{-8}	2.74×10^{-8}
Resistivity ratio, $(\rho_{min}/\rho_{300°K})\times10^2$		1.37	2.44	0.583
C. Lorenz ratios L observed for $T<4$°K (ohm watt deg^{-2})		2.49×10^{-8}	2.64×10^{-8}	2.60×10^{-8}

[a] Reference 14.
[b] Johnson-Matthey and Company, London, England. It is noted that the laboratory number of our Mg(Fe) specimen is the same as for the specimens used by Kemp (reference 15) and by G. B. Yntema, Phys. Rev. 91, 1388 (1953).
[c] Dow Chemical Company, Magnesium Division, Midland, Michigan. The authors wish to thank Mr. C. Sheldon Roberts of the Dow Company for making available to us a number of dilute alloy specimens.
[d] The analysis is in each case that provided by the supplier. All values are in weight percent.
[e] VFV = spectral line very faintly visible.
[f] FV = spectral line faintly visible.

Apparatus and Technique

The calorimeter was similar in many respects to those recently described in detail by White and Woods[19] and by Alers.[20] As the calorimeter was originally constructed for measurements of the thermal conductivity of superconductors in low magnetic fields,[21] it was constructed entirely of nonferromagnetic materials, and gold-ring gaskets[22] were used as vacuum seals instead of solder.

During all of the measurements reported here, the calorimeter was surrounded by a bath of liquid helium. As usual, one end of the specimen rod was thermally isolated by the calorimeter vacuum. The other end of the specimen, instead of being fixed in intimate contact with the bath, was attached to a chamber which, when evacuated, provided a relatively poor thermal contact to the bath. Through the use of an auxiliary heater it proved possible to fix the specimen temperature at any point in the range 4.2°K to 25°K. For measurements below 4.2°K, the isolating chamber was flooded with liquid from the bath by puncturing a rupture disk.

The temperature at two points along the specimen was determined by carbon resistance thermometers,[23] the resistance being measured by means of a modified dc Wheatstone Bridge circuit which permitted the absolute resistance of one thermometer, and the difference between the resistances of the two thermometers to be measured in rapid sequence.[24] A high-impedance dc breaker amplifier was employed as the null detector.

The gradient thermometer elements were Allen-Bradley carbon-composition 1-watt radio resistors of nominal value 56 ohms. The thick plastic material normally covering the resistors was removed with a centerless grinder and a thin, baked coating of G. E. Formvar insulation was added. To provide good thermal contact between the resistor body and the thermometer holder, a liberal coating of G. E. Glyptal was applied to each resistor before it was slipped into the copper

[19] G. K. White and S. B. Woods, Can. J. Phys. **33**, 58 (1955).
[20] P. B. Alers, Phys. Rev. **101**, 41 (1956).
[21] R. T. Webber and D. A. Spohr, Phys. Rev. **84**, 384 (1951); **91**, 414 (1953).
[22] Wexler, Corak, and Cunningham, Rev. Sci. Instr. **21**, 259 (1950).
[23] J. R. Clement and E. H. Quinnell, Rev. Sci. Instr. **23**, 213 (1952).
[24] H. A. Fairbank and J. Wilks, Proc. Roy. Soc. (London) **A231**, 545 (1955).

holder attached to the specimen. In the liquid hydrogen and liquid helium temperature ranges, the resistors were calibrated against the vapor pressure of a few cm³ of the respective liquid condensed into a bulb attached to the bath end of the specimen. In the intermediate range they were calibrated against a constant volume He gas thermometer using the same bulb as the sensitive element. The two resistors used in these measurements had calibration curves identical to within one percent over the entire range; the absolute values of resistance at a given temperature varied by less than one-tenth percent from run to run.

The electrical resistivity measurements were carried out in a subsequent run using currents up to 5 amperes. The potential drop was measured with a microvolt potentiometer and a photoelectric galvanometer capable of detecting an unbalance of about 0.002 microvolt.

The probable error in the measurements of thermal resistivity is of the order of 1%; that of the electrical resistivity ~0.3%.

RESULTS AND DISCUSSION

Electrical and Thermal Resistivities

The electrical resistivity of the two specimens as a function of temperature is given in Fig. 1. The resistance minimum phenomenon is evident in both specimens, and is quite pronounced in the dilute manganese alloy, Mg(Mn). The resistivities of these specimens at the lower temperatures are closely proportional to the logarithm of the temperature[2,6] and can be represented by:

$$\begin{aligned}&\rho=(6.621-0.244\log T)\times10^{-8}\ \text{ohm-cm};\\ &\qquad 1.3^\circ\text{K}<T<4^\circ\text{K};\ \text{Mg(Fe)},\\ &\rho=(14.79-3.15\log T)\times10^{-8}\ \text{ohm-cm};\\ &\qquad 1.3^\circ\text{K}<T<5^\circ\text{K};\ \text{Mg(Mn)}.\end{aligned}\tag{1}$$

The absolute values of the thermal conductivity ($K=1/w$) as a function of temperature are shown in Fig. 2. The solid curves in this figure represent equations of form $1/K=w=\alpha T^2+\beta/T$, as discussed in the following section. Below about 5°K, the function wT for the specimen Mg(Mn) fits reasonably well an equation of the form:

$$wT=5.58-1.19\log T\ \text{watts}^{-1}\ \text{cm deg}^2.\tag{2}$$

"Normal Behavior" of the Resistivities

For metals not exhibiting the resistance minimum effect, the electrical resistivity at low temperatures can generally be represented[25] by an equation of the form suggested by theory:

$$\rho=\rho_0+AT^n\tag{3}$$

where ρ_0 is the temperature independent "residual resistivity" associated with electron-impurity scattering, and AT^n is the "ideal resistivity" associated with electron-phonon scattering.

For most metals which do not exhibit an electrical resistivity minimum, the thermal resistivity w at low temperatures is well represented[26] by an equation of the form:

$$w=\beta/T+\alpha T^2,\tag{4}$$

where α and β are constants; and the terms on the right have the same association with the theoretical scattering mechanisms as do the respective terms in Eq. (3).

In our magnesium specimens it has proved possible to fit the data at temperatures above that of the resistance minimum to Eqs. (3) and (4). Standard graphical techniques (for example, see Fig. 3) were employed and yielded the values of the constants ρ_0, A, n, α, and β shown in Table II. The units of the constants are those appropriate to yield ρ in ohm-cm and w in watts.

Deviations from "Normal Behavior"

At temperatures below that of the resistance minimum, it is evident (see Figs. 1 and 3) that the values of ρ and w calculated from the "normal behavior" Eqs. (3) and (4), using the parameters in Table II, deviate from the values of ρ and w actually measured. If we designate these calculated resistivities by ρ_n and

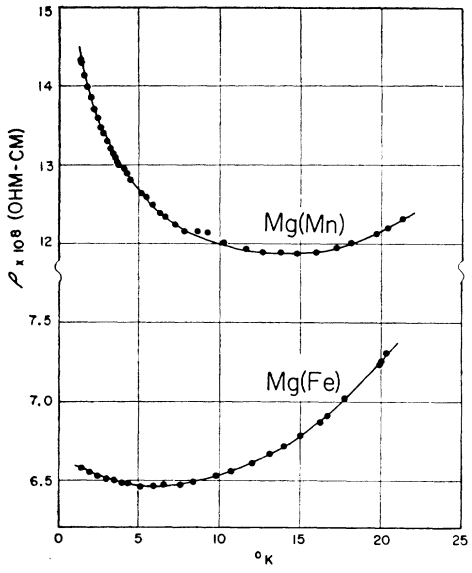

FIG. 1. Electrical resistivity of the two magnesium specimens as a function of temperature.

[25] D. K. C. MacDonald and K. Mendelssohn, Proc. Roy. Soc. (London) **A202**, 103 and 523 (1950).

[26] Extensive summaries of measurements of the thermal conductivity of metals at low temperatures are given by R. L. Powell and W. A. Blanpied, Nat. Bur. Standards Cir. 556 (U. S. Government Printing Office, Washington, D. C., 1954), and by H. M. Rosenberg, Trans. Roy. Soc. (London) **A247**, 441 (1955).

w_n, and designate the measured resistivities by ρ_m and w_m, we can then define the deviation of the resistivities from normal behavior by

$$\Delta\rho=\rho_m-\rho_n, \quad \Delta w=w_m-w_n. \tag{5}$$

In order to determine the correlation between the anomalous behavior in ρ and w, it is instructive to compare the relative deviations from "normal behavior," $\Delta\rho/\rho_n$ and $\Delta w/w_n$. This is done in Fig. 4.

In the dilute manganese alloy specimen, Mg(Mn), it is quite evident from Fig. 4 that the relative deviations of ρ and of w are completely equivalent to within experimental error. So in this specimen we can claim an exact correlation in the electrical and thermal aspects of the resistance minimum.

In the case of the relatively pure magnesium specimen, Mg(Fe), it is seen in Fig. 4 that the relative deviations are not only an order of magnitude smaller than those of Mg(Mn), but are of a rather more complex form. At temperatures between 1.5°K and 4°K, a direct correlation between the deviations of the electrical and thermal resistivities is apparent. However, at temperatures between 4°K and 15°K an entirely new phenomenon is manifested. Whereas the deviation of the electrical resistivity in this range of temperatures remains close to zero, indicating good agreement with the "normal behavior" of Eq. (3), the deviation of the thermal resistivity becomes quite large and negative. This negative deviation is in good agreement with that found by Kemp.[15] We believe that here we have evidence for the existence of two separate and distinct mechanisms: one ($T<4$°K) associated with the resistance minimum and having equal effect on both the electrical and the thermal resistance, and the second ($4°<T<15$°K) arising from other causes and influencing only the thermal resistance. We shall return to this point in the following section.

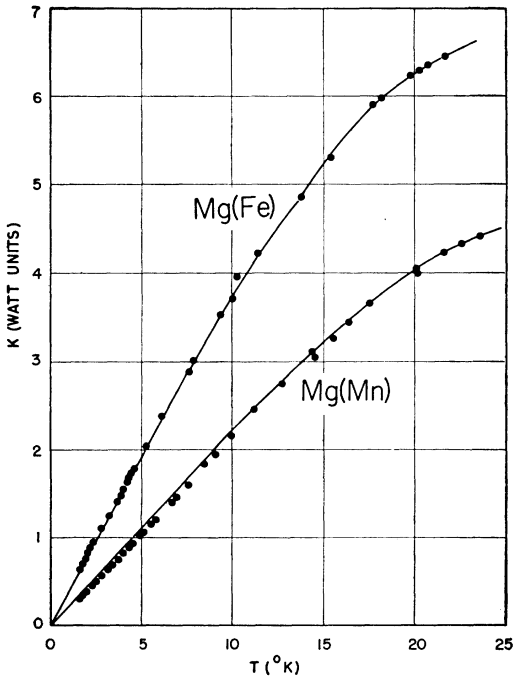

FIG. 2. Thermal conductivity of the two magnesium specimens as a function of temperature. The solid curves are of the form $w=\beta/T+\alpha T^2$.

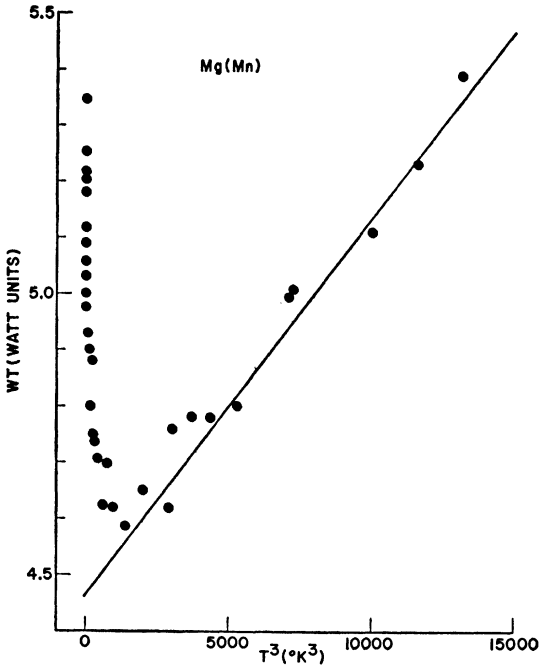

FIG. 3. wT *vs* T^3 for the Mg(Mn) specimen. The solid line demonstrates that the data at the higher temperatures fit the "normal" behavior equation $wT=\alpha T^3+\beta$. See Eq. (4) and Table II.

Lorenz Ratio

A more common method of comparing the electrical and thermal resistivities of metals is through the Lorenz ratio, $L=\rho/wT$. At high temperatures, and again at very low temperatures (where impurity scattering is dominant) the Lorenz ratio is usually found to be a constant, of value fairly close to the classical value[27] 2.443×10^{-8} watt-ohm/deg^2.

TABLE II. Values of the parameters of electrical and thermal resistivity appearing in Eqs. (3) and (4) for the two magnesium specimens.

	Mg(Fe)	Mg(Mn)
ρ_0	6.475×10^{-8}	11.75×10^{-8}
A	1.74×10^{-13}	8.00×10^{-16}
n	3.60	5.15
α	7.10×10^{-5}	6.70×10^{-5}
β	2.60	4.46

[27] A. Sommerfeld, Z. Physik **47**, 1 (1928). The numerical evaluation used constants tabulated by J. W. DuMond and E. R. Cohen, Revs. Modern Phys. **25**, 691 (1953).

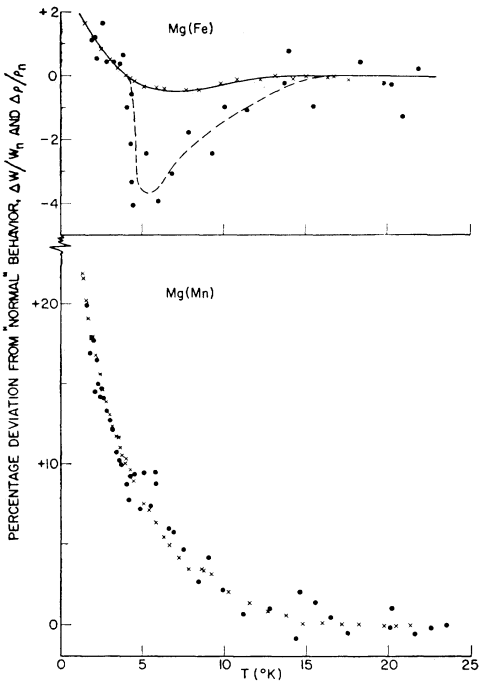

FIG. 4. Percentage deviation of the observed electrical and thermal resistivities from the "normal" behavior [as defined by Eqs. (3) and (4) and Table II] plotted as a function of temperature for the two magnesium specimens. The crosses give $\Delta\rho/\rho_n$; the circles give $\Delta w/w_n$.

The Lorenz ratios of our specimens are given in Fig. 5. Smoothed values of the electrical resistivity were combined with measured thermal resistivities to obtain the data points plotted in this figure. The dashed curve gives the Lorenz ratios reported by Rosenberg.[14] It is evident that for both specimens the values of L at $T<4°$K are, within experimental error, independent of T. At the higher temperatures (where the scattering of the electrons by phonons becomes important) the values of the Lorenz ratio decrease.[28]

The solid curves appearing in Fig. 5 are calculated from Eqs. (3) and (4) using the constants given in Table II. The fact that these calculated curves agree well with the measured points in the relatively pure specimen Mg(Fe) (for $T<4°$K and $T>15°$K) and in Mg(Mn) (for all T), can be expressed [using the notation of Eq. (5)] by the equation

$$L=\frac{\rho_n}{w_nT}=\frac{\rho_m}{w_mT}=\frac{\rho_n+\Delta\rho}{(w_n+\Delta w)T}. \tag{6}$$

So we come again to to the conclusion (see Fig. 4) that

$$\Delta\rho/\rho_n=\Delta w/w_n,$$

except for the case of the Mg(Fe) specimen in the temperature range $4°\text{K}<T<15°\text{K}$.

The hump in the Lorenz ratio in the Mg(Fe) specimen at intermediate temperatures corresponds to the depression in thermal resistance in this specimen shown in Fig. 4. These related anomalies might be explained either by (1) a contribution to the total thermal conduction due to the appearance of an appreciable lattice conduction at these temperatures, or (2) a decrease in the portion of the thermal resistance due to phonon scattering of the electrons arising from the precipitous decline in the value of the Debye θ in magnesium[29] at these temperatures.

In analyzing the second suggestion, we can see from Eqs. (3) and (4) and Table II that, at 6°K in theMg(Fe) specimen, the portion of the thermal resistance due to phonon scattering, αT^2, is only about 0.5% of the total thermal resistance, while the portion of the electrical resistance due to phonon scattering, AT^n, is less than 0.2% of the total. The ratio $\alpha T^2/w$ is nearly an order of magnitude smaller than the observed relative deviation in w at 6°K (see Fig. 4). Since $\alpha T^2/w$ necessarily represents an upper limit to the relative deviation of the thermal resistance which could arise from the temperature variation of θ, we can support this explanation only at the cost of sacrificing our confidence in the validity of Eq. (4).

We therefore conclude that lattice conduction is the more probable cause of the 5° to 15°K minimum in thermal resistivity in Mg(Fe) specimen. This mechanism would also explain the similar minima appearing in the measurements of thermal resistance of pure magnesium by other investigators,[14,15] but leaves us with the unresolved question of why there seems to be no similar manifestation of lattice conduction in our dilute manganese alloy specimen, Mg(Mn).

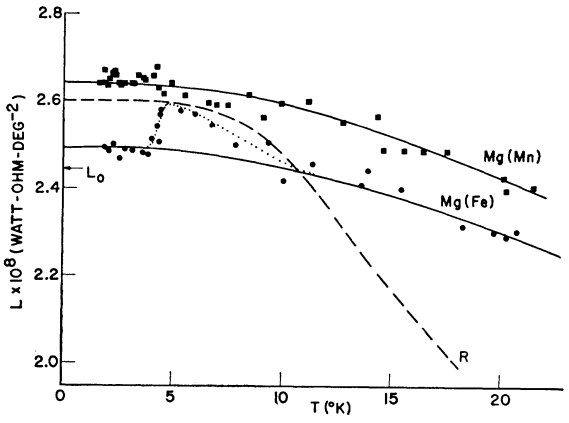

FIG. 5. Lorenz ratios ($L=\rho/wT$) as a function of temperature for the two magnesium specimens. The solid curves are calculated from the "normal" behavior given by Eqs. (3) and (4) and by Table II. The plotted points represent observed values. The dashed curve represents the results of Rosenberg (reference 14).

[28] The conditions determining the temperature dependence of L are discussed by P. G. Klemens, *Encyclopedia of Physics* (Springer-Verlag, Berlin, 1956), Vol. 14, pp. 231–2.

[29] P. L. Smith, Phil. Mag. 46, 744 (1955).

CONCLUSIONS

(1) The electrical resistance of a specimen of magnesium containing 0.043% manganese in solid solution, Mg(Mn), passed through a minimum at 14.5°K and then increased approximately 20% in value as the temperature was lowered to 1.5°K. Within experimental error, the temperature dependence of the thermal resistance was exactly equivalent to that of the electrical resistance.

(2) The electrical resistance of a relatively pure and severely cold-worked specimen of magnesium, Mg(Fe), passed through a minimum at 5°K and then increased about 1.7% in value as the temperature was lowered to 1.5°K. The temperature dependence of the thermal resistance of this specimen was equivalent to that of the electrical resistance at temperatures below 4°K. However, in the temperature range 4°K to 15°K, the thermal resistance showed an additional minimum (with a maximum of 4% at 5°K) which did not correspond to any similar behavior in the electrical resistance. This 4°K to 15°K anomaly in the thermal resistivity is tentatively ascribed to the presence of a small amount of lattice conduction.

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Resistance Minimum of Magnesium: Electrical Resistivity below 1°K

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Measurements of the electrical resistivity of the two magnesium specimens described by Spohr and Webber in the preceding paper have been extended to temperatures below 1°K. At the lowest temperature at which data were obtained (0.22°K), the electrical resistivities of both specimens were continuing to increase. The ratio $\rho_{0.22°K}/\rho_{4.22°K}$ was found to be 1.05 for the purer specimen and 1.24 for the less pure (manganese alloy) specimen.

INTRODUCTION

IN the preceding paper,[1] referred to as I, the electrical resistivity of two specimens of magnesium in the temperature range 1.3°K to 25°K is reported. As the temperature was lowered in both specimens, the electrical resistivity passed through a minimum and was still increasing at 1.3°K.

Earlier experiments on the electrical resistance, at very low temperatures, of specimens exhibiting the resistance minimum revealed three types of behavior: (1) Croft *et al.*[2] found in nearly pure gold that the resistance continued to increase monotonically [ρ was linear in $\log(1/T)$] at temperatures as low as 0.007°K, (2) Gerritsen and Linde[3] have reported that in dilute alloys of manganese in the noble metals, the resistance minimum is usually followed at lower temperatures by a resistance maximum, and (3) White[4] has observed in a dilute alloy of tin in copper as the temperature was lowered below that of the resistance minimum (~11°K) that the resistance increased by about 5% and then, over the whole temperature range 0.02°K to 1°K, the resistance remained quite strictly constant.

It was felt to be of interest to extend the resistance measurements on the magnesium specimens used in I to temperatures below 1°K to determine whether their behavior fell into any of the three classes described above.

EXPERIMENTAL DETAILS

The specimens used in the present investigation were the ones with which the thermal and electrical resistivity data reported in I and the magnetoresistance data[5] reported in a later paper of this series were obtained. Details concerning their history and purity can be found in I.

The techniques of cooling the specimen and of determining the resistance were in most respects similar to those previously described by one of us.[6] A noteworthy modification was that thermal contact between the specimen and the salt pill was achieved by cementing (GE Adhesive 7031) one end of the sample into an

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[6] R. A. Hein, Phys. Rev. **106**, 1511 (1956).